# OBSERVATIONALARCHAEOASTRONOMY AT STONEHENGE: WINTER SOLSTICE SUN RISE AND SET LINES ACCURATE TO $0.2^{\circ} \operatorname{IN} 4000 \mathrm{BP}$ 

Gordon R. Freeman and Phyllis J. Freeman


#### Abstract

For several centuries it has been assumed that observations of Solstitial Sun Rises and Sets, if they were made at Stonehenge, were made from inside the Sarsen Circle looking outward. The outlying Heel Stone is in approximately the Summer Solstice Sun Rise direction when viewed from the centre of the Sarsen Circle. The fact that the Solstitial Sun rises Northerly of this line, and 4000 years ago rose still more Northerly of it, has been excused by the assumption that the observations were ceremonial, and the inaccuracy of $2^{\circ}$ did not matter. Our long-term experience at a site in Alberta has shown that Sun lines were accurate to $0.1^{\circ}$ to $0.2^{\circ}$, so we wondered whether the Stonehenge people in Britain had been as fussy. They had been. During three study visits to Stonehenge, in the Decembers of 1995 and 1997, and June 1999, we discovered that accurate lines in 4000 BP were obtained by standing far outside the Sarsen Circle, and looking through narrow gaps between the Circle and Trilithon Stones, to the Sun Rise or Set on the horizon beyond. The Heel Stone was not a foresight for the Summer Solstice Sun Rise, but was an observation position for the Winter Solstice Sun Set into the side of a burial mound 1 km away, on the far side of the Sarsen Circle. The Rise and Set lines crossed over the middle of the Altar Stone, which 4000 years ago was lying flat in essentially the same position as now. The Winter Solstice alignments are reported in the present paper. Those for the Summer Solstice are reported in the Observational Archaeoastronomy web site : www.ualberta.ca/~gfreeman.


#### Abstract

Résumé

On a supposé pendant des siècles que si on relevait à Stonhenge la position du lever et du coucher du soleil aux solstices d'hiver et d'été, ce devait être depuis l'intérieur du cercle de monolithes en direction de l'extérieur. Le menhir qu'on appelle Heel Stone se trouve à peu près dans la direction du lever solsticial d'été quand on le regarde à partir du centre du cercle de monolithes. On préférait oublier qu'au solstice, le soleil se lève au nord de cette ligne et qu'il se levait plus au nord encore il y a 4000 ans, et on attribuait cette imprécision de $2^{\circ}$ au fait que les observations avaient un but cérémoniel. Ayant constaté depuis longtemps dans un site de l'Alberta que les alignements solaires présentaient une précision de $0,1^{\circ}$ à $0,2^{\circ}$, nous nous sommes demandé si par hasard la population de Stonehenge, en Grande-Bretagne, n'avait pas fait preuve de la même minutie. Dans le cadre de trois voyages d'étude à Stonehenge, en décembre 1995, en décembre 1997 et en juin 1999, nous avons découvert qu'en 4000 A.A., on avait pu obtenir des alignements exacts en se tenant bien à l'extérieur du cercle de monolithes et en regardant le soleil se lever ou se coucher à l'horizon dans l'intervalle étroit qui sépare les monolithes du cercle et les trilithes. La Heel Stone n'était pas un point de visée pour le lever solsticial d'été, mais un point d'observation pour le coucher solsticial d'hiver qui coïncidait avec le flanc d'un monticule funéraire situé à un kilomètre de là, du côté éloigné du cercle de monolithes. Nous avons relevé la position des levers et couchers solsticiaux à Stonehenge en décembre et en juin. Les lignes d'il y a 4000 ans se situaient à $0,94^{\circ}$ de l'est et de l'ouest francs et traversaient le cercle à la manière que nous allons montrer. Les lignes du lever et du coucher se croisaient au milieu du dolmen.


## Introduction

Our studies since 1980 of Solstice and Equalnight Sun Rise and Set alignments at an ancient site in southern Alberta, the Majorville Medicine Wheel Complex (MMWC) (Freeman and Freeman 1990a,b), have drawn our attention to Stonehenge (Atkinson 1979; Burl 1976, 1993). While there might have been no ideological or religious similarities between societies in North America and Britain 5000 years ago, we know of no evidence that there was not. Indeed, Sun worship was world-wide at that time. To avoid distraction from the subject of this article, we state without elaboration that the mean diameter of the Stone Circle at the focus of the MMWC is 29 m (inside diameter of the Sarsen Circle is 30 m ), and there are three Cairns and one South Rock a few tens of meters from the Stone Circle (two Station Stones and two Barrows a few tens of meters from the Sarsen Circle). Both structures are at $51^{\circ} \mathrm{N}$ latitude, which has solar/lunar astronomical significance (Hawkins 1965).

At the MMWC the positions of the first and last flashes of the Sun at the Summer and Winter Solstices are marked to within $0.1^{\circ}$ by lines up to 2 km long; those at the Equalnights are marked with similar accuracy by lines up to 1 km long. Furthermore, the Equalnight lines mark the Sun Set and Rise that bracket the 11.97-12.03 hour night, not the equatorial "Equinox" when the Sun passes over the Earth's equator (Freeman and Freeman 1990b). The former occurs three nights before the latter in the spring, and three nights after it in the fall, due to light refraction by the atmosphere, and the diameter of the Sun's disc. At the "Equinox", the night is 0.3 hour shorter than the day, so they are not equal.

Lines at Stonehenge that purportedly mark the Summer and Winter Solstice Sun Rises and Sets range from about 33 m to 85 m in length (Thom 1971; Hoyle 1977; Cleal et al. 1995). The lines under 50 m long have not been demonstrated to be accurate to better than about two degrees, and longer ones have either been shown to be inaccurate (from the circle center to the Heel Stone for the Summer Sun Rise) or were not tested by observation (for example, station 93 to hole H for the Winter Rise (Hawkins 1965), which we show herein to be inaccurate by $4^{\circ}$ counterclockwise).

The latitude of the MMWC is $50.586^{\circ} \mathrm{N}$, which is only 66 km south of Stonehenge latitude, $51.178^{\circ}$ N. Sun and Moon Rise and Set characteristics at MMWC are somewhat similar to those reported for Stonehenge. Both monuments date back about 5000 years and evolved over a long period. Based on our studies at MMWC, in 1988 we suggested that there might be outliers, at about 1 km distance from Stonehenge circle, that accurately mark the Solstices. During a trip to England in April 1989, one of us (GRF) obtained permission to enter Stonehenge to look for outliers. There is a Bell-barrow 1 km southwest of the henge, and we suggested that it would mark the Winter Solstice Sun Set from a backsight at Sarsen 1 or Bluestone 31 in the Sarsen Circle. The Bell-barrow is the one labelled no. 15, one km SW of Stonehenge in the Barrow distribution map (Royal Commission 1979), and the summit is 1020 m from the Heel Stone. There is now a backdrop of trees that probably did not exist 4000 years ago (Cleal et al. 1995, p. 484). Without the trees, Bell-barrow 15 would penetrate the horizon viewed from the Sarsens.

