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Estimating Structural Breaks in the Volatility of Canadian Output Growth

by

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Abstract

The presence of a structural break in the volatility of GDP growth has important implications for economic modelling and econometric techniques. Recently, McConnell and Quiros (1997, 2000) have identified a structural decline in the U.S. GDP growth volatility in 1984, and they attributed the cause of the decline to the reduction in business inventory investment growth volatility. They have also reported a similar break in Canadian GDP growth volatility occurring in 1991. In this paper we build on the work of McConnell and Quiros but use a more flexible form of Markov-switching and the likelihood framework of the Andrews-Ploberger test to identify structural breaks in the Canadian GDP growth volatility. Our estimation results show that the volatility of GDP growth in Canada has shifted to a lower regime in 1987, not in 1991 as reported by McConnell and Quiros. When applying our methodology to U.S. data, we obtain the same break date as McConnell and Quiros. However, we have detected an additional break in the regression coefficients of the AR process occurring at the end of 1991. Our attempts to identify the source of the break in the Canadian GDP growth volatility yield mixed results. We found structural declines in the volatility of many GDP components' contributions to growth, but none of the break dates is the same as that of the aggregate GDP. However, our finding of a break in the volatility of business inventory's contribution to growth in 1984Q1 is consistent with the results of McConnell and Quiros for the U.S., suggesting that the structural change in inventory management might also have played a role in reducing GDP growth volatility in Canada.

Résumé

La présence d'un bris structurel dans la volatilité de la croissance du PIB a d'importantes répercussions sur la modélisation et les techniques économétriques utilisées. McConnell et Quiros (1997, 2000) ont récemment décelé un déclin permanent de la volatilité du PIB aux États-Unis à partir de 1984, et ils l'ont imputé à une baisse de la volatilité de la croissance de l'investissement en inventaires. Ils ont également signalé un bris structurel semblable dans la volatilité de la croissance du PIB au Canada en 1991. La présente étude s'appuie sur les travaux de McConnell et Quiros mais utilise une forme plus souple des changements de régime de Markov ainsi que l'approche du maximum de vraisemblance du test Andrews-Ploberger pour identifier des bris structurels dans la volatilité de la croissance du PIB au Canada. Les résultats de nos estimations démontrent que la volatilité du PIB au Canada est passée à un régime plus faible en 1987, et non pas en 1991 tel que signalé par McConnell et Quiros. Lorsque nous appliquons notre méthodologie aux données américaines, par contre, nous obtenons la même date du bris structurel que McConnell et Quiros. Nous avons toutefois décelé un bris additionnel dans les coefficients de régression du processus AR à la fin de 1991. Nos efforts visant à identifier la source du bris structurel dans la volatilité du PIB canadien donnent des résultats mitigés. Nous avons trouvé une baisse structurelle dans la volatilité de la contribution à la croissance pour plusieurs composantes du PIB, mais aucune date de bris ne

correspond à celle du PIB lui-même. Toutefois, notre découverte d'un bris structurel dans la volatilité de la contribution à la croissance de l'investissement en inventaires au 1er trimestre de 1984 est compatible avec les résultats de McConnell et Quiros pour les États-Unis, ce qui indique que le changement structurel dans la gestion des inventaires a peut être également contribué à atténuer la volatilité dans la croissance du PIB au Canada.

1. Introduction and Summary

Output growth volatilities in the U.S. and Canada appear to have stabilized considerably in recent years. For example, the standard deviation of quarterly growth rates of U.S. real GDP from 1950 through 1983 was more than twice as large as that for 1984 through 1999, and the standard deviation of Canadian real GDP growth has declined by more than half since 1987 compared to that over the period from 1961 to 1986. McConnell and Perez-Quiros (1997, 2000) examined this observation of reduced volatility empirically in two recent studies. They have uncovered a structural break in the volatility of U.S. GDP growth occurring in the first quarter of 1984 and a break in the Canadian GDP growth volatility occurring in the second quarter of 1991.

The presence of a structural break in the volatility of GDP growth has important implications for economic modelling and econometric techniques. Specifically, a volatility break means that linear models of GDP growth that span the period over which the break occurs are misspecified. Using non-linear models such as Markov-switching models, however, does not necessarily guarantee improvements over linear models if the break is not accounted for explicitly. In addition, the presence of a break will also invalidate the use of some of the commonly used test statistics in hypothesis testing. For example, a structural break in volatility means that test statistics requiring constant underlying variance, such as F-test of linear constraints and Chow's test of structural stability in the classical least squares theory, are incorrect. Alternative test statistics must be used in order to account for the change in the underlying variance.

The presence of a break in the GDP growth volatility has led to a substantial literature investigating the causes of the break. By analysing disaggregated U.S. data, McConnell and Quiros (1997, 2000) attribute the source of the reduction in GDP growth volatility to a decline in the volatility of durable goods production. They also show that the break in durable goods volatility is roughly consistent with a break in the proportion of durable output accounted for by inventory investment. Consequently, they argue that the break in growth volatility results from the implementation of just-in-time inventory-management techniques. Although Kahn, McConnell and Quiros (2001) have reinforced this explanation, some controversies still exist. Alternative explanations, such as more efficient monetary policy, have been proposed by Taylor (2000) and have gained support from Kim, Nelson and Piger (2001) and Blanchard and Simon (2001).

Applying the same method of McConnell and Quiros (1997) to Canadian constant-1992 dollar Laspeyres data, Debs (2001) discovers a structural break in the real output growth volatility occurring in the first quarter of 1991. This is similar to the finding of McConnell and Quiros (1997) on Canadian GDP growth volatility. Based on disaggregated data, Debs attributes this break to both a break in the growth volatility of investment in residential structures and personal expenditures on goods. He suggests that the move towards a more service-oriented economy, improved inventory management, and a change in monetary policy are possible explanations for the break in the data.

In this paper, we build on the research of McConnell and Quiros (1997, 2000) but using quite different test procedures and estimation techniques to analyse Canadian GDP growth volatility. Specifically, we use a more flexible form of Markov-switching model than McConnell and Quiros (henceforth referred to as MQ) to investigate the probability of a change in the GDP growth process. Also, instead of using generalized method of moments (GMM) to detect structural breaks as in MQ, we use the likelihood function framework of the Andrews-Ploberger (1994) tests together with sequential maximum likelihood estimation to identify break dates. Our purpose is to re-estimate the break dates in the volatility of Canadian real GDP growth using our methods.

Using Fisher chained data (with 1997 as reference year), we found a structural decline in the volatility of real GDP growth in Canada occurring in the first quarter of 1987, not in 1991 as reported in McConnell and Quiros (1997) and Debs (2001). This discrepancy in the break date is not caused by the use of different data types (Fisher chained data in our analysis versus constant-1992 dollar Laspeyres data by MQ and Debs). Using constant-1992 dollar Laspeyres data, Liu and Painchaud (2002) have also identified a break in the Canadian GDP growth volatility occurring in the first quarter of 1987. Rather, the cause of the discrepancy in the break date is caused by differences in estimation techniques and test procedures.

We have applied our methodology to U.S. data to verify the break date. Our results confirm the break date in the U.S. GDP growth volatility as obtained by MQ. However, we have also detected the presence of another structural break that is caused by the break in regression parameters in the fourth quarter of 1991. The fact that this break was missed by MQ shows the efficiency of our procedures.

We have tried to identify possible sources of the break in Canadian real GDP growth volatility by examining the volatility of its components' growth contributions. Again, contrary to Debs' results, we are not able to identify exactly a specific final demand component that is at the source of the break. We found structural declines in the growth volatility of many GDP components, but none of the break dates is the same as that of the aggregate GDP. Our results suggest that breaks in the covariance between components might have played an important role in reducing aggregