NAOMI/US

A small-scale model of the U.S. economy

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Abstract

This paper presents NAOMI/US (**N**orth **A**merican **O**pen economy **M**acro-econometric **I**ntegrated model/US), the U.S. counterpart to NAOMI/Canada, developed with the objective of contributing to EAFD's analytical and forecasting tools. It is a tractable system built around three main equations: IS curve, Phillips curve and monetary policy rule. The model exhibits a meaningful steady state, is forwardlooking and consistent with the structure of NAOMI/Canada. Despite its parsimony, it displays credible dynamics that are comparable to those from larger, more complex models.

Résumé

Ce papier introduit MIOAN/US (modèle **M**acro-économique **I**ntégré de l'économie **O**uverte de l'**A**mérique du **N**ord/US), la contrepartie américaine de MIOAN/Canada. Ce modèle a été développé dans l'objectif de contribuer aux outils d'analyse et de prévisions macroéconomiques relatifs à l'économie américaine de la division de l'analyse et de prévisions économiques. MIOAN/US est un modèle à attentes rationnelles qui repose essentiellement sur trois équations : une courbe IS, une courbe de Phillips et une règle de politique monétaire. Malgré sa parcimonie, les propriétés de simulation du modèle sont comparables à ceux des modèles plus complexes et plus détaillés.

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1 Introduction

Model building process can be aimed at many different and sometimes mutually incompatible objectives. For example, while a small-scale model is desirable for its simplicity and tractability, it often prohibits the forecaster from conducting detailed or disaggregated analyses. NAOMI/US is not exempt from such compromises. However, we tried to meet a certain number of basic objectives throughout the building of the model.

First of all, because NAOMI/US is ultimately meant to work in conjunction with its Canadian counterpart (hereafter NAOMI/CAN), the two quarterly models are based on a similar structure: both are specified around three main estimated equations (IS curve, Phillips curve, monetary policy rule), and they exhibit mutually consistent dynamics.[1] Second, we have attempted to build a parsimonious, tractable, and if necessary expandable model that is easy and fast to execute. It also has to be fit for analytical as well as forecasting purposes, and consequently include explicit steady-state properties. Finally, the system is expected to exhibit credible dynamics when compared to priors or other models; whenever possible, we make sure to draw comparisons between the properties of NAOMI/US and those from FRB/US, a large-scale model of the U.S. economy developed by the Federal Reserve.[2]

[1] For a description of NAOMI/CAN, see Murchison (2001).

[2] Good overviews of the structure and properties of FRB/US can be found in Reifschneider *et al.* (1999) and Brayton *et al.* (1997). For a more complete description of the model, see "A Guide to FRB/US: A Macroeconomic Model of the United States" (Brayton and Tinsley, 1996).

This model has been widely tested and used throughout numerous studies.

The paper is organized as follows. Section 2 presents the empirical work and other issues leading to the building of the model, while Sections 3 and 4 deal with the single-equation and full-model properties of NAOMI/US. Section 5 discusses the forecasting performance of the aggregate demand function and the Phillips curve, and in Section 6 we conclude.

2 The Model

In this document, we present a version of the system that appears interesting on the basis of theory, empirical results, and simulation dynamics. Expectations play an important role in this model: the monetary rule is forward-looking, the Phillips curve uses long-term inflation expectations, the real exchange rate is based in part on a forward-looking uncovered interest rate parity condition and the expectations hypothesis is central to the long-term interest rate specification.

A great deal of emphasis has been placed on obtaining estimated parameters based on historical data, and ensuring that the whole system is easy to modify (e.g. to take into account changes in prior) and exhibits credible dynamics and steady-state properties. Given the parsimony of the model, we are particularly pleased with the results of the impulse responses.

The following sections discuss the specification and estimation of each equation, as well as the properties of the model.

2.1 Phillips curve

Throughout the model-building process, it appeared that adopting a fully backward-looking, unit-root specification for the Phillips curve would create problems, notably an oversensitivity of inflation to shocks. Consequently, we turned our attention towards a specification that would yield more credible dynamics. The inclusion of a nominal anchor as proxied by long-term inflation expectations appeared to give particularly interesting results. As a general form, our Phillips curve can be expressed as:

$$\pi_t = lambdaf[(L)\pi_{t-1}] + (1-\lambda)\pi_t^{LT} + g(...) + \epsilon_{\pi,t} \quad (1)$$

where π is the inflation rate, π^{LT} are the long-term inflation expectations, $g(...)$ is a function of additional explanatory variables that will be defined later and the subscript t denotes the quarter.

The inclusion of a nominal anchor (long-term inflation expectations) in our Phillips curve comes from the plausible assumption that in the presence of a credible monetary policy, agents who are less inflation-sensitive decide to avoid the time-consuming task of updating regularly their forecasts of future inflation, and instead base their expectations on the perceived inflation target of the monetary authorities.[3]

[3] We acknowledge that the Federal Reserve does not have an explicit inflation target, and will use long-term inflation expectations as a proxy for the perceived value of the target in the Phillips curve. Note that defining an explicit inflation target for the monetary policy rule will be necessary for simulation purposes (see section 2.3) and will determine the steady-state values of the nominal variables.

Indeed, in an environment of low and stable inflation, the costs of the systematic processing of new information from economic agents could become too high compared to the benefits. We do however acknowledge that such specification can be difficult to derive from a theoretical framework based on fully rational agents and that it renders some particular issues more complicated to analyze (e.g. disinflation policies). Ideally, one would want to model with greater details the expectations-formation process, a subject beyond the scope of this paper.

In order to estimate our Phillips curve equation, a measure of long-term inflation expectations is therefore necessary. We want an indicator of the perceived rate of inflation that would prevail at an horizon long enough to ensure that the "steady state" was achieved, i.e. where the effects of monetary policy and automatic stabilizers are completed. In this preliminary version, we use results from the long-term inflation forecasts survey series published by the Federal Reserve of Philadelphia. The 10-year ahead expectations meet our requirements in terms of horizon. This particular series starts in late 1970s, but is not available on a regular quarterly basis before 1990. To circumvent this problem, we use linear interpolation for the missing quarters. This technique can be acceptable if one makes the assumption that long-term expectations are relatively stable over time. This assumption seems more difficult to accept during the period of high and variable inflation of the early 1980s. Consequently, our Phillips curve is estimated starting in 1985.[4]

[4]Concerning the reliability of the series we use for inflation expectations, we are com-

The following equation has been estimated using instrumental variables (lags of the variables used in the Phillips curve, plus a constant and inflation expectations from the Michigan Survey of Consumers) to take into account the measurement error in the inflation expectations series. We include a measure of commodity prices (Producer Price Index) and the first difference of the log of real exchange rate.[5] The output gap enters contemporaneously, and the first difference in the output gap proves to be non-significant considering the specification and the sample chosen.

$$\begin{aligned}\pi_t &= \lambda[\rho\pi_{t-1} + (1-\rho)\pi_{t-2}] + (1-\lambda)\pi_{t-1}^{LT} + \beta y_t^G \\ &\quad +\gamma\pi_{t-1}^{PPI} + \delta\sum_{i=2}^{4}\frac{\Delta z_{t-i}}{3} + \epsilon_{\pi,t} \qquad (2)\end{aligned}$$

where π is CPIXFE (Consumer Price Index excluding food and energy) inflation, π^{LT} is the 10-year-ahead CPI inflation expectations series, y^G is the output gap[6], π^{PPI} is inflation of the relative Producer Price Index, and Δz is the first difference (annualized) of the log trade-weighted real exchange rate.[7] All the inflation rates are defined as annualized quarter-over-quarter rates.

The fact that π^{LT} and π do not make reference to the same measure of inflation (CPI and CPIXFE) does not appear relevant in this case. Long-

forted by the fact that data from the Michigan Survey of Consumers concerning 5-to-10-years inflation expectations (available since 1980) follow very closely the Fed series.

[5]The real exchange rate is defined as the price of one U.S. dollar in terms of foreign exchange units. An increase in z implies a real exchange rate appreciation.

[6]We use the Department of Finance's estimate of potential real GDP to construct the output gap. See Collins (1998) for a description of the methodology.

[7]Unless otherwise stated, lower case letters represent logs of variables.

term expectations of both measures should be very similar, since the impact of food and energy shocks will have vanished over a 10-year horizon.

Table 1
Phillips curve, Nonlinear Instrumental variables, 1985Q1 to 2000Q4

	λ	ρ	β	γ	δ	$\bar{R}^2$	σ_ϵ	GNR*
Eqn (2)	0.68	0.26	0.20	0.03	-0.01	0.83	0.44	0.13
	(0.000)	(0.051)	(0.002)	(0.066)	(0.117)			

* Throughout the document, *GNR* refers to the p-value of a GNR-based test for autocorrelation of order 4, the null referring to no serial correlation. Numbers in parentheses are p-values from a *t*-test.

Note that the coefficient λ is statistically different from 0 (p-value=0.000) and 1 (p-value=0.008), supporting the assumption that both past inflation and long-term expectations play a significant role in the inflation process. While the coefficients for π^{PPI} and the exchange rate are small and not statistically significant over that sample, the variables are kept as part of the model.

2.2 Aggregate demand

2.2.1 Basic specification

In this section, we develop an IS curve expressed initially in terms of output growth. The basic specification is:

$$\Delta y_t = \rho \Delta y_{t-1} + (1-\rho)\Delta y_t^p + f(...) + \epsilon_{y,t} \tag{3}$$

where Δy is real GDP growth, Δy^p is potential output growth, and $f(...)$ is a function of other explanatory variables that will be introduced later. It relates current production to potential output and past production through a lag term. The restriction that the sum of the coefficients on these two variables is equal to one is strongly supported by the data, and we impose it throughout the rest of the section.

From specification (3) emerges the first noteworthy characteristic of our IS curve. If ρ is lower than 1, as it is the case in NAOMI/US, this means that a shock to potential growth does not translate immediately into a one-for-one change in real output growth. Also, the unit-sum restriction ensures that a permanent shock to the *level* of potential will be matched entirely by an equivalent (gradual) shift in the level of real output, without any intervention from policymakers. This feature is consistent with forward-looking agents ultimately behaving according to the permanent income hypothesis, but using some kind of rule of thumb in their convergence towards the new equilibrium.

Also, we can modify (3) in order to express it in terms of the output gap or, together with a certain assumption, the change in output gap. These formulations will appear particularly useful when introducing certain additional explanatory variables. First, by rearranging (3) we can show that:

$$y_t^G = y_{t-1}^G + \rho(\Delta y_{t-1} - \Delta y_t^p) + f(...) + \epsilon_{y,t} \tag{4}$$

where y^G is the output gap defined as the difference between (the logs of) real GDP and potential GDP. If one makes the assumption that potential output growth is constant over time (Δy_t^p=Δy_{t-1}^p=...), we obtain a unit-root specification for the level of output gap:

$$\Delta y_t^G = \rho \Delta y_{t-1}^G + f(...) + \epsilon_{y,t} \quad (5)$$

It was noted earlier that the economy in this model can adjust by itself to a *supply* shock. An additional characteristic of our IS curve arises implicitly from (5). In the case of a (permanent) *demand* shock, actions from policy-makers are necessary in order to bring back output to its potential. This feature stems from the unit-root form of (5).

While we will use equation (3) for estimation and forecasting purposes, specification (5) will be an exact formulation for all the shocks and simulations that are implying no change in the value of potential output growth. In addition, most simulation results will be presented in terms of the output gap to facilitate comparisons with the dynamics of other models.

2.2.2 The role of interest rates

Considering that interest rates represent a key economic channel in the model, we want to ensure that any future work is based on a sound and meaningful specification, providing us with plausible dynamic properties. We have retained the specification that appears to fit best these informal criteria.

Following the basic specification presented in the previous section, we estimate the following relationship:

$$\Delta y_t = \rho \Delta y_{t-1} + (1-\rho)\Delta y_t^p + \alpha(SPR_{t-2} - S\bar{P}R) + \beta \Delta r_{t-2}^s + \epsilon_{y,t} \quad (6)$$

where r^s is the level of the real Federal funds rate (inflation in the previous quarter is used as a proxy for expected inflation), SPR is the yield spread (defined as Federal funds rate minus 10-year bond rate), and $S\bar{P}R$ is the equilibrium level of the spread (equal to the negative of the average level of the term premium over history). This specification is similar to the one from MPMOD, a small model used for the analysis of Canadian monetary policy (see Charron, Fillion and Fougère, 2001). The results from the regression are presented in Table 2.[8]

Table 2
IS curve, Restricted OLS, 1979Q1 to 2000Q4

	ρ	α	β	$\bar{R}^2$	σ_ϵ	GNR
Eqn (6)	0.31	-0.10	-0.11	0.38	0.60	0.35
	(0.004)	(0.017)	(0.000)			

By including Δr^s, we emphasize the simple, well-known relation between real interest rates and output; β summarizes changes in the cost of capital, wealth, and substitution effects. As a measure of the instantaneous real rate of return, it captures the effect of an *absolute* shift in current (or lagged) interest rates on economic activity.

However, one could argue that the level of the short rate *relative* to a long-run equilibrium maturity is also of relevance. In this case, the theory predicts that when the short-term rate is higher than its equilibrium value, consumers

[8]Real and potential output growth are expressed as *non-annualized*, quarter-over-quarter rates.

will decrease consumption and increase savings. This framework, based on the consumption capital asset pricing model (CCAPM), thus relates the term structure to the path of consumption growth through the trade-off faced by agents between saving and consumption.[9] In NAOMI/US, the variable SPR is used as a proxy for the effect from such channel.[10]

2.2.3 The external sector

As commonly observed in empirical studies, the roles of the exchange rate and foreign activity are poorly identified when included directly in the IS curve. An alternative method is therefore necessary if one wishes to estimate some external sector pass-through effect in the model.

One possibility would be to adopt Rae and Turner's (2001) method which involves modeling separately the internal (domestic demand) and external (net exports) components of total output. While this method might appear very interesting to obtain a more detailed model, for the purpose of parsimony we prefer to use a simpler approach. First, recall that our IS curve can be approximately described in terms of the change in the output gap (see (5)).

[9] See Harvey (1988) and Robidoux and Wong (1996) for applications.

[10] In NAOMI/CAN, it is instead the first difference of the term spread that is included in the IS curve. The choice of the level of the spread in NAOMI/US is based on various reasons. Notably, this specification provides the best in-sample results: the first difference of the spread is either non-significant or yields a poorer fit. Also, the current specification clearly outperforms alternatives in terms of model dynamics.

Also, let us express the total output gap as:

$$
\begin{aligned}
y_t^G &= (\frac{Y_t}{Y_t^p} - 1) * 100 \\
&= ((\frac{DD_t + NX_t}{Y_t^p}) - 1) * 100 \\
&= ((dd_t^G + nx_t^G) - 1) * 100
\end{aligned}
\tag{7}
$$

where dd^G and nx^G are the ratios of domestic demand and net exports with respect to potential output. Because we cannot find a significant role for the real exchange rate or foreign activity in the total output gap specification, we isolate nx^G, and estimate the following linear function:

$$
\Delta nx_t^G = f[(L)\Delta nx_{t-1}^G, (L)\Delta y_{t-1}^G, \Delta \hat{z}_{t-1}, (\Delta y_{t-1}^f - \Delta y_{t-1}^p)] \tag{8}
$$

where $\Delta \hat{z}$ is the 8-quarter moving average of the change in the log 35-country real exchange rate (quarterly annualized), Δy^f is the quarter-over-quarter annualized growth rate in a 35-country index of real foreign GDP and Δy^p is the U.S. potential output growth.[11] From regression (8), we obtain a coefficient of -0.02 with a t-statistic of 3.62 for the exchange rate. While this value is significantly higher than in some other models (notably Lalonde, 2000), we will show later that it yields results quite similar to those from FRB/US. Concerning the foreign output variable, its coefficient is equal to 0.15, with a t-statistic of 2.30. This is relatively close to the share of exports in total U.S. output. Both parameters will be imposed in subsequent estimations of the IS curve specification.

[11] It could be argued that output also affects net exports contemporaneously. Using an IV estimation procedure to deal with the endogeneity problem, it appears however that such specification change does not alter significantly our results.

2.2.4 Complete specification

The IS curve specification is completed by adding two more variables. In our attempts to include a fiscal side to the model, we started by including lags of the government budget balance, a specification often found in small-scale models. However, this form imposes a symmetric effect from changes in government spending and receipts, while further examination of the results showed that the explanatory power comes exclusively from the revenue side.

Given these results, we decided to adapt the FiPS (Fiscal Policy Stance) approach proposed by Murchison and Robbins (2002) to the U.S. case, with mixed success. While the IV methodology failed to identify any significant contemporaneous effect of taxes or transfers to persons on output, government consumption and investment appeared highly significant with a coefficient not statistically different from one, as expected. In addition, on the basis of regression results, we also included the third lag of the change in government revenues.[12] Consequently, the fiscal variable is defined as:

$$GOV_t = 1 * \Delta gsp_t - 0.35 \Delta grev_{t-3} \tag{9}$$

where gsp and $grev$ are respectively the ratios of government consumption and investment and government revenues over potential GDP.

We also include the second difference of the S&P500 stock market in-

[12]Note that the fiscal variables are not adjusted for the business cycle in order to preserve their characteristics as automatic stabilizers. Arguably, a complete specification would somehow endogenize the cyclical portion of government spending and revenues. For reasons of parsimony however, we did not go as far in this version of the model.

dex.[13] This formulation indicates that an increase in the growth rate of the index would have only a temporary effect on aggregate output growth. The rationale is that a long-lasting increase/decrease in the rate of return of the stock market would be implicitly captured in the growth rate of potential output, already present in our aggregate demand specification. An abnormal rate of return for 2 or 3 quarters that does not affect the potential of the economy would consequently have a very short effect on the output gap, since consumers would not change much their consumption pattern. The estimated regression is:

$$\begin{aligned} \Delta y_t &= \rho \Delta y_{t-1} + (1-\rho)\Delta y_t^p + \alpha(SPR_{t-2} - \bar{SPR}) + \beta \Delta r_{t-2}^s + GOV_t \\ &\quad + \gamma \sum_{i=3}^{5} \frac{\Delta S\dot{P}500_{t-i}}{3} + \lambda \sum_{i=1}^{8} \frac{\Delta z_{t-i}}{8} + \theta(\Delta y_{t-1}^f - \Delta y_{t-1}^p) + \epsilon_t \qquad (10) \end{aligned}$$

where Δy and Δy^p are real GDP growth and potential output growth respectively (quarterly, non-annualized rates), $S\dot{P}500$ is the annualized quarterly growth rate of the deflated S&P500 index, GOV is a fiscal variable defined by (9), Δz is the annualized change in the log 35-country real effective exchange rate and Δy^f is foreign output growth. All variables are expressed in percentage terms. Table 3 summarizes the results obtained for the complete IS curve.

[13] We obtain comparable results by using alternatively the Wilshire 5000 index or a measure of household holdings of corporate equities.

Table 3
IS curve, Restricted OLS, 1979Q1 to 2000Q4

ρ	α	β	γ	λ	θ	$\bar{R}^2$	σ_ϵ	GNR
0.29	-0.09	-0.14	0.015	-0.02	0.15	0.50	0.54	0.98
(0.004)	(0.010)	(0.000)	(0.005)	-	-			

From the results above, we see that the extra variables increase the adjusted R^2 by approximately 30%, while the standard error of the residuals falls by 10% compared to the specification including only interest rate variables (see Table 2). Note also that on the basis of a F-test the imposed restrictions cannot be rejected by the data, with a p-value of 0.43.

2.3 Monetary policy rule

The specification of the forward-looking monetary policy rule is similar to Clarida, Gali, and Gertler (1998a). We express the nominal Federal funds rate as a function of past interest rate (interest rate smoothing), expected inflation (forward-looking rule) and current output gap. We estimate the equation using Generalized Method of Moments (GMM, Hansen 1982), with four lags of the following variables as instruments: CPIXFE inflation, output gap, Federal funds rate, and PPI inflation.[14] We use Hansen's optimal weighting matrix that accounts for possible serial correlation in the distur-

[14]Because of the difference in estimation samples, we cannot use long-term inflation expectations from surveys as an instrument. Instead, we include variables that seem to predict inflation relatively well since 1979.

bance term. The following equation is estimated over the 1979Q3-1999Q4 sample(Volcker-Greenspan era), allowing enough observations in the data to accommodate for the inclusion of the lead in inflation.

$$R_t^s = \rho R_{t-1}^s + (1-\rho)(\lambda + \beta E_t \pi_{t+6} + \gamma y_t^G) + \epsilon_{R,t} \tag{11}$$

where R^s is the nominal Federal funds rate, E is the expectations operator, π is CPIXFE inflation (year-over-year rate in this case), and y^G is the output gap. Recent research has provided theoretical support for this type of monetary policy rule. Empirical results are presented in Table 4. They are relatively robust in terms of instruments included and lead of inflation, but quite sensitive to the sample chosen.[15]

Table 4

Monetary policy rule, GMM estimation, 1979Q3 to 1999Q4

	ρ	λ	β	γ	$\bar{R}^2$	σ_ϵ	GNR
Eqn (11)	0.83	0.46	1.81	1.13	0.91	1.06	0.20
	(0.000)	(0.760)	(0.000)	(0.003)			

For the purpose of simulation and forecasting, the second term of (11) can be re-specified as the target nominal Fed funds rate expressed in deviation from the inflation target. Thus, we obtain:

$$R_t^s = \rho R_{t-1}^s + (1-\rho)(\bar{r}^s + \pi^* + \alpha(E_t \pi_{t+6} - \pi^*) + \gamma y_t^G) \tag{12}$$

[15]Making the monetary policy rule forward-looking in terms of the output gap does not modify significantly the dynamics of the model, except for a somewhat lower sacrifice ratio.

where $\bar{r}^s$ is the equilibrium real Federal funds rate and π^* is the inflation target. Comparing (11) and (12), we obtain the following relationships:

$$\begin{aligned}\beta &= \alpha \\ \lambda &= \bar{r}^s + \pi^* - \alpha\pi^* \end{aligned} \tag{13}$$

Moreover, using the average real interest rate over the sample (2.9%) to proxy $\bar{r}^s$ and the values for λ and β found in Table 4, the implicit inflation target is estimated to have been 3% on average over our sample (Clarida *et al.*, 1998b, found an average inflation target of 4%).[16]

2.4 Long-term interest rate

As a starting point, we model the theoretical yield on 10-year government bonds using the expectations hypothesis:

$$R_t^{l,*} = \psi + E_t \sum_{i=0}^{39} w_i R_{t+i}^s \tag{14}$$

where ψ is a (constant) term premium. The weights w_i decline geometrically at a quarterly rate of 2% (8% annually) to reflect an average yield to maturity on the coupon bond (see Appendix 1).

This specification, however, does not appear totally satisfactory. The long rate reacts too little in response to shocks to short-term rates relative

[16]Clearly, this formulation does not allow the identification of a time-varying inflation target. Moreover, as discussed in footnote 15 in Clarida *et al.* (1998b), by fitting this particular policy rule to data and using the sample average real rate, the estimate of the inflation target is expected not to be too different from the sample average of the inflation rate.

to historical data. Because we want a model that mimics reasonably closely the dynamics of historical data, we adopt a hybrid specification commonly used in forecasting/policy analysis models:

$$R_t^l = \lambda(\psi + R_t^s) + (1 - \lambda)R_t^{l,*} \tag{15}$$

where R^l is the yield on a 10-year government note, R^s is the Federal funds rate, and the parameter λ is used to match the extra variability of actual data. We use a simple iterative expectations method to determine λ since the contemporaneous short-term interest rate also influences the second term of (15). From historical data properties, we obtain $\lambda = 0.25$, a calibration that seems to yield plausible dynamics for long-term interest rates.

2.5 Exchange rate

The real exchange rate can be endogenised through a simple uncovered interest rate parity form (UIP). The contemporaneous value is a weighted average of forward-looking and adaptive UIP specifications, comparable to the one adopted in other models, notably the Bank of Canada's Quarterly Projection Model (QPM, see Colletti *et al.*, 1996). By using this formulation, we avoid excessive "jumps" in the real exchange rate (Dornbusch overshooting hypothesis), and instead opt for a pattern of delayed overshooting (see Eichenbaum and Evans, 1995).

$$z_t = 0.6z_{t-1} + 0.3z_{t+1} + 0.1z^* + \frac{(r_t^s - r_t^{s,w})}{400} \tag{16}$$

where z is the logarithm of the 35-countries trade-weighted real exchange rate, $r^{s,w}$ is the real interest rate in the rest of the world, and z^* is the terminal condition. A weight on the terminal condition is necessary to obtain a plausible convergence rate for the real exchange rate. The weights used are very similar to those found in QPM. Following further research, a fuller specification could include a trade channel in addition to the capital flows effect, as well as other concepts tested empirically in NAOMI/CAN.

2.6 Full model

From the equations developed in previous sections, the full model is:

$$\begin{aligned}
\pi_t &= 0.68[0.26\pi_{t-1} + 0.74\pi_{t-2}] + 0.32\pi^{LT} + 0.2y_t^G \\
&\quad +0.03\pi_{t-1}^{PPI} - 0.01\sum_{i=2}^{4}\frac{\Delta z_{t-i}}{3} \\
\Delta y_t &= 0.29\Delta y_{t-1} + 0.71\Delta y_t^p - 0.09(SPR_{t-2} - S\bar{P}R) - 0.14\Delta r_{t-2}^s + GOV_t \\
&\quad +0.015\sum_{i=3}^{5}\frac{\Delta S\dot{P}500_{t-i}}{3} - 0.02\sum_{i=1}^{8}\frac{\Delta z_{t-i}}{8} + 0.15(\Delta y_{t-1}^f - \Delta y_{t-1}^p) \\
R_t^s &= 0.83R_{t-1}^s + 0.17(\bar{r}^s + \pi^* + 1.81(E_t\pi_{t+6} - \pi^*) + 1.13y_t^G) \\
R_t^l &= 0.25(\psi + R_t^s) + 0.75[\psi + E_t\sum_{i=0}^{39} w_i R_{t+i}^s] \\
z_t &= 0.6z_{t-1} + 0.3z_{t+1} + 0.1z^* + \frac{(r_t^s - r_t^{s,w})}{400} \qquad (17)
\end{aligned}$$

where all variables have been defined previously. Recall from section 2.3 that the monetary rule can be expressed in terms of deviation from the inflation target. This would ensure that the model will converge to a meaningful steady state.[17] Finally, the system is completed with appropriate identities.

[17] In this model, the following relationships hold in steady state: $y = y^p$, $\pi = \pi^*$, $SPR = S\bar{P}R = -\psi$, $R^s = \bar{r}^s + \pi^*$, $R^l = R^s + \psi$, $z = z^*$.

3 Single-equation properties

In this section, we perform some simple shocks on different equations of the system. Since we only want to analyze the reaction of output and inflation to certain exogenous shocks, the dynamics are observed on a single-equation basis: *there are no responses from other channels* (monetary policy, exchange rate, etc.). Full-model dynamics will be studied in Section 4.

3.1 IS curve

The graphs in Figure 1 of Appendix 2 show the effects of five types of shocks on output growth. Results are presented in terms of output growth, i.e. the formulation we use to engineer the impulse responses is similar to equation (3). Output growth is expressed as quarter-over-quarter rates, non-annualized.

3.1.1 Interest rate and exchange rate shocks

The first experiment simulates a one percentage point permanent increase in short-term interest rates (see Figure 1.1). The shock operates through two channels: first, it increases the real short-term interest rate (one-for-one, since we assume inflation stays constant). Second, the term spread widens by 1 percentage point (long-term interest rates do not react in this experiment). A trough of -0.23% occurs at $t+2$, with the combined effects of the yield spread and the change in the real rate. Because of the sustained positive value of the term spread (remember we assume unrealistically that long-term interest rates are constant), the impact on output growth is permanent, with a long-run elasticity of -0.12%.

Figure 1.2 plots the impact of a real exchange rate shock: z is assumed to experience a 1% permanent appreciation. Given that the IS curve is specified as a function of Δz, the effect will be temporary on output growth, but permanent on the output gap. Because of the lag structure used, Δy declines gradually until $t + 8$, and reaches a minimum of about -0.014% relative to basecase before heading back to the level of potential growth. In a separate exercise (no graph reported), we compare the impact of a permanent 10 percent reduction in z on output gap to the results from FRB/US presented in Reifschneider *et al.* (1999). With no monetary policy reaction, real GDP in FRB/US is 0.4 and 1.6 percent above baseline after one and two years respectively, compared to 0.5 and 1.2 for NAOMI/US. These similarities bring support to our estimate of the exchange rate pass-through. In the longer run, however, output continues to grow at a higher rate in the Fed's model.

3.1.2 Shocks to potential output

The specification of the IS curve implies that disturbances to the *growth rate* or the *level* of potential output are eventually matched entirely by an equivalent shift in real GDP. This property is illustrated by Figures 1.3 and 1.4. The first plots the reaction of output growth (solid line) following a shock of 1 percentage point to potential growth (broken line). Given the form of the aggregate demand, about two thirds of the adjustment occurs in the period of the shock and convergence is completed after one year.

The second graph plots the evolution of the *level* of output given two shocks to the level of potential output. As witnessed previously, the shift is gradual, and is compatible with a progressive adjustment of the behaviour of economic agents.

3.1.3 Fiscal shocks

The first shock (Figure 1.5) simulates a permanent exogenous increase in government spending by 1% of potential GDP. Output growth jumps at the moment of the shock, and stays positive for a few quarters due to the multiplier effect. Without any action from policymakers, the cumulative increase in real GDP is equal to 1.4%. In FRB/US, a comparable shock increases the aggregate demand by 1.4% compared to baseline by the end of the first and second years, similar to NAOMI/US (see Reifschneider *et al.*, 1999).

In the second scenario (Figure 1.6), government tax receipts are considered exogenous and are increased by 1% of potential output. The impulse response function shows that aggregate demand reacts with a lag to the change in fiscal policy, with a peak effect of -0.35% on the growth rate of output relative to basecase. In this case, the long-run elasticity is significantly lower than 1 (-0.5), illustrating the presence of Ricardian behaviour from economic agents.[18] Comparisons with FRB/US are more difficult in

[18]This represents however only an average reaction. For example, modifications in the tax system that are perceived as permanent would be expected to yield more important

this case, since it distinguishes explicitly between tax categories and the nature of the change in tax policy (permanent vs. temporary). In this model, a permanent shock to personal income taxes equal to 1% of potential GDP causes output to fall by 0.4% after one year (similar to NAOMI/US), but long-run effects are more important.

3.1.4 Stock market shock

The initial experiment regarding the stock market simulates a fall of 5 percentage points in the annualized growth rate of the S&P500 (see Figure 1.7). Recall that the stock market enters as a second difference, i.e. a change in the return on the index has only a temporary effect on output growth, as long as potential growth remains unchanged. In this case, Δy falls to -0.035% relative to basecase, then goes back to equilibrium.

While this impact seems low, our specification is not necessarily inconsistent with the well-known rule of thumb stating that a $1 decrease in share value leads to a decrease of 3 to 5 cents in consumption. To test this assumption, we engineered a 1-quarter shock to simulate a fall of 1% in the S&P500. The cumulative loss of output resulting from this shock is 0.10% (the integral under the shocked output gap curve). Now, from historical data, it appears that on average a change in the S&P500 is matched by a very similar fluctuation in household wealth in corporate equity (in percentage terms). Consequently, we consider that the 1% fall in the stock index is matched by a similar 1% decrease in share market wealth (denoted W).

long-run impacts than temporary measures.

We compute the ratio $\frac{0.10\%}{1\%}\frac{Y}{W}$, where Y and W correspond to their average values for 2000, and obtain 0.06%.

If we assume that the lost output comes entirely from foregone consumption, it means that a \$1 decrease in stock market value will cause a 6 cents decrease in consumption. Alternatively, if we consider the impact on output coming from consumption is proportional to its share of GDP (about 60%), then the decrease in consumption amounts to 3.5 cents. This is consistent with the rule of thumb stated above. Also, the peak negative effect is about 1/12 of the one from the interest rate shock.

3.2 Phillips curve

We perform similar exercises with the Phillips curve equation by creating shocks to the different explanatory variables. Note that our specification does not include a unit root, and inflation will return to its target by itself.

Figure 2.1 shows the impact on inflation from a temporary increase to the output gap (one percentage point, for four quarters). Because this variable has a contemporaneous effect, inflation jumps initially by 0.2% compared to the basecase, then continues to rise up to a maximum of 0.38% at $t = 3$. From then on, inflation returns steadily to the level of long-term expectations, here equal to the inflation target. Without any reaction from monetary authorities, and assuming constant long-term expectations, the inflation rate is almost back to its baseline value after 3 years.

The second graph (Figure 2.2) shows the impact a temporary change in the inflation rate of the relative Producer Price Index. Note that a shock of 5 percentage points to this variable (relatively to the core inflation rate) is quite large by historical standards. Core inflation reaches a modest peak of 0.3% relative to basecase, before declining back towards the target rate. The dynamic pattern is very similar for the real exchange rate shock (see Figure 2.3), and we notice that the reaction to a 1% depreciation is very small (0.023 percentage points).

Finally, Figure 2.4 plots the evolution of the inflation rate following a change in the long-term inflation expectations (e.g. from an announced change in the inflation target). Inflation converges toward the new equilibrium relatively quickly, with 95% of the convergence achieved at the end of the 4^{th} year.

4 Full-model properties

To test the model as a full system, we simulate different shocks and analyze the impulse responses for the main variables. To facilitate comparisons with FRB/US and other models, aggregate demand dynamics are presented in terms of output gap. Note that we present shock-minus-control simulations, and that the following exogenous variables are kept equal to their baseline values, unless otherwise stated: commodity prices, stock market, inflation target, fiscal policy stance, and rest-of-world interest rates.

4.1 Demand shock

We start with a simple one-quarter shock of half a percentage point to the output gap (Figure 3.1, Appendix 3). The excess demand causes the inflation rate to depart from the target, and to reach a peak close to 0.30%. The increase in the Federal funds rate necessary to bring back the output gap to zero and inflation to its target is about 80 basis points above the baseline level. The real exchange rate appreciates by 1% before going back to its initial value as the real interest rate converges back to the basecase.

4.2 Interest rate shock

Our second simulation is concerned with a one-quarter, 100 basis-points exogenous increase in the Federal funds rate (see Figure 3.2). The tightening in monetary policy initially creates a situation of excess supply for four years. The inflation rate drops slightly for a longer period, with a trough at 0.18% below the target. In order to counter the slowdown in the economy, the monetary authorities lower the short-term interest rate below its equilibrium. The aggregate demand starts reacting to the easing in the stance of monetary policy about one year and a half after the shock, from the successive declines in the real interest rate and the inversion of the term structure. The cumulative loss of output over 10 years is 3.3 percentage points. As expected, the real exchange rate initially jumps up with the rise of the real interest rate.

The dynamics from this shock are very similar to those reported from a comparable exercise conducted with FRB/US (see Brayton and Tinsley, 1996). Under model-consistent expectations, the inflation rate for the Federal Reserve's model falls by 0.20 percentage points (similar to NAOMI/US), the output gap reaches -0.45% (-0.40% for NAOMI/US), and the long-term interest rate climbs to 0.25% (similar).

4.3 Price shock

We engineered a shock to inflation spread over three quarters (0.25%, 0.20%, and 0.15%). The inflation rate, after peaking at almost 0.35%, returns relatively quickly to the basecase, due to the weight on long-term expectations (see Figure 3.3). The output gap, after blipping shortly in positive grounds, reaches a modest trough of -0.09%. The nominal short-term interest rate rises at first, than decreases in order to invert the term spread and stimulate the economy, but the fluctuations are very small. No similar shock is available for FRB/US.

The rapid adjustment of inflation and the relatively low cost in lost output (1%) are due to the fact long-term inflation expectations are mostly unaffected by the price shock: economic agents see monetary policy as fully credible and believe core inflation will go back to its target quickly. Other assumptions could be made. For example, if we assume the target is half-credible (i.e. long-term expectations are a weighted average of the target and last period's inflation rate), the loss in output for the same price shock becomes 1.8%.

4.4 Reduction of inflation target

The graphs in Figure 3.4 and 3.5 show the responses of the main variables to a reduction of the inflation target by 1 percentage point.[19] As a first exercise, we consider the target as fully credible, i.e. long-term inflation expectations adjust instantly to the new announced target (see Figure 3.4). This announcement effect explains the immediate fall in the inflation rate. With full credibility, the inflation rate reaches its new steady state relatively quickly (2 years). The output gap falls to -0.40%, before heading back to zero, even if the nominal Federal funds rate does not increase relative to its initial equilibrium. This is because the long-term interest rate falls by more at the beginning (-0.80%, mainly from the expectations hypothesis), causing a positive yield spread for about 2 years. Note that the Federal funds rate 'overshoots' in order to fight the excess supply in the economy. The exchange rate exhibits a small, very temporary appreciation due to the positive real Fed rate, before depreciating to a trough of -0.6%.

The sacrifice ratio in terms of lost output is 4.6 over 10 years, within the 3.4-5.1 range obtained from FRB/US under the assumption of full credibility (see Bomfim *et al.*, 1997).[20] Also, if we refer to the results available from

[19] Here we make the assumption that the Federal Reserve follows an implicit inflation target. Note that this model is not primarily designed to study disinflation shocks. The presence of long-term expectations in the Phillips curve means that the monetary authorities could in fact achieve lower inflation without any output loss. To better handle this issue, improved characterization of the inflation expectations process would be necessary.

[20] The range is obtained by multiplying employment sacrifice ratio from FRB/US by Okun's Law coefficients generally cited, i.e. 2 and 3.

FRB/US for this type of shock (Brayton and Tinsley, 1996), it is interesting to note that the dynamics are very comparable: the inflation rate reaches its new target after 2 years; Federal funds rates drop from the beginning and overshoot; and long-term rates experience a one-quarter fall to -0.7% before continuing heading toward their equilibrium.

Figure 3.5 depicts a second, more realistic scenario. Inflation expectations do not change before the second year, and the new inflation target becomes fully credible by the 8th quarter; markets participants follow the same learning process, and long-term rates move accordingly; and finally the monetary authority establishes its credibility by first targeting the real short-term rate. This scenario yields more plausible impulse responses. Initially, the Federal Reserve tightens monetary policy in order to signal that it is committed to achieve its new target. Second, there is no announcement effect, and the inflation rate starts falling significantly only after one year. As a consequence of the learning process, the cumulative loss of output (sacrifice ratio) is now higher at 8%.

4.5 Stock market shock

As discussed previously, the stock market enters the IS curve as a second difference, i.e. changes in the growth rate of the S&P500 index have only a temporary effect on the output gap as long as they do not modify (or are not the consequence of a change in) the growth rate of real potential GDP. In this section, we illustrate the effects of a contraction in the stock market.

We draw a shock in which the quarter-over-quarter annualized growth rate of the S&P500 index is 20 percentage points lower than its basecase value for two quarters in a row. This pattern implies that the sum of $\Delta \dot{SP}500$ equals 0 over the simulation horizon.

Simulation results are presented in Figure 3.6. The output gap becomes negative following the lagged depressing effect of the fall in the S&P500, and reaches -0.25%, five quarters after the beginning of the shock. The forward-looking monetary authorities detect the future effect of the shock on inflation, and therefore lower the short-term interest rate right at the beginning of the exercise. The ease in the stance of monetary policy creates temporary excess demand after 2 years, before the model goes back to its steady state.[21] During the process, the inflation rate is only slightly affected, varying between -0.06% and 0.04% relative to basecase, and the reaction of the Federal funds rate is muted (total range of fluctuations of less than 25 basis points).

The shock in the stock market being very short, we assumed potential growth rate was not modified as a result of this disturbance. As is obvious from Figure 3.6, the combination of the temporary shock and actions from monetary policy means that there is no significant net loss in output. Finally, evidence from the shocks performed in Reifschneider *et al.* (1999) for

[21]The positive output gap is entirely created by the effect of monetary policy. Recall from section 3.1.4 that from the point of view of the IS curve alone, a *temporary* negative change in the rate of return on the S&P500 implies a temporary negative (never positive) output gap, and a net loss of output.

FRB/US appears to give relatively similar results in terms of magnitude, but with longer-lasting effects. It is however difficult to draw clear comparisons, since the stock market is treated as endogenous in the Fed's model, while the monetary policy reaction is exogenously determined for this particular shock, and the expectations formation process is somewhat unclear. Nevertheless, impulse responses from NAOMI/US do not appear to be off-key.

4.6 Real exchange rate shock

Figure 3.7 presents the evolution of the main variables following a one-quarter shock to the real exchange rate resulting in a depreciation relative to basecase of 5%. The initial depreciation stimulates output as well as inflation. The monetary authorities react accordingly, and rise the Fed funds rate by 20 basis points. Later, higher interest rates and the return of the real exchange rate to its basecase value have a negative impact on the economy, before variables go back to their respective initial levels.

To complete the analysis of exchange rate dynamics, we look at the effect of a one-time, 5% permanent depreciation of the U.S. currency (Figure 3.8). In this case, z is simply treated as exogenous. As expected, this shock has an expansionary effect, causing the output gap and inflation to peak at 0.5% and 0.25% respectively. In response to the excess demand and inflationary pressures, the Federal funds rate is increased by 70 basis points, 3 years after the beginning of the shock.

The only relatively similar simulation we have from FRB/US is reported in Reifschneider *et al.*. The description of the shock being very tenuous, comparisons should be taken with caution. Nevertheless, our reduced-form system produces interesting results. The reaction of output gap under the Taylor-rule case seems almost identical to results from NAOMI/US (peak at 1.2 for comparable 10% permanent depreciation of z); the pass-through effects of exchange rate on output are in line with FRB/US'.[22] The impact on inflation (coming at 90% from the increase in the output gap) and Fed funds rates is however higher in NAOMI/US.

5 Forecast performance

This section evaluates the forecasting performance of the two main equations of the model, the aggregate demand function and the Phillips curve. Because of sample considerations, we do not conduct a pure out-of-sample forecast exercise. While a new dynamic forecast is performed every quarter, equations are not re-estimated. We use the coefficient values presented in Section 2.

Table 5 presents the results for the IS curve, over two different samples. In both cases, the Theil's U statistic shows that our specification improves the forecast performance significantly over a no-change model, with an increase in accuracy ranging from 40% to 54%. The fact the RMSE measure is not increasing with the forecast horizon is due in a large part to the importance of

[22]We tried the same shock by replacing our monetary rule by the Taylor rule, and obtained similar results.

the exogenous, *ex post* potential growth term in our specification. However, potential growth as estimated today relies heavily on the historical values of real GDP growth, the same variable we are trying to forecast. This problem clearly implies a downward bias in the RMSE statistics. It would be in fact more informative to compare these results with those from a forecast in which the predicted values are simply equal to *ex post* potential growth.[23] This way, we can isolate the contribution of the other explanatory variables. This is presented in the last column of Table 5. The ratios of the RMSEs show that the forecasting accuracy is increased by 25% on a one-quarter ahead basis, 21% for an horizon of one year, 17% for 2 years, but only 7% for 3-years ahead forecasts.

[23]Even better, a real-time exercise could use the predicted values for potential output growth as assessed at each quarter in the past, therefore not including in the analysis subsequent revisions made to our assessment of the potential of the economy. Also, the fact we use realized foreign output growth improves only marginally the forecast performance.

Table 5
Forecasting performance, IS curve
Real GDP growth, Q/Q rate

	1980Q1-2000Q4 Eqn (10)			1990Q1-2000Q4 Eqn (10)			1980Q1-2000Q4 $\Delta y = \Delta y^P$	
Horizon	MAE	$RMSE_1$	U	MAE	$RMSE_2$	U	$RMSE_3$	U^*
1	0.42	0.54	0.60	0.41	0.48	0.69	0.72	0.75
2	0.42	0.52	0.55	0.40	0.49	0.72	0.67	0.78
4	0.42	0.52	0.49	0.38	0.45	0.61	0.66	0.79
8	0.41	0.50	0.46	0.36	0.43	0.59	0.60	0.83
12	0.41	0.50	0.46	0.38	0.45	0.58	0.54	0.93

Note: Estimation sample is 1979Q1-2000Q4; U is Theil's U; U^* is the ratio of $RMSE_1$ and $RMSE_3$

A similar exercise is conducted for the Phillips curve, and results are reported in Table 6. Because of the inclusion of long-term inflation expectations as exogenous variable, the specification captures the disinflation of the 1990s even for longer horizons (the Theil's U statistic decreases with the forecast horizon over the 1985Q1-2000Q4 sample). In order to draw better comparisons, we applied the same method as the one for the IS curve: we computed a forecast in which the predicted values for inflation are equal to long-term expectations, and calculated the ratio of the RMSEs, denoted by U^*. Compared to this new benchmark, our Phillips curve increases forecast accuracy by 26% to 32%. Also, it continues to outperform a no-change model even over the recent sample, characterized by a more stable inflation rate.

Table 6
Forecasting performance, Phillips curve
CPIXFE inflation, Q/Q annual rate

	1985Q1-2000Q4				1993Q1-2000Q4		
Horizon	MAE	$RMSE_1$	Theil's U	U^*	MAE	$RMSE_2$	Theil's U
1	0.34	0.42	0.65	0.69	0.26	0.32	0.79
2	0.34	0.42	0.76	0.68	0.27	0.32	0.72
4	0.34	0.46	0.62	0.74	0.24	0.30	0.60
8	0.33	0.45	0.48	0.70	0.20	0.25	0.57
12	0.33	0.44	0.39	0.68	0.20	0.27	0.44

Note: Estimation sample is 1985Q1-2000Q4. U^* is the ratio of $RMSE_1$ and the RMSE obtained from a model in which forecast values are equal to long-term inflation expectations.

6 Conclusion

In this paper, we have introduced NAOMI/US, a small-scale model of the U.S. economy. This model is built around three estimated equations (IS curve, Phillips curve, monetary rule), is consistent with NAOMI/Canada, incorporates forward-looking behaviour and exhibits a meaningful steady-state. Simulations of various shocks to NAOMI/US produce credible dynamics, consistent with those drawn from a larger, thoroughly tested macroeconomic model (FRB/US). Because of its parsimony, NAOMI/US presents the advantage of being highly tractable and easy to execute.

Despite our satisfaction with the current state of the model, time and experience will likely provide the ultimate and most important test. Once again, parsimony will prove to be an asset in future development of the model, allowing us to modify one or a few equations without altering seriously the desirable properties of the original specification. Ultimately, we believe that future versions of the model will remain based on a similar structure, providing a useful tool for forecasting and analyzing the U.S. economy.

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Appendix 1

Long-term interest rate specification

First, recall the hybrid specification chosen to model the long-term interest rate:

$$R_t^l = \lambda R_t^s + (1-\lambda) E_t \sum_{i=0}^{39} w_i R_{t+i}^s \tag{18}$$

We abandon the term premium for now, since we want to focus on the main relationship between short- and long-term interest rates. By taking the expectations at time t of the one-period lead of equation (18), we get:

$$E_t R_{t+1}^l = \lambda E_t R_{t+1}^s + (1-\lambda) E_t \sum_{i=0}^{39} w_i R_{t+1+i}^s \tag{19}$$

We then subtract (18) from (19) and obtain an expression in first difference:

$$E_t \Delta R_{t+1}^l = \lambda E_t \Delta R_{t+1}^s + (1-\lambda) \sum_{i=0}^{39} \omega_i E_t \Delta R_{t+1+i}^s \tag{20}$$

In order to identify the parameter λ, we describe ΔR^s as an AR(1) process such that $E_t \Delta R_{t+1+i}^s = \rho^i E_t \Delta R_{t+1}^s$. The weights are defined in the usual manner, i.e. $\omega_i = \beta^i(1-\beta)/(1-\beta^n)$ where $\beta = 0.98$ (discount rate of 2% quarterly, 8% annually) and corresponds to the average yield to maturity on a coupon bond. The following relationship follows:

$$\begin{aligned} \sum_{i=0}^{39} \omega_i E_t \Delta R_{t+1+i}^s &= E_t \Delta R_{t+1}^s \frac{(1-\beta)}{(1-\beta^{40})} \sum_{i=0}^{39} \beta^i \rho^i \\ &= \phi E_t \Delta R_{t+1}^s \end{aligned} \tag{21}$$

where ρ is the autocorrelation coefficient between ΔR_t^s and ΔR_{t+1}^s.

By using historical data for long rate and Federal funds rate, we find that historically $\Delta R_t^l = 0.28 \Delta R_t^s$. With this result, we can identify λ:

$$\begin{aligned} E_t \Delta R_{t+1}^l &= \lambda E_t \Delta R_{t+1}^s + (1-\lambda)\phi E_t \Delta R_{t+1}^s \\ \lambda + (1-\lambda)\phi &= 0.28 \\ \lambda &= \frac{0.28-\phi}{1-\phi} \end{aligned} \tag{22}$$

From our calculation of the parameter ϕ, we obtain $\lambda = 0.25$. That is, most of the short-term variation in long rates will come from the contemporaneous change in Federal funds rates. This specification appears to give dynamics closer to what can be observed from actual data.

Figure 1 : Shocks to aggregate demand / Single-equation properties

Output growth, quarter-over-quarter rates (unless otherwise stated)

Figure 1.1: Short-term interest rate shock

1 p.p. permanent increase

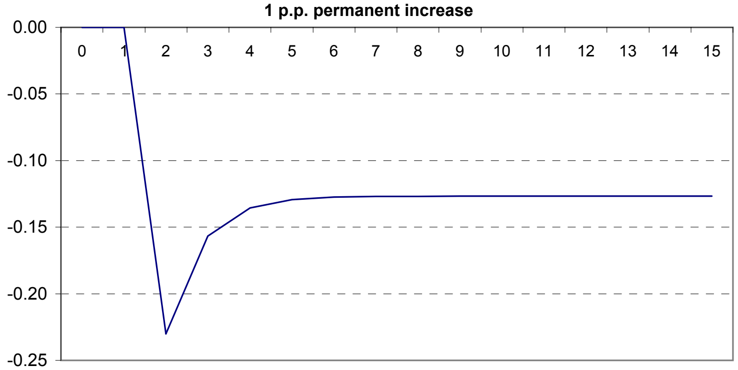

Figure 1.2: Real exchange rate shock

1 % permanent appreciation

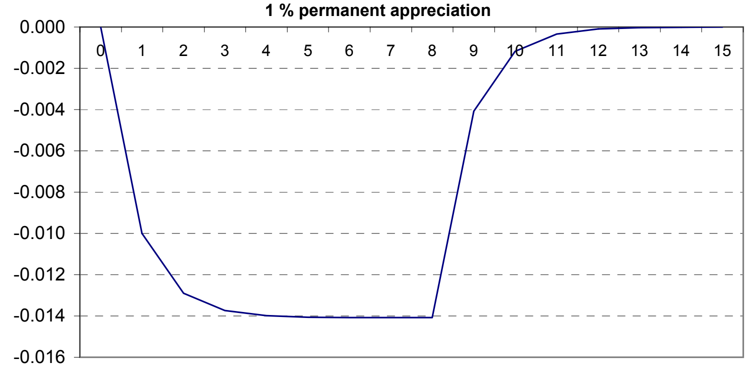

Figure 1.3: Shock to potential output growth

1 p.p. permanent increase in growth rate

Solid is real GDP, broken is potential GDP

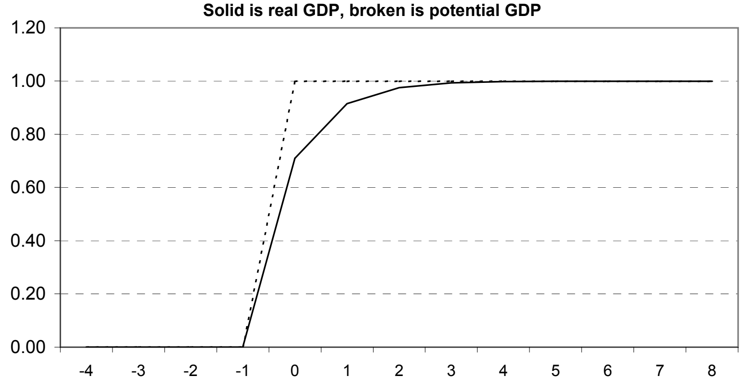

Figure 1.4: Shock to level of potential output

1 % and -0.5 % shocks (in logs)

Solid is real GDP, broken is potential GDP

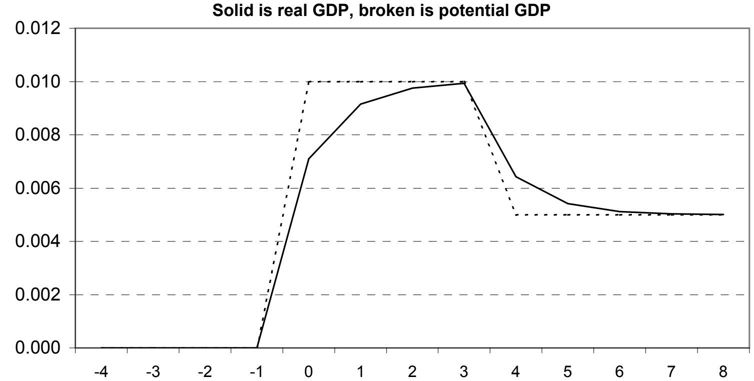

Figure 1.5: Fiscal shock #1

Permanent increase in government spending of 1% of potential output

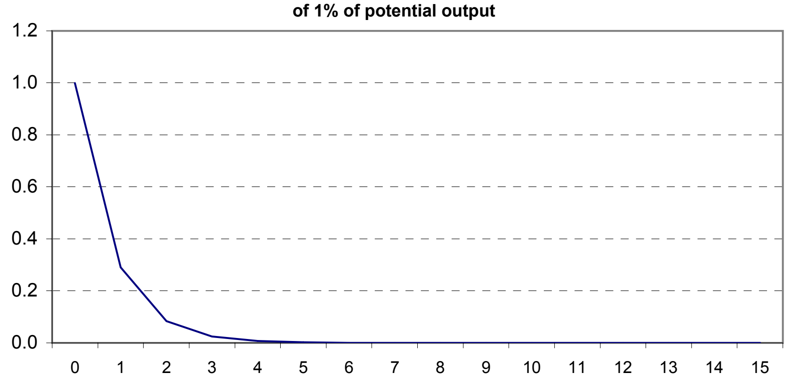

Figure 1.6: Fiscal shock #2

Permanent increase of government tax receipts of 1% of potential output

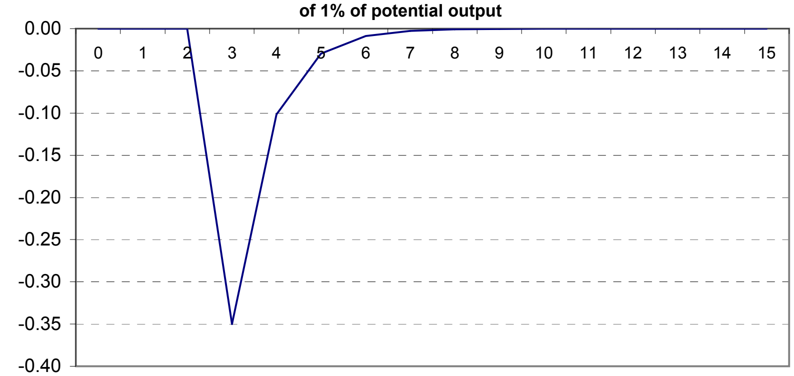

Figure 1.7: Stock market shock

5 p.p. drop in return on S&P500 (q/q ann.)

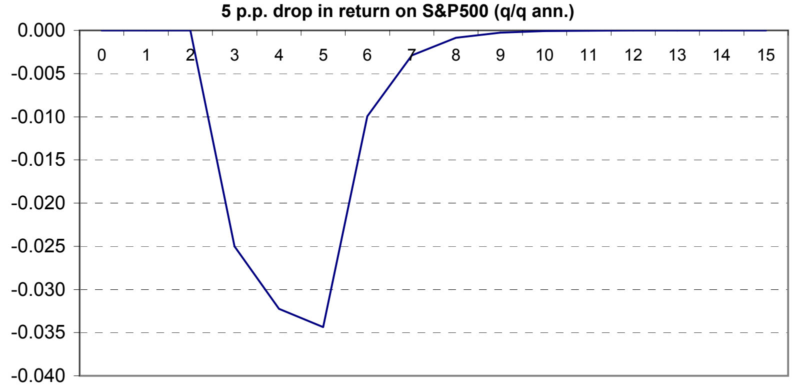

Figure 2 : Shocks to Philips curve / Single-equation properties

Core inflation, quarter-over-quarter annualized rates

Figure 2.1: Output gap shock

1 p.p. temporary increase (4 quarters)

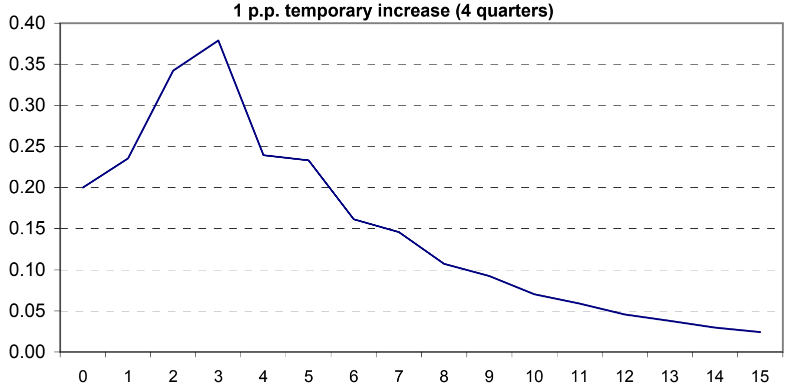

Figure 2.2: Producer Price Index shock

5 p.p. temporary increase in PPI inflation (4 quarters)

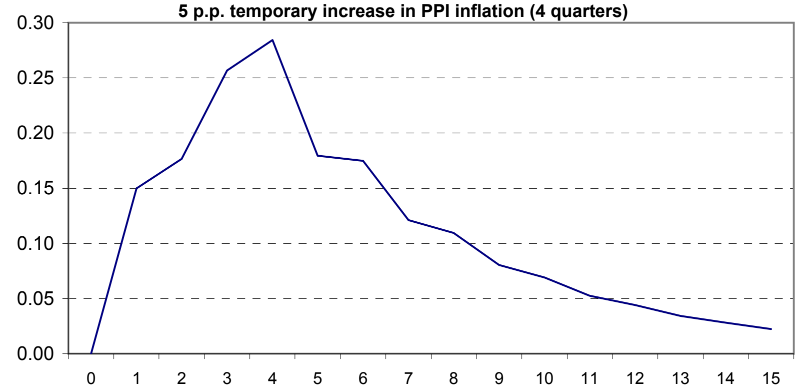

Figure 2.3: Real exchange rate shock

1 % permanent depreciation

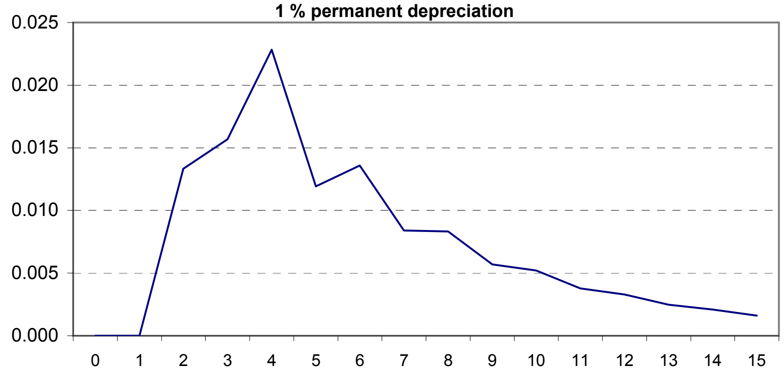

Figure 2.4: Long-term inflation expectations shock

1 p.p. permanent increase in long-term expected inflation

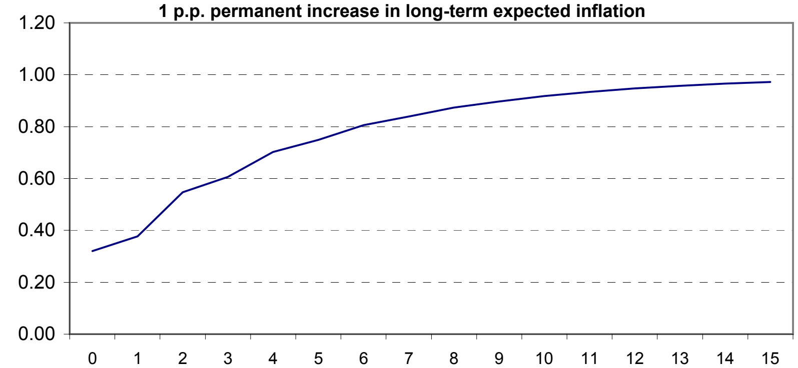

Figure 3.1 : Demand shock

One-quarter shock of 0.5 p.p to the output gap

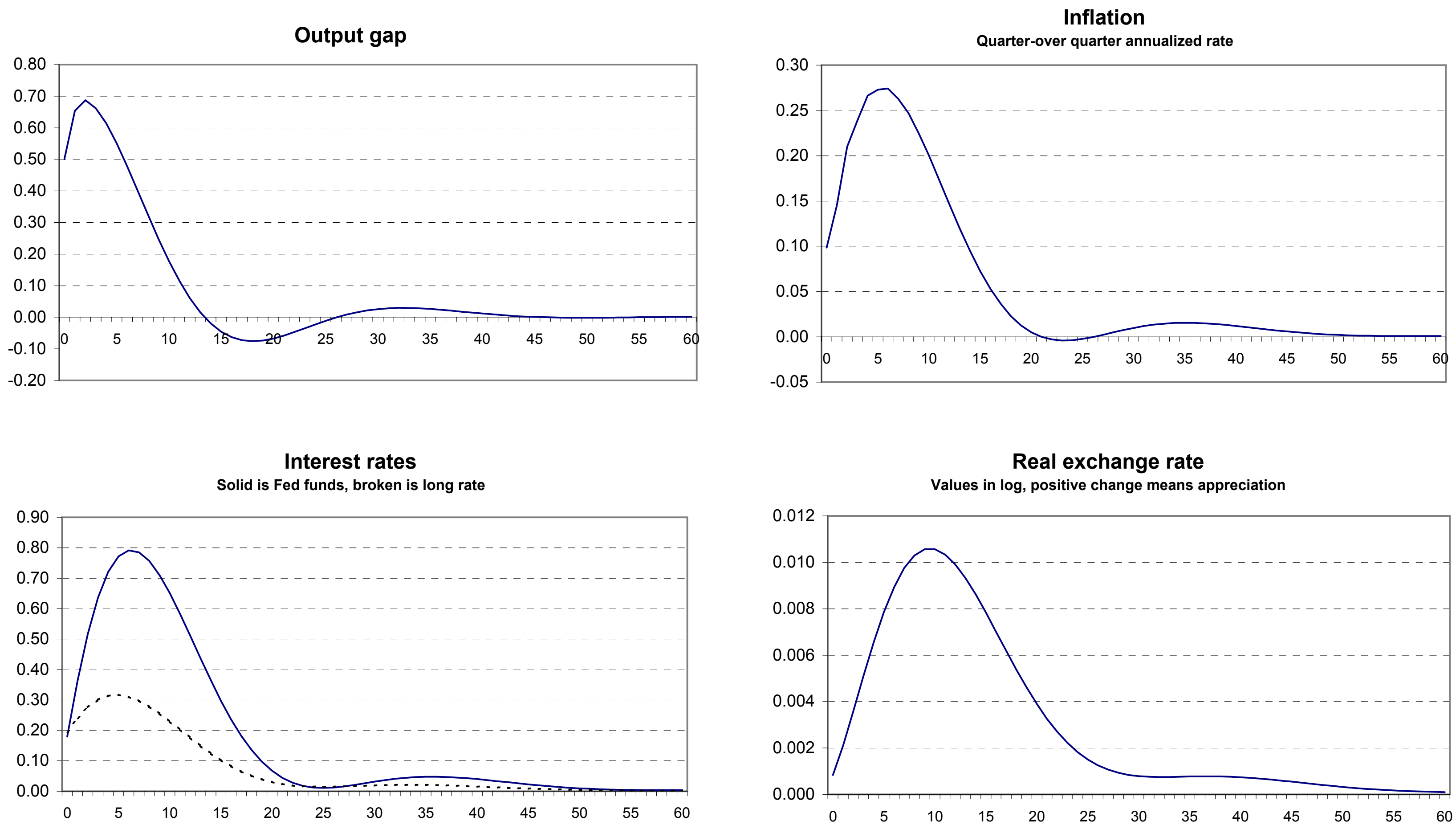

Figure 3.2 : Interest rate shock

One-quarter shock of 100 b.p. to the Federal funds rate

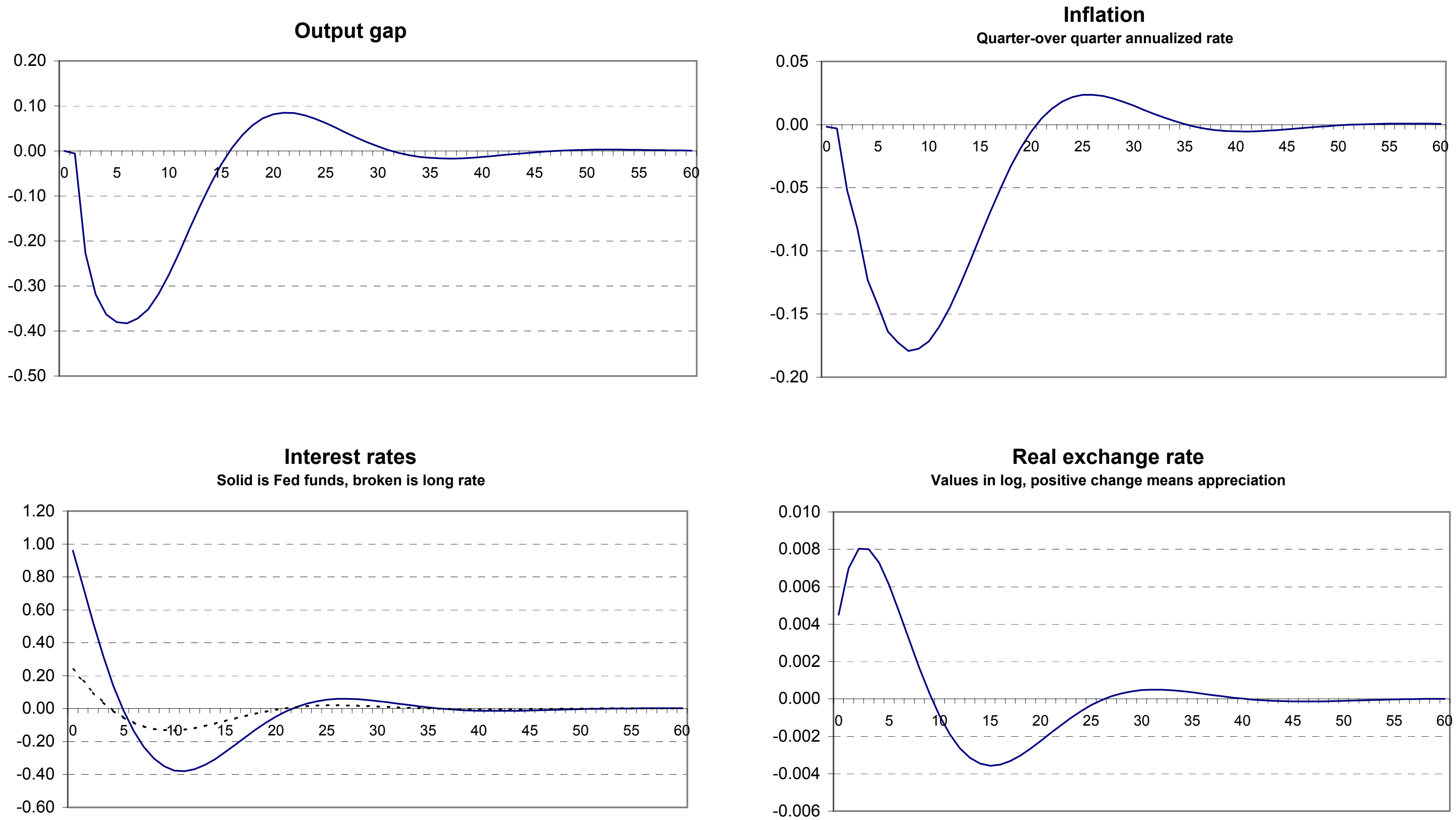

Figure 3.3 : Price shock

Temporary shock to the inflation rate

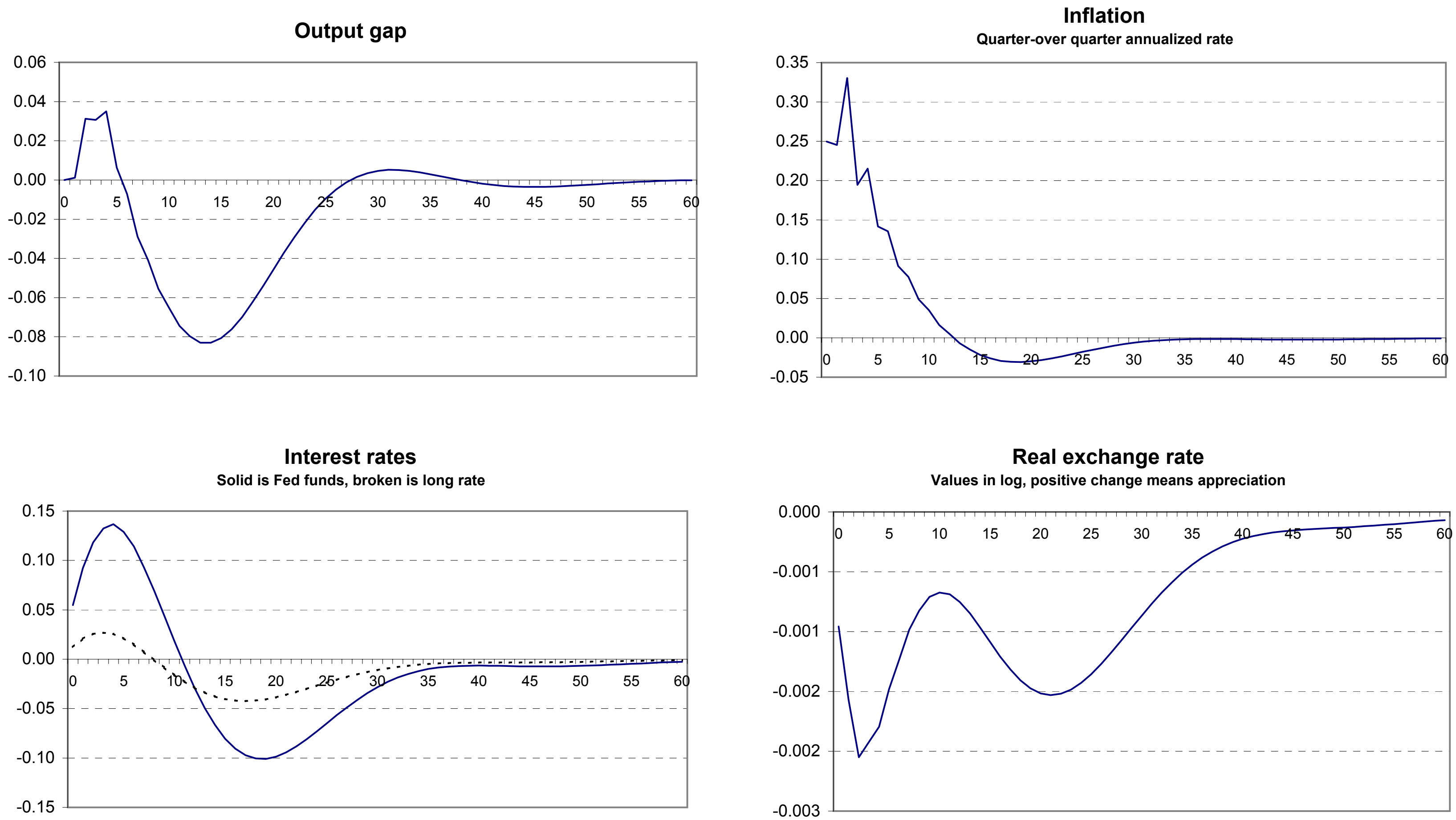

Figure 3.4 : Disinflation shock #1

1 p.p. reduction of the inflation target, fully credible

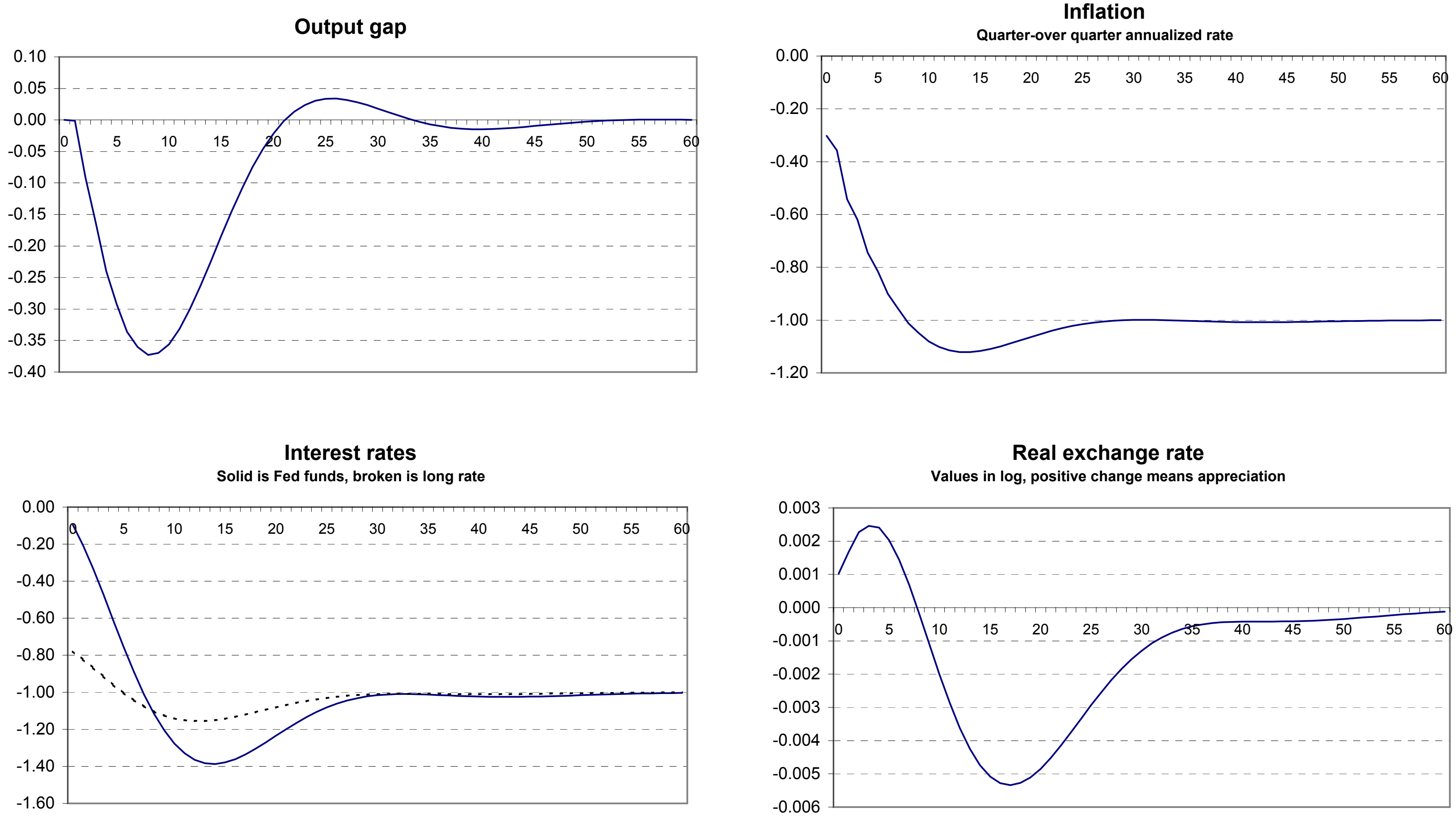

Figure 3.5 : Disinflation shock #2

1 p.p. reduction of the inflation target, fully credible after 2 years

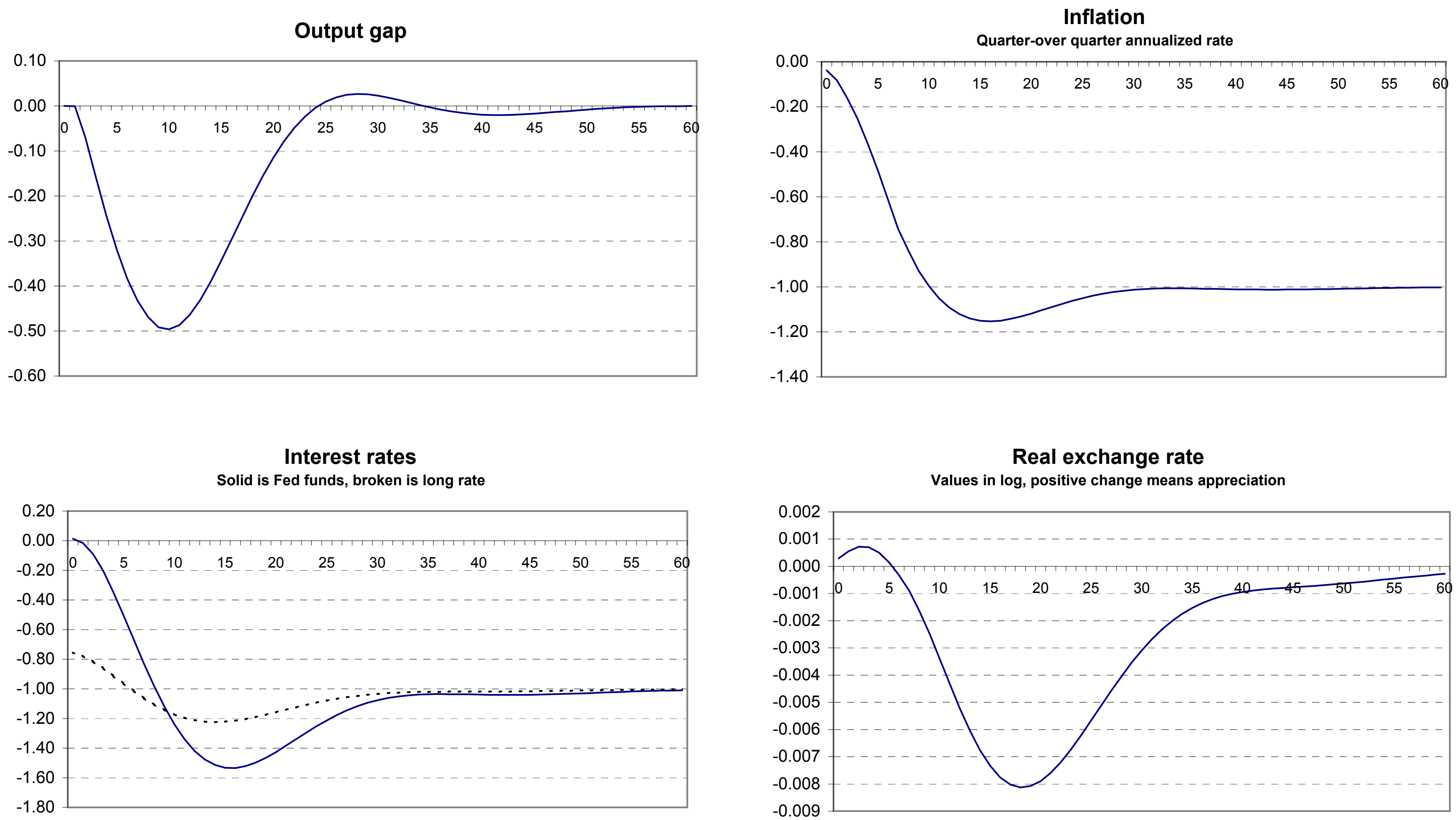

Figure 3.6 : Stock market shock

Two-quarter shock of -20 p.p. to the growth rate of the S&P500 index

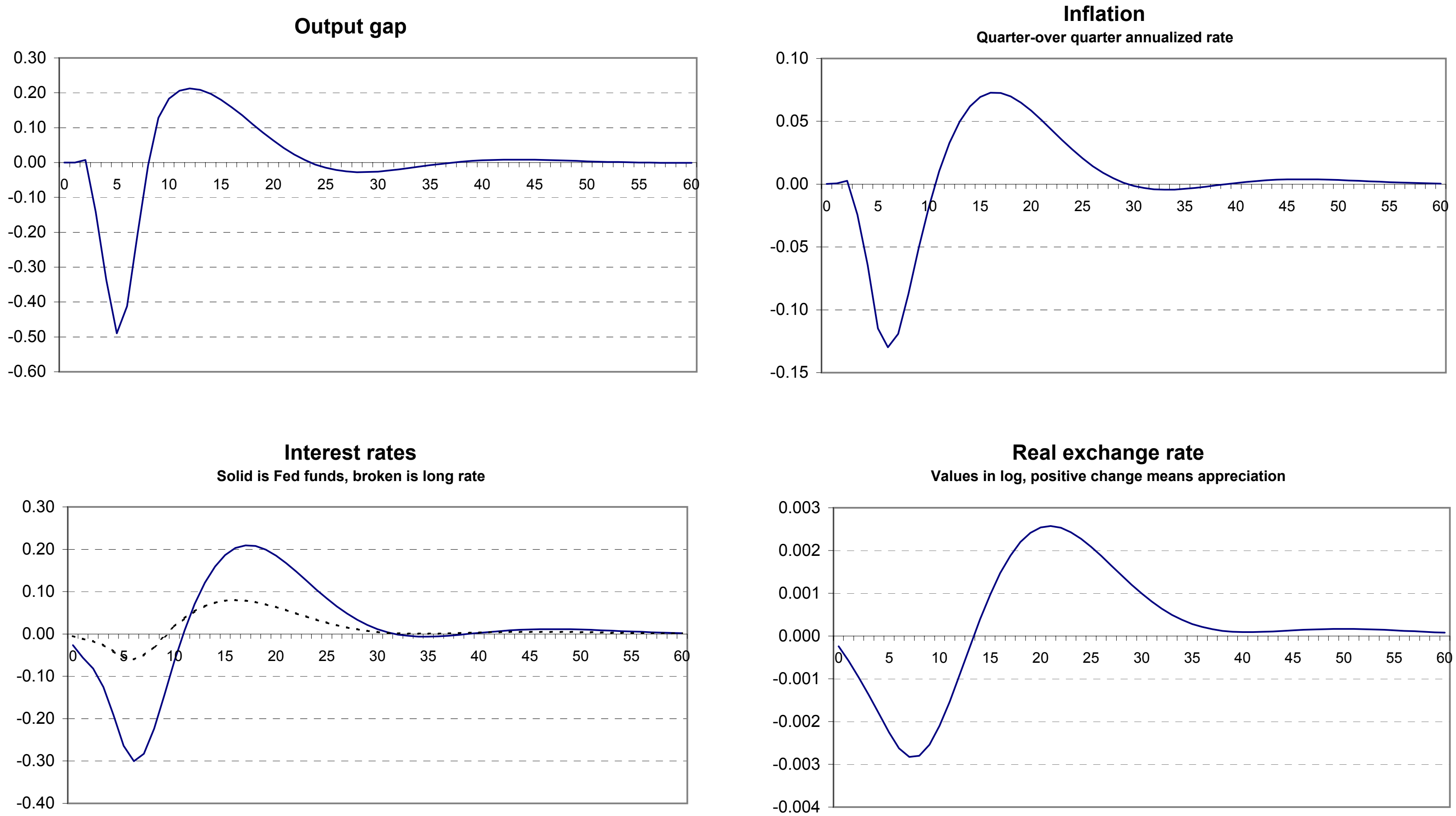

Figure 3.7 : Real exchange rate shock (temporary)

Temporary depreciation of 5 %

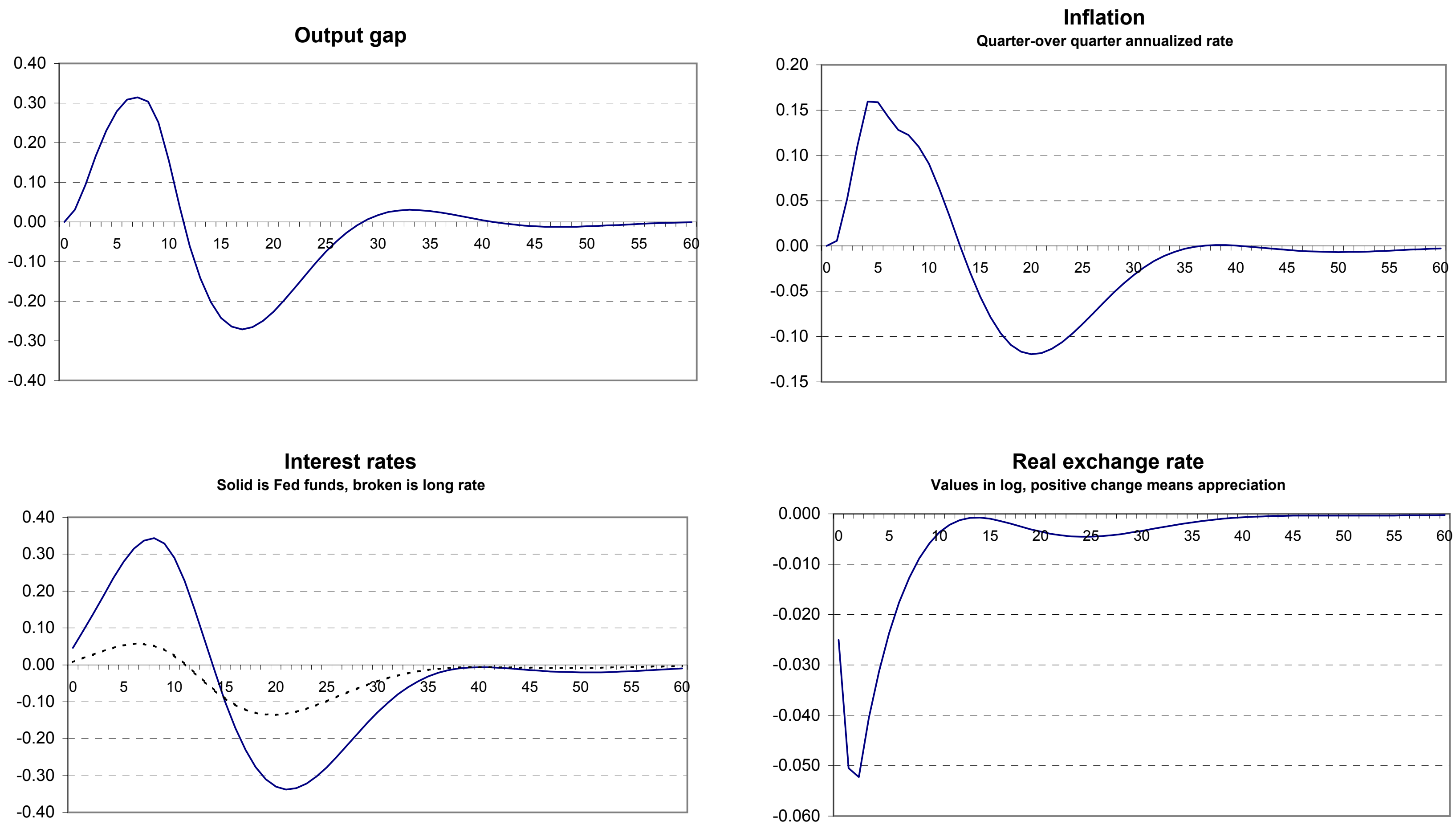

Figure 3.8 : Real exchange rate shock (permanent)

Permanent depreciation of 5 %

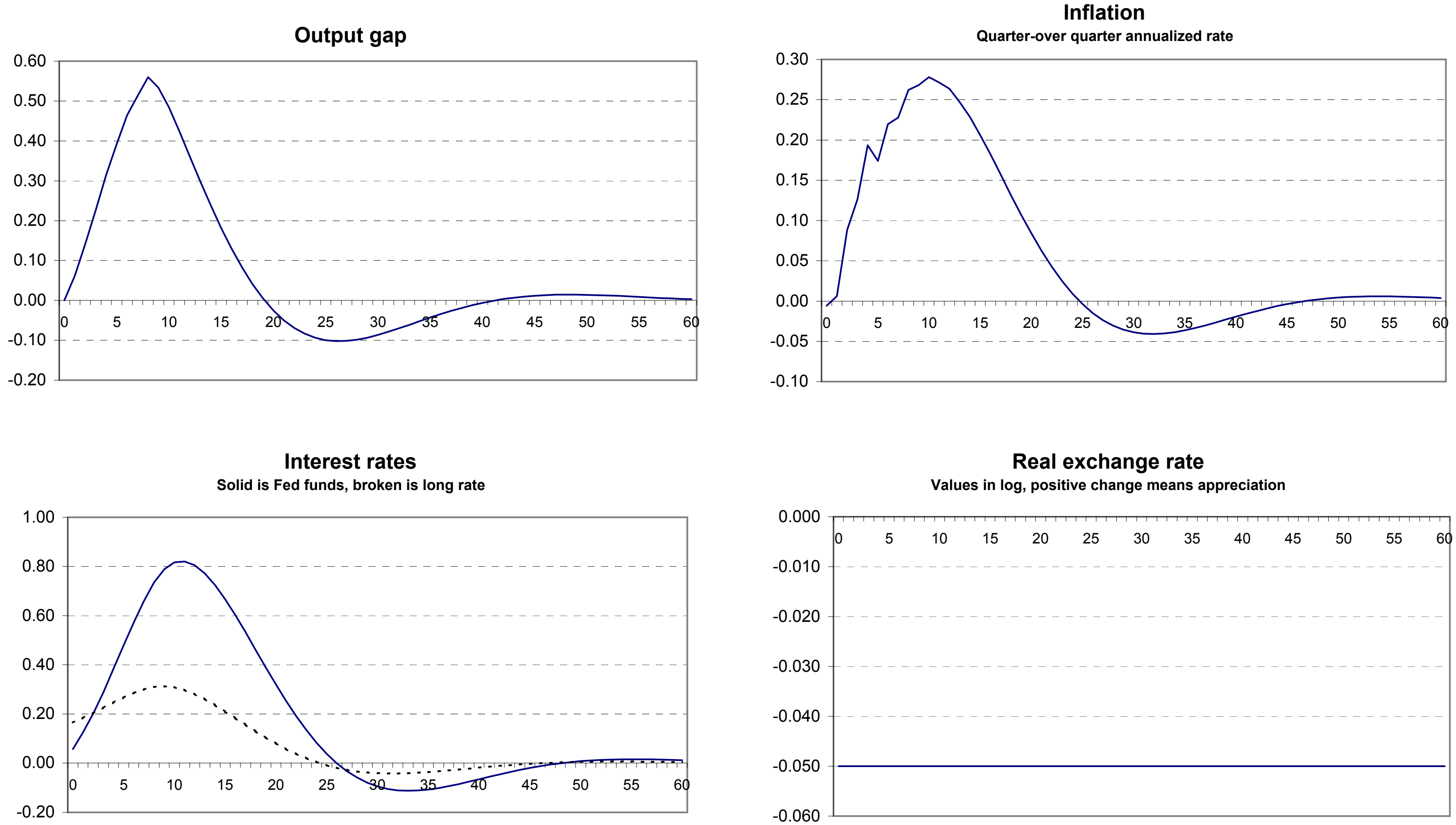

Figure 3.9 : Shock to potential growth