Attempts by Brian K. Davison, Inspector of Ancient Monuments of English Heritage, to photographically verify the Winter Solstice Sun Set line during two subsequent Decembers were foiled by clouds. In December 1995 we verified the line. It has characteristics similar to lines at the MMWC. This article reports the observed line and calculational extrapolations to the day of the Solstice, 22 December 1995, and to the Winter Solstice of 4000 BP.

In December 1997 we recorded a Winter Solstice Sun Rise line that passes through the Sarsen Circle. To allow the passage of light a large notch had been hammered into the SW side of Trilithon Sarsen 58. The line passes along the Altar Stone, which evidently was not standing on end during Phase 3v (Phase classification of Cleal et al. 1995). The observed line and calculational extrapolations to the Solstice, 21 December 1997, and to the Winter Solstice of 4000 BP, are reported below.

## Statistical Analysis in Archaeoastronomy

The exact angles of the Sun Rise and Set sight lines are modified by local topography, and to a smaller extent by changes of the refraction with changing air density gradient (mainly due to vertical gradient of temperature, Sampson 1994).

There has been much discussion of whether the position of the Sun Rise was taken as that of the first flash (upper limb), or where the center of the Sun disc reaches the horizon (mid orb), or where the lower limb touches the horizon (when the Sun disc is tangential to the horizon as it finishes rising) (Hawkins 1965; Newham 1972; Thom et al. 1974; Hoyle 1977; Chippindale 1994; North 1996). However, we have not been able to locate any accurate report of mid orb or lower limb Rise positions. Distortions of the Sun's image by atmospheric variations near the Earth's surface sometimes make it impossible to determine when mid orb or lower limb touch the horizon (Freeman and Freeman 1998).

We have found that the position of the first or last flash can be measured reproducibly to within $\pm 0.1^{\circ}$ under adverse atmospheric conditions (Freeman and Freeman 1998), and to within $\pm 0.03^{\circ}$ under good conditions. The atmosphere tends to be more uniformly heated at Sun Set time than at Rise time, so distortions are less and Set positions are usually more accurately measurable than are Rise positions. Measurements of 244 Sun Rise times and 125 Set times from positions in Edmonton ( $53.53^{\circ} \mathrm{N}, 113.50^{\circ} \mathrm{W}$, altitudes $714 \pm 14 \mathrm{~m}$ above mean sea level) over a 37 month period were transformed to apparent refraction angles at $0.00^{\circ}$ observational elevation: the average estimated refractions and standard deviations were $0.71^{\circ} \pm 0.18^{\circ}$ for Rises and $0.58^{\circ} \pm 0.11^{\circ}$ for Sets (Sampson 1994). The deviations reported by Sampson were large because corrections were not made for sky glow or mirage (R.D. Sampson and G.R. Freeman, personal communication 1996). Abnormally large advances of apparent Sun Rise times are called the Novaya Zemlya effect (Sampson 1994, pp 59, 60), because they were first observed on that Arctic Island $\left(76^{\circ} \mathrm{N}\right)$ in winter. The advances are due to mirage, reflection, not to refraction, and are detected and eliminated by experienced observers (Freeman and Freeman 1998).

Calculations are used to extrapolate observed Rise or Set points, recorded a week or two away from a Solstice or a few days away from an Equalnight, to the positions on the required day. Calculations also allow the extrapolation of an observed position to the analogous position a few millennia ago (Smart 1965; Hoyle 1977, Appendix).

Some of the previously suggested archaeoastronomical sight lines have been subjected to statistical analysis by comparing the reported directions with those calculated by ignoring local topography (Haack 1987) and by assuming experimental azimuth uncertainties of up to $2.5^{\circ}$ (Haack 1987 ; Ruggles 1984). This type of analysis has been superceded by the present on-site observational technique, over the actual topography, with azimuth uncertainties of $0.03^{\circ}$ to $0.1^{\circ}$.

## Equipment and Technique

The camera was a Nikon F3 with Nikkor Zoom 35-105 mm and 80-200 mm lenses. Nikon L37 uv filters were used to protect the lenses. The tripod was a Manfrotto 055 with a 141 head. The usual height of the camera above the ground was 1.5 m . A cable release was used for multiple exposures of the same frame, and for lower shutter speeds. The 35 mm film was Fujicolor SuperG 100, processed and printed by Colorfast Corp., Edmonton.

The location of the tripod was recorded for each photo or sequence of photos. The date, time and focal length of each exposure were recorded; the lens aperture and exposure duration were occasionally recorded.

The arc and angle of descent of the setting Sun were measured by taking multiple exposures on the same frame of film; each shot was appropriately underexposed by one or two stops. The descent path was needed because the horizon in the Sun Set direction was not quite horizontal, and a short extrapolation was needed to project the path to the surface of the tumulus from where the Sun disappeared behind trees that now screen the tumulus from the last flash of the Sun.

The position of the last flash on the tumulus on 11 December 1995 was extrapolated by calculation to the position on the Solstice, 22 December 1995, and to the WSS position in 4000 BP. The method of calculation is given in the Appendix.

The horizon in the Sun Rise direction is a cultivated field, with no obstruction of observation of the first flash.

The orientation of the horizontal, 4.9 m long Altar Stone (Atkinson 1979) is along the Winter Solstice Sun Rise (WSR) and Summer Solstice Sun Set (SSS) directions. It has often been suggested that the Altar Stone was originally vertical, and that it fell over (Atkinson 1979; Hawkins 1965; Chippindale 1994; Richards 1990; Cleal et al. 1995; North 1996), but our results make that improbable. Furthermore, no hole has been found in which the Altar Stone could have stood in Phase 3v (Atkinson 1979; Hawkins 1965; Cleal et al. 1995). We began our WSR photography at the NW end of the Altar Stone. After photographing the first few minutes of Sun Rise we moved the camera tripod NW along the WSR line, outside the gap between Sarsens 57 and 58, then outside the gap between Sarsens 21 and 22, then back near the Bank, about 7 m NE of Station Stone 93.

The positions of the first flash on 12 and 13 December 1997 were extrapolated by calculation to the Solstice, 21 December 1997, and to the WSR position in 4000 BP.

## 1020 metre Winter Solstice Sun Set line

The setting Sun was photographed from behind the SE and NW edges of Bluestone 31 (Fig. 1a), so that the edges of the stone were visible as reference positions in the photos. The camera was aimed at Bellbarrow 15 (Fig. 1b). Moving the camera from the SE to the NW side of Bluestone 31 changed the angle at Bell-barrow 15 by only 1.5 mrad , or $0.09^{\circ}$. Details of the tree-top line behind Bell-barrow 15 (Fig. 1c) serve as a measuring scale, because the trees now prevent the direct observation of the Sun setting behind the Barrow (the Sun entering the Barrow).

Table 1. Line of the tip of the rising or setting Solstitial Sun, seen through refracting standard air.

| Measured elevation $\left({ }^{\circ}\right):$ | -0.50 | 0.0 | 1.0 | 2.0 | 3.0 | 5.0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Refraction $\left({ }^{\circ}\right)^{\circ}$ | -0.71 | -0.59 | -0.41 | -0.31 | -0.24 | -0.18 |
| True elevation $\left(\mathrm{TE}^{\circ}{ }^{\circ}\right)$ : | -1.21 | -0.59 | +0.59 | 1.69 | 2.76 | 4.82 |
| Horizontal axis $\left({ }^{\circ}\right)=$ TE/tan $31^{\circ}:^{\circ}$ | -2.01 | -0.98 | +0.98 | 2.81 | 4.59 | 8.02 |

a Hoyle (1977,p.141). Thom (1971,p.26), adjusted slightly to match Hoyle in the overlap region; aire at $10^{\circ} \mathrm{C}, 100 \mathrm{kPa}$.
$b \quad$ The observed line of descent in Fig. 3a was fitted by assuming a true angle of descent of $31^{\circ}$ in this zone. The angle calculated for zero elevation at the Solstices, using eqn (16) of North (1996, p.563) is $31.9^{\circ}$, which is consistent with our $31^{\circ}$ at elevations of $0.6^{\circ}$ to $3^{\circ}$.


Figure 1. Line NE_SW along which we observed the Sun Set. (a) Looking NE from about the center of the Sarsen Circle to the Heel Stone (Sarsen 96); the bracketing Sarsens and Bluestones are labelled; the small circle on the Heel Stone image marks 1.6 m above ground; negative 11 April/89, 51 (date, bold-face film role no., negative no.). (b) Looking SW from 0.5 m behind the NW side of Bluestone $31,2 \mathrm{~m}$ in front of Sarsen 1 and 1.5 m above the ground, to Bell-barrow 15, 950 m away; focal length 105 mm , negative 11 Dec. 195,30 ; the trees are about 50 m behind the Barrow. (c) Bell-barrow 15 and tree line viewed from about 110 m SW of Bluestone 31, looking about $840 \mathrm{~m}, 231^{\circ}$ to the Barrow; focal length 105 mm , negative 4 Dec./95,17. The inner diameter of the circular ditch around the Barrow, and therefore the outer diameter of the flare of the Bell, is 40 m , and the diameter of the dome of the Bell, where the dome meets the flare, is 30 m .


Figure 2. Sun Set on 11 December, 1995, eleven days before the Solstice. All focal lengths are 105 mm ; camera was 1.5 m above ground. (a) Camera was 0.3 m behind the SE edge of Bluestone 31; negative 312 at times $15 \mathrm{~h} 33 \mathrm{~m} 45 \mathrm{~s}, 15 \mathrm{~h} 41 \mathrm{~m} 00 \mathrm{~s}, 15 \mathrm{~h} 49 \mathrm{~m} \mathrm{00} \mathrm{s}, 15 \mathrm{~h} 49 \mathrm{~m} 20 \mathrm{~s}$, each shot two stops underexposed. (b), (c) Camera was 0.5 m behind the NW edge of Bluestone 31, negative 3 15, time $15 \mathrm{~h} 52 \mathrm{~m} ; 322,15 \mathrm{~h} 54 \mathrm{~m} 50 \mathrm{~s}$; normal exposures but printed dark to see the Sun circle. The observed last flash was at about 15 h 55 m 00 s , due to the trees.

The angle of descent of the Sun was recorded by multiple exposures of one frame over a 16 minute period (Fig. 2a). The final stages of the Sun setting were recorded as single shots from a fixed position, in this case behind the NW edge of Bluestone 31 (Figs. 2b,c). The Barrow is not visible in Figure 2, because of the backdrop of trees, so the event that would be seen in the absence of trees is reconstructed in Figure 3.

The apparent angle of descent decreases slightly as the Sun approaches the horizon, due to the increasing refraction caused by the increasing mass of air that the light passes through before reaching the observer. The curvature of the descent line cannot be calculated analytically (Smart 1965, p. 68), but it can be reconstructed from Tables based on observational data. The curve in Figure 4 was obtained by using the refraction corrections listed in Tables of Hoyle (1977, p. 141) and Thom (1971, p. 26) to calculate the horizontal distance of Sun motion required to give each successive degree of altitude above the horizon. Our observed line of descent was fitted by taking a true angle of descent of $31^{\circ}$ for the final $3^{\circ}$ of elevation, then finding the horizontal distance between the measured elevations as shown in Table 1.

The angular altitude from 1.5 m above the base of the Heel Stone (or of Bluestone 31) to the summit of Bell-barrow 15 is $0.51^{\circ}\left(0.42^{\circ}\right)$, and to the base of the dome of the Barrow is $0.34^{\circ}\left(0.24^{\circ}\right)$. These are the lower limits of the measured altitudes to use for the descent curve of Figure 4 when applied to Figure 3.


Figure 3. (a) setting Sun on 11 Dec./95, images of photos 3 12, 15 and 22 projected onto 30 , focal leng th 105 m m. (b) setting Winter Solstice Sun paths of 11 and 22 Dec./95 and 4 ka BP projected onto Dominey photo 11, 20 May/96, focal length 200 mm ; camera was 1.4 m in front of Bluestone $31,1.4 \mathrm{~m}$ above ground. The path of the upper limb on 11 Dec./95 is extrapolated to the last flash at the Bell-barrow using Fig. 4. The paths calculated for the Solstices 22 Dec./95 (---) and 4 ka BP (-), are also shown.


Figure 4. Refraction by the average atmosphere $\left(10^{\circ} \mathrm{C}, 100 \mathrm{kPa}\right)$ for the rising and setting Sun, and the line followed by the top of the setting Solstitial Sun, plotted from Table 1.

Extrapolation of Set Point From 11 Dec. to 22 Dec., 1995
To calculate the small shift in Set point during the eleven days to the Solstice, the orbit of the Earth is taken as a quarter circle during the 90 days from Autumnal Equinox to Winter Solstice. Thus each day represents one degree. The total shift of the Sun Set point during these 90 days at Stonehenge is $40^{\circ}$ (Hoyle 1977, p. 70). The counterclockwise shift $\Delta \theta$ from 11 Dec. to 22 Dec./95 was

$$
\Delta \theta=40^{\circ}\left(1-\cos 11^{\circ}\right)=0.73^{\circ}(1)
$$

On a $102 \mathrm{~mm} \times 152 \mathrm{~mm}$ print of a $24 \mathrm{~mm} \times 36 \mathrm{~mm}$ negative with $2 \%$ discarded from each edge, photographed at focal length $105 \mathrm{~mm}, 1^{\circ}$ represents 8.1 mm . The Set line for the 1995 Winter Solstice, viewed from the NW side of Bluestone 31, is shown as dashes in Figure 3.


Figure 5. Views in both directions along the 4 ka BP Winter Solstice Sun Set (WSS) line, taken on 1 April, 1997 with equipment of Derek Dominey. (a) View from a ladder on the walkway, about 5 m in front of the Heel Stone, camera 2.64 m above ground to see over fallen Sarsen 55, focal length 200 mm , negative 20A; the SE junction of the dome and flare of Bellbarrow 15 is behind the bush in the centre of the photo, seen between Sarsens 1 and 56. (b) View from 17 m SW of bank top to Dominey on ladder in position for (a), camera 1.75 m above ground, focal length 200 mm , neg. 21A. (c) View backwards from the SE junction of dome and flare of Bell-barrow 15, looking $50^{\circ}, 1020 \mathrm{~m}$ to the Heel Stone in the gap between Sarsens 56 and 1, focal length 200 mm , neg. 12A. (d) Position of tripod for (c), focal length 80 mm , neg. 16A.

## Extrapolation of Winter Solstice set point from AD 1995 to 4000 BP

Details of the calculation are given in the Appendix. The position of the last flash of the WSS in 4 ka BP was $0.92^{\circ}$ counterclockwise from the present WSS position. The line of descent of the upper limb of the Sun in 4 ka BP is drawn on the photos in Figure 3. The Sun disappeared into the angle on the SE side of Bell-barrow 15 where the dome meets the flare of the Bell; the Sun entered a Barrow groin.

The position of the last flash disappearing into the dome-flare angle was visible from the center of the Heel Stone (Fig. 5), from where most of the Bell-barrow was hidden behind Sarsen 56.

## Speculation

We suggest that as the Winter Solstice approached, the daily southward progress of the last flash of the setting Sun was observed from the centre of the Altar Stone and then from Bluestone 31. A 30 cm gap between Trilithon Sarsens 55 and 56 would give a viewing angle of $2.9^{\circ}$ from the Altar Stone, and $0.9^{\circ}$ from


Figure 6. Heel Stone viewed from the same direction as in Fig. 1a, but from a smaller distance. 11 April/89, 51.

Bluestone 31. On a particular evening the top of the Sun would slide down the NW side of the dome of the Bellbarrow. On subsequent evenings the last flash would gradually cross the dome, and would finally reach the angle between the dome and the flare on the SE side. The last flash would then reverse its path and begin its annual northward trek. Viewed from Bluestone 31, the horizontal angle between the NW and SE bases of the dome of Bell-barrow 15, 30 m , is $1.8^{\circ}$. The Sun would have taken 17-18 days to cross the dome (eqn (1) with $\Delta \theta=1.8^{\circ}$ ), from sliding down the NW slope to the turn around point, and another 17-18 days to again slide down the NW slope. The total time that the last flash spent in the turn-around angle on the SE side of the Bellbarrow, corresponding to motion of 0.1 Sun diameters in each direction, would be about six days, a fifth of a Moon cycle. The actual Solstice could have been determined as half way between the two slidings down of the NW slope of the dome. The total counting period would have been about 35 days, nearly five quarterphases of the Moon.

For reasons of weather and planning, it seems probable that counting the days from a Sun Rise or Set point approaching a Solstice position to the return there after the Solstice, was an important part of determining the time of a Solstice in ancient times (Atkinson 1979; Hoyle 1977). The position of the last flash on the Barrow dome would serve as a calendar, so the counting of days to the Solstice did not depend on the actual observation of the last flash on each day. Clouds probably intervened then as now. The last flash would have been visible from the Heel Stone for nearly two weeks. If the standing Sarsen 55 or 15 further restricted the view of the SE edge of Bell-barrow 15 from the Heel Stone, it is unlikely that the view was narrowed to less than $0.2^{\circ}$, corresponding to a 30 cm gap SE of Sarsen 56, in which case the last flash would have been visible from the Heel Stone for at least ten days.

The distance from the Heel Stone to the NE base of the dome of Bell-barrow 15 is 1020 m , so $0.2^{\circ}$ corresponds to a view of a horizontal distance of 4 m at the Barrow.

We agree with R.S. Newall (Atkinson 1979, p. 96) and A. Burl $(1976,1993)$ that a major focus of the Sarsen structure was the midWinter (Winter Solstice) Sun Set. An observation line that is an order of magnitude more accurate than any previously demonstrated is here recorded.

## Heel Stone 96, Altar Stone 80, sightline altitude

From a detailed contour map of Stonehenge (Thom et al. 1974, Fig. 6) we estimate that the altitude of the line from ground level at the base of the Heel Stone to the present top of the Altar Stone 80 m away is $1.3^{\circ}$, and from the base of the Heel Stone to the top of the SW Bank 127 m away in the WSS direction is $1.0^{\circ}$. However, the altitude of a line from the base of the Heel Stone to the SE junction of the


Figure 7. Winter Solstice Sun Rise. (a) Tripod at NW end of Altar Stone, viewed from between Sarsens 57 and 58, focal length 80 mm , negative 12 Dec. 97,3 25, at time 08 h 25 m . (b) First flash viewed from tripod in (a), 08 h 04 m 15 s , focal length 200 mm , negative 37 ; the positions of the first flash on the Winter Solstices of 1997 and 4 ka BP are marked. (c) 4 ka BP WSR line viewed from just inside the Bank, about 7 m NE of Station Stone 93, focal length 200 mm , negative 13 Dec./97, $410,08 \mathrm{~h} 13 \mathrm{~m} 00 \mathrm{~s}$; the WSR in 4 ka BP would have appeared on the horizon at the position marked
dome and flare of Bell-barrow 15 is only $0.4^{\circ}$, estimated from another contour map (Ordnance Survey Explorer 130,1998 ). Therefore, the last flash of the WSS would have to be observed from at least 0.6 x $127 / 57=1.3 \mathrm{~m}$ above ground level at the Heel Stone.

Seasonal Sun Rises and Sets at the Majorville Medicine Wheel Complex in Alberta were observed from fixed eye levels, usually from a rock top or from the bottom of a $V$ formed by two rocks (Freeman and Freeman 1990a,b). A suggestive place on the SW face of the Heel Stone is the lower end of the "mouth line" (Fig. 6), which is marked by a small circle in Figure 1a. This place is 1.6 m above ground; from here the altitudes to the Altar Stone top, Bank top, and dome-flare junction are $0.1^{\circ}, 0.3^{\circ}$ and $0.3^{\circ}$, respectively. The WSS last flash would therefore have been observable from the "corner of the


Figure 8. WSR line in 4 ka BP. (a) Notch pecked into the SW side of Sarsen 58, focal length 35 mm , negative 5 Dec. $/ 97,127,08^{\mathrm{h}} 53^{\mathrm{m}}$. (b) The Sarsen structure through which the WSR line passes, seen from inside the Bank, about 7 m NE of Station Stone 93 , focal length 80 mm , negative 12 Dec. $/ 97,323,08^{\mathrm{h}} 23^{\mathrm{m}}$. (c) Reverse view of the WSR line from NE of Sarsen 53, looking NW along the Altar Stone 80 to the gap between Bluestone 69 and Sarsen 58, and to PJF about 7 m NE of Station Stone 93, focal length 80 mm , negative 12 Dec./97, $330,08^{\mathrm{h}} 34^{\mathrm{m}}$; the line passes between Circle Sarsens 21 and 22, which are behind Trilithon Sarsens 57 and 58.
mouth", and the position of it has probably not altered much during the last 4 ka . The $0.3^{\circ}$ altitude is slightly lower than that used for the calculations in the Appendix, so visibility over the Bank was not a problem.

The Altar Stone could have been 0.3 m higher than its present level without interfering with the observation of the WSS from the suggested position at the Heel Stone.

## Slaughter Stone 95

The WSS sightline would have passed close to the NW side of a vertical Sarsen 95 (Fig. 9). There is no evidence to suggest that Sarsen 95 would have blocked the view from Sarsen 96 of the Solstice Sun entering the groin of Bell-barrow 15.

## Bell-barrow 15

Table 2. Geographical altitudes h of sight lines, and true altitudes hs of the Sun's center for Winter Solstice Sun Rise and Set seen from Stonehenge ( 1.5 m above ground), with Rise and Set directions $\theta_{\mathrm{s}}$ in AD 1995 and 4 ka BP.

|  | WSR |  | WSS |  |
| :---: | :---: | :---: | :---: | :---: |
|  | mrad | deg | mrad | deg |
| $\mathrm{h}^{\text {J }}$ | 4.7 | 0.27 | $\begin{array}{\|l} \hline 8.5 \\ (7.3 \end{array}$ | $\begin{aligned} & \hline 0.49 \\ & 0.42) \\ & \hline \end{aligned}$ |
| air refraction ${ }^{\text {c }}$ | 9.35 | 0.536 | $\begin{aligned} & 8.55 \\ & (8.80 \end{aligned}$ | $\begin{aligned} & \hline 0.490 \\ & 0.504) \end{aligned}$ |
| $h_{5}{ }^{\text {P }}$ | -9.3 | -0.53 | $\begin{array}{\|l\|} \hline-4.7 \\ (-6.1 \end{array}$ | $\begin{aligned} & -0.27 \\ & -0.35) \end{aligned}$ |
| " AD 1995 | 2243 | 128.53 | $\begin{aligned} & 4033 \\ & (4035 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 231.05 \\ & 231.17) \\ & \hline \end{aligned}$ |
| 4.0 ka BP | 2259 | 129.45 | $\begin{aligned} & \hline 4016 \\ & (4019 \\ & \hline \end{aligned}$ | $\begin{aligned} & 230.12 \\ & 230.25) \\ & \hline \end{aligned}$ |

a Recorded to two or three decim al places when necessary to avoid ro und-off errors in the calculations.
1 radian $=180^{\circ} / \pi=57.296^{\circ}$.
$b$ WSR from the notch in Sarsen 58 at 104 m MSL to the horizon 1500 m away at 111 m MSL.
WSS from the Heel Stone at 102 m MSL to the horizon 4330 m away at 139 m MSL; (from Blue-
stone
31 at 104 m MSL to top of Bell-barrow 15955 m away at 111 m MSL).
$c$ From Figure 4. Sun's ra dius $=4.65 \mathrm{~m}$ rad $=0.266^{\circ}$.
$d$ Calculated from eqn (4).
e Azimuth for the firs t or last flash, c alculated from eqn (7) or (8). $1995: \varepsilon=0.4091 \mathrm{rad}=23.44^{\circ}$.
4.0 ka
$B P: \varepsilon=0.4176 \mathrm{rad}=23.93^{\circ}$.

Bell-barrow 15 is a far less imposing structure than are the Sarsen Trilithons and Circle. The Heel Stone was erected before the Sarsen Trilithons (Atkinson 1979; Cleal et al. 1995). The Heel Stone to Bell-barrow WSS line might have been established before the Sarsen Circle was constructued, or perhaps Bell-barrow 15 was constructed after the Trilithons to refine the accuracy of the WSS line and to honor its occupant. What is known about Bell-barrow 15?

Bell-barrow 15 was numbered by Goddard and is sometimes labelled as G15, but in Victoria History: Wiltshire I (1957) it is listed as Bell-barrow Amesbury 15, where bold face numbers indicate that they were in Goddard's list. The diameter of the dome at its base is 31 m , the height is 3.4 m , the outside diameter of the flare (berm) of the Bell is 42 m , the ditch is 0.8 m deep and 5 m wide.

The Barrow was excavated by R.C. Hoare in the early 19th century (Victoria History 1957). It contained a primary interment of an adult male (skeleton on an elm plank), with a grooved dagger in a wooden case, a small dagger, a richly ornamented drinking cup, and antlers. Traces of three wooden poles extended from the primary interment to the top of the Barrow. The wooden poles extending from the interment indicate a ritual significance (Victoria History 1957).

We do not know details of the poles in the mound, or whether the burial was of a king or of a sacrifice. It is possible that the poles were Sun markers, and considered to be transmitters to the crypt. It has also been suggested that the poles were part of a mortuary house (Victoria History 1972, p. 357), although it is not clear that such poles would extend to the surface of the mound.

The date of Bell-barrow 15 is not known, but related Armorican barrows have dates about 2300-2000 BC (Burl 1999). The era of stone construction and rearrangements at Stonehenge was about 2400-1900 BC (Cleal et al. 1995). This favors the suggestion that Bell-barrow 15 was placed in that position on the horizon to refine the WSS line. However, one cannot completely rule out the possibility that the Heel Stone to Bell-barrow groin line existed before the Sarsen Circle was constructed.

It is difficult to determine from site maps (North 1996; Ruggles 1997) whether an outlying object is sufficiently close to a particular line extrapolated from the Stonehenge structure. Bell-barrow $\mathbf{1 5}$ is not on an extension of the axis of the Sarsen Circle. We have measured that the summit of $\mathbf{1 5}$ is 45 m NW of the extended axis, and the groin into which the Winter Solstice Sun Set in 4 ka BP is 30 m NW of the extended axis.

## The shapes of the Heel Stone 96 and Bluestone 31

The profile of the Heel Stone seen from the Sarsen Circle has the shape of a paleolithic lanceolate projectile point (Fig. 1a). From a smaller distance in this direction the Heel Stone looks like a turtle head (Fig. 6). The turtle is an ancient totem for water.

Bluestone 31 has a profile somewhat similar to that of the Heel Stone (Fig. 1a). The neighboring Bluestones 49 and 48 (Fig. 1a), and indeed the other remaining Bluestones (Cleal et al. 1995), have shapes quite different from that of the Heel Stone. The shape similarity is consistent with the suggestions that Bluestone 31 was a Sun Set monitoring position during the three or four weeks that bracket the Winter Solstice, and that the Sun Set during the ten or twelve days that bracket the Solstice was observed from the Heel Stone.

## Winter Solstice Sun Rise line along Altar Stone

The tripod at the NW end of the Altar Stone, seen through the gap between Sarsens 57 and 58, is shown in Figure 7a. The first flash of the Sun on 12 December 1997, photographed from the tripod in Figure 7a, is shown in Figure 7b.

On the Winter Solstice, 21 December 1997, the first flash would be $0.49^{\circ}$ to the right (eqn 1, for 9 days; two decimal places are carried through the calculations to avoid round-off errors). The WSR first flash in 4 ka BP would have been a further $0.92^{\circ}$ to the right (Table 2). These positions are marked in Figure 7b.


Figure 9. the observed Sun Rise and Sun Set lines extrapolated by calculation to WSR and WSS in 4 ka BP, plotted on a map of Stonehenge. The site map is a composite of Fig. 13 of Cleal et al. (1995) and Fig. 4 of Chiipendale (1994), mainly the former.

When viewed from just inside the bank, about 7 m NE of Station Stone 93 and 26 m from Sarsen 21, the 4 ka BP WSR position is visible in the gap between Sarsen 58 and Bluestone 69 (Fig. 7c). A large notch had been hammered into the SW side of Sarsen 58 at this level (Figs. 7c and 8a) to broaden the viewing angle. We have not found a previous mention of this diagnostic notch.

The second clear sunrise, this one through slight haze, was on 13 December. We recorded the first flash from the NW end of the Altar Stone, then moved the camera progressively NW as before. The first flash position extrapolated 8 days to the Solstice was $0.03^{\circ}$ to the left of the solstice position obtained from the Rise on 12 December, corresponding to 0.5 mm on a 152 mm wide copy of Figure 7 b .

In Figure 7c the angular width of the 58-to-69 gap at the widest part of the pecked notch in Sarsen 58 is $0.4^{\circ}$; above the notch the gap is $0.2^{\circ}$. Lowering the camera 10 cm would have put the horizon at mid notch level.

A general view of the Sarsens from the Bank near the WSR observation position is shown in Figure 8b. A view backwards along the WSR line, from the NE side of Sarsen 53, is shown in Figure 8c. Phyllis Freeman is standing just in front of the Bank.

The WSR line in 4 ka BP would have passed between Sarsens 8 and 9. Perhaps a notch had also been pecked into the SW side of Sarsen 8 ; only the relatively narrow upper portion of Sarsen 8 remains, so we don't know how broad the lower portion was. However, the position of the Stone Hole for Sarsen 8 (Cleal et al. 1995, foldout Plan 2 in back cover pocket) indicates that the WSR line passed on the SW side of that stone.

All of the stones that currently restrict the light path, to define the $4 \mathrm{ka} \mathrm{BP} \mathrm{WSR} \mathrm{line} ,\mathrm{have} \mathrm{been} \mathrm{straight-}$ ened or re-erected during the present era (Cleal et al. 1995, p. 344). They might be in positions a few cm different from those 4000 years ago. The size of the notch pecked into Sarsen 58 allows adequate sighting of the WSR across the Sarsen Circle and could tolerate slightly different Stone positions.

If a distant outlier marked the WSR 4000 years ago, we found no remaining evidence of it.
The ritual importance of the WSR was evidently much less than that of theWSS.

## Mapped WSR and WSS lines

The Winter Solstice Sun Rise and Set lines for 4 ka BP are plotted on a Stonehenge site map (Fig. 9). The map is a composite of Figure 13 of Cleal et al. (1995) and Figure 4 of Chippendale (1994). The two maps, after scaling to the same size of Aubrey Hole circle, were not precisely superimposable, so the more recent (Cleal et al. 1995) was selected as the principal one.

The angle between the WSR and WSS lines is $80^{\circ}$, in agreement with expectation, so the map is accurate to this extent. The WSR and WSS lines intersect at the centre of the Altar Stone (Fig. 9).

A line from Station Stone 93 to the doubtful Stonehole H (bush hole, Cleal et al. 1995, pp. 289-290) is $4^{\circ}$ counterclockwise from the WSR direction; the angle is calculated from the distances of 93 and H from the WSR line and the distance between them.

## Summary

At Stonehenge, a 1020 m long line from the Heel Stone to the NE groin between the dome and the flare of Bell-barrow 15 accurately marks the Winter Solstice Sun Sets of the period around 4000 BP. The line of view is now narrowly defined by the NW edge of Sarsen 1 and the SE edge of Sarsen 56, giving an angular width of $0.5^{\circ}$ (Fig. 5a). The original line was probably further restricted by the NW edge of Sarsen 55 ; if the gap between Sarsens 55 and 56 at this level was 0.3 m , the angular width viewed from the Heel Stone was $0.2^{\circ}$. Observation of a wider view from the Altar Stone and Bluestone 31 would have permitted the counting of the 17-18 days from the time that the upper limb of the Sun slid down the NW slope of the dome of Bell-barrow 15 to when it Set into the angle between the SE base of the dome and the flare of the Bell-barrow. About eighteen days after the Winter Solstice the upper limb of the Sun again slid down the NW slope of the dome as the Set point repeated its annual northward journey.

A Winter Solstice Sun Rise observation line crosses the Sarsen Circle between Sarsens 21 and 22, through the Trilithon gap 57-58, past the NE edge of Bluestone 69, above and parallel to the Altar Stone 80, past the NE edge of Trilithon Sarsen 53 and the SW edge of Circle Sarsen 8. Viewed from the NW Bank, the angular width of view restricted by Bluestone 69 and a large notch pecked into the SW edge of Sarsen 58 is $0.4^{\circ}$; viewed at levels above the notch the angular width is $0.2^{\circ}$ (Fig. 7c).

The fact that the WSR and WSS lines intersect above the centre of the Altar Stone (Fig. 9), make it virtually certain that the Altar Stone is still in its original orientation. Stone 80 was probably pushed downward when 55 fell on it.

## Appendix

The directions of the Solstice Sun Rises and Sets are drifting slowly toward the Equalnight directions, due to a gradual decrease in the obliquity of the ecliptic (the angle e between the plain of the Earth's equator and the plain of the Earth's orbit around the Sun). The equation for $\varepsilon$ and its time dependence (Smart 1965; Astronomical Almanac 1996) can be transformed to

$$
\varepsilon\left({ }^{\circ}\right)=23.439-0.130 \mathrm{~T}+0.00050 \mathrm{~T} 3(2)
$$

or

$$
\varepsilon(\mathrm{rad})=0.40909-0.00227 \mathrm{~T}+8.7 \times 10^{-6} \mathrm{~T}^{3}
$$

where T in thousands of years (kiloanni, ka ) is measured algebraically from AD 2000. There is a small $\mathrm{T}^{2}$ term that can be neglected for T smaller than 10 ka . In AD 1995 the value of $\varepsilon$ was $23.44^{\circ}(0.4091 \mathrm{rad})$, while in 4 ka BP, the time when the Sarsen Horseshoe and Circle were constructed, $\varepsilon$ was $23.93^{\circ}(0.4176$ $\mathrm{rad})$.

The relation between the azimuth $\theta_{s}{ }^{R}$ of the first flash of the rising Sun (angle measured clockwise from north), the angular distance $\mathrm{S}_{0}$ of the Sun along its apparent orbit from the Vernal Equalnight, the obliquity $\varepsilon$, the latitude $\lambda$ of the position of the observer on the Earth, and the true altitude $h_{s}{ }^{R}$ of the center of the Sun at the moment of the first flash, with all quantities expressed in radians, is (Smart 1965; Hoyle 1977, pp. 138145)

$$
\sin \varepsilon \sin \mathrm{S}_{0}=\cos \mathrm{h}_{\mathrm{s}}^{\mathrm{R}} \cos \lambda \cos \theta_{\mathrm{s}}^{\mathrm{R}}+\sin \mathrm{h}_{\mathrm{s}}^{\mathrm{R}} \sin \lambda(3)
$$

The true altitude of the Sun's center at the moment of the first flash is lower than the horizon, due to the angular radius of the Sun, which is $4.65 \mathrm{mrad}=0.266^{\circ}$, and to the refraction of the Earth's atmosphere, which is on average $8.6 \mathrm{mrad}=0.49^{\circ}$ at Stonehenge when the geographical altitude $\mathrm{h}^{\mathrm{R}}=8.7 \mathrm{mrad}=0.50^{\circ}$ in the direction of observation (Fig. 4). The general equation for $h_{s}$ of the Sun's center is

$$
\mathrm{h}_{\mathrm{s}}=\mathrm{h} \text { Sun's radius - refraction (4) }
$$

For the preceding example of Sun Rise we have

$$
\mathrm{h}_{\mathrm{s}}^{\mathrm{R}}=0.0087-0.0047-0.0086=-0.0046 \mathrm{rad}, \text { or }-0.26^{\circ}
$$

The values of $h_{s}{ }^{R}$ for Sun Rise or $h_{s}{ }^{s}$ for Sun Set at Stonehenge are in the range -5 to -9 mrad (Table 2), so within an accuracy of $10^{-2} \%$ we can put $\cos h_{s}=1$ and $\sin h_{s}=h_{s}\left(h_{s}\right.$ must be in radians). Then eqn (3) becomes

$$
\sin \varepsilon \sin S_{0}=\cos \lambda \cos \theta_{\mathrm{s}}{ }^{\mathrm{R}}+\mathrm{h}_{\mathrm{s}}{ }^{\mathrm{R}} \sin \lambda(5)
$$

At the Summer Solstice $S_{0}=\pi / 2$ radians so $\sin S_{0}=1$, whereas at the Equalnightes $S_{0}=0$ or $\pi$ with $\sin \mathrm{S}_{0}=0$, and at the Winter Solstice $\mathrm{S}_{0}=3 \pi / 2$ with $\sin \mathrm{S}_{0}=-1$. Thus for the Winter Solstice Sun Rise,

$$
-\sin \varepsilon=\cos \lambda \cos \theta_{\mathrm{s}}^{\mathrm{R}}+\mathrm{h}_{\mathrm{s}}^{\mathrm{R}} \sin \lambda(6)
$$

and

$$
\begin{equation*}
\theta_{\mathrm{s}}^{\mathrm{R}}=\cos ^{-1}\left(\frac{-\sin \varepsilon-\mathrm{h}_{\mathrm{s}}^{\mathrm{R}} \sin \lambda}{\cos \lambda}\right) \tag{7}
\end{equation*}
$$

At Stonehenge the value of $\lambda$ is $51.178^{\circ}$ ( 0.8932 rad$)$.
The azimuth $\theta_{\mathrm{s}}{ }^{R}$ of the last flash of the setting Sun would be $\left(2 \pi-\theta_{\mathrm{s}}{ }^{\mathrm{R}}\right)$ radians if $\mathrm{h}_{\mathrm{s}}{ }^{R}=h_{s}{ }^{\mathrm{S}}$, but in general the two geographic altitudes $\mathrm{h}^{\mathrm{R}}$ and $\mathrm{h}^{\mathrm{S}}$ are different. Therefore, for the Winter Solstice Sun Set,

$$
\begin{equation*}
\theta_{\mathrm{S}}^{\mathrm{S}}=2 \pi-\cos ^{-1}\left(\frac{-\sin \varepsilon-\mathrm{h}_{\mathrm{S}}^{\mathrm{S}} \sin \lambda}{\cos \lambda}\right) \tag{8}
\end{equation*}
$$

The values of $\theta_{s}, \varepsilon$ and $\lambda$ in eqns (5) - (8) may be expressed in degrees, but $\mathrm{h}_{\mathrm{S}}$ must always be expressed in radians to use the simplification $\sin h_{s}=h_{s}$. If one wishes to express $h_{s}$ in degrees one simply replaces $h_{S}$ by $\sin h_{s}$ in eqns (5) - (8).

One can estimate the value of the geographical elevation $h$ by calculating a value of $\theta_{\mathrm{s}}$ from eqn (7) or (8), using $\mathrm{h}=0$ in eqn (4), then drawing a line in that direction from the observation location to the horizon on a contour map such as Ordnance Survey Explorer 130 (1998), and estimating the value of h along that line: $h(r a d)=$ elevation dzetween the point of observation and the horizon. The value of $h$ is inserted into eqn (4), along with the Sun's radius and the refraction angle for this $h$ read from Figure 4, to obtain $h_{s}$. Then $\theta_{\mathrm{s}}$ is recalculated. If necessary the calculation cycle is repeated (method of successive approximations).

The transfer of this method to the actual landscape, using a compass to find the line from a particular observation point, requires accurate knowledge of the local value of the magnetic declination (direction of Magnetic North from True North). The magnetic declination in the vicinity of Stonehenge in 1997 was about $4.8^{\circ} \mathrm{W}$, decreasing by about $0.13^{\circ}$ per year, estimated from the 1998 Ordnance Survey map. Local variations in the magnetic declination, and uncertainties in its rate of drift, make this method inadequate for the present level of accuracy of determining the Sun Set and Sun Rise lines.

The observed Sun Set line was from Bluestone 31 to the upper SE side of the Bell-barrow 15. We drew this line on a 2.50x magnification of the Ordnance Survey 130 map and measured the direction angle, $231^{\circ}$. The value of h from 1.5 m above the base of Bluestone 31 to near the top of Bell-barrow $\mathbf{1 5}$ was estimated to be $7 \mathrm{mrad}\left(0.42^{\circ}\right)$. The refraction correction for a geographical altitude of 7 mrad is -8.8 mrad $\left(-0.50^{\circ}\right)$, read from Figure 4. Inserting these values into eqn (4) along with the correction from the Sun's radius, -4.7 mrad , gives $\mathrm{h}_{\mathrm{s}}{ }^{\mathrm{S}}=-6 \mathrm{mrad}\left(-0.35^{\circ}\right)$. These quantities are listed in Table 2; more digits are given to minimize roundoff errors. The value of $\theta_{\mathrm{s}}{ }^{\mathrm{S}}$ then calculated from eqn (8) is 4.035 rad , or $231.2^{\circ}$. The
value of $\mathrm{h}^{\mathrm{s}}$, and therefore of $\mathrm{h}_{\mathrm{s}}{ }^{\mathrm{S}}$, for the sight line at $231.2^{\circ}$ is the same as that listed, so another cycle of calculation is not needed.

At the time of construction of the Sarsen Circle and Horseshoe, 4 ka BP , the value of $\varepsilon$ was 0.4176 rad. The values of $h^{s}$ and $h_{s}{ }^{S}$ would have been the same as now. The value of $\theta_{s}$ calculated from eqn (8) for 4 ka BP is $4.019 \mathrm{rad}\left(230.3^{\circ}\right)$, which is $0.9^{\circ}$ counterclockwise from the present direction. The line of descent of the upper limb of the setting Sun in 4 ka BP is drawn on the photos in Figure 3. The last flash of the setting Sun would have disappeared into the angle (groin) on the SE side where the dome meets the flare of the Bell. (The flare of a Bell-barrow is often referred to as a berm, which is actually a ledge or path between a moat and rampart, or between a canal and its embankment.)

## Acknowledgments

We are grateful to the personnel of English Heritage who assisted us at various times in this project. The warm hospitality of Prof. Norman H. March of Egham, and the use of his car, during our vigils for the Sun during December 1995 and 1997 are much appreciated. We thank Dr. and Mrs. Derek A. Dominey of Bristol for their warm hospitality and for several trips to Stonehenge to take photos, including those of Figures 3b and 5. Prof. Douglas P. Hube has for the past decade done astronomical calculations for us and guided us through intricacies of spherical astronomy, to archaeo-extrapolate our seasonal Sun measurements. GRF is grateful to Dr. R. William Hummel for hospitality and taking him to Stonehenge and environs in April, 1989.

## References Cited

Astronomical Almanac
1996 Washington, DC: US Government Printing Office, p. B18. The ratio of durations Julian century/ Georgian century $=1.00002$ is negligibly different from unity in the present work (Smart, 1965, pp. 144-145).
Atkinson R.J.C.
1979 Stonehenge, 2nd edition. Penguin, Harmondsworth.
Burl A.
1976 Stone circles of the British Isles. Yale University Press, New Haven, USA.
1993 From Carnac to Callanish. Yale University Press, New Haven, USA.
1999 Great Stone Circles. Yale University Press, New Haven, USA.
Chippindale C.
1994 Stonehenge complete this time?, revised edition.Thames \& Hudson, London.
Cleal R.M.J., Walker K.E., and Montague R.
1995 Stonehenge in its landscape. English Heritage, London.
Freeman G.R., and P.J. Freeman
1990a Majorville Medicine Wheel Site: I Distant outliers and accurate Solstice alignments, 23rd Chacmool Conference, Calgary, Canada, November.
1990b Majorville Medicine Wheel Site: II Marking the 12.00 hour day, not the Solar Equalnight, 23rd Chacmool Conference, Calgary, Canada, November.
1998 Variation in estimated positions of Sun Rise and Set due to atmospheric conditions, SEAC Conference, Dublin, 31 August - 2 September.

Haack S.C.
1987 A critical evaluation of medicine wheel astronomy, Plains Anthropologist, 32-115, 72-82 Hawkins G.S.
1965 Stonehenge decoded. Doubleday, Garden City, N.Y.
Hoyle F.
1977 On Stonehenge. Freeman, San Francisco.
Newham C.A.
1972 Astronomical significance of Stonehenge. Blackburn, Leeds.
North J.D.
1996 Stonehenge: Neolithic man and the cosmos. Harper Collins, London.
Ordnance Survey
Explorer 130, Salisbury and Stonehenge, Southampton, England. Scale 1:25,000; contour interval 5 m .
Richards J.
1990 The Stonehenge environs project. Archaeological Report 16. English Heritage, London. Royal Commission on Historical Monuments (England)
1979 Stonehenge and its environs: Monuments and land use, Lord Adeane, Chairman, Edinburgh, University Press, Folding map 2; 1:20,000; contours at 7.6 m intervals
Ruggles C.L.N.
1984 Megalithic astronomy: A new archaeological and statistical study of 300 western Scottish sites. British Archaeological Reports, Series 123, Oxford.
Sampson R.D.
1994 Atmospheric refraction and its effects on Sunrise and Sunset, MSc Thesis, Department of Geography, University of Alberta, Edmonton, Canada
Smart W.M.
1965 Spherical astronomy, 5th edition. Cambridge University Press, chapters 1 and 2, p. 420.
Thom A.
1971 Megalithic Sites in Britain, corrected reprint of 1967 edition. Clarendon Press, Oxford.
Thom A., A.S. Thom, and A.S. Thom
1974 Stonehenge. Journal of the History of Astronomy, 5, 71-90
Victoria History of the Counties of England
1957,1972 A History of Whiltshire, I, London, Oxford University Press: Part 1, 1957, pp 207, 246; Part 2, 1972, pp 333-358.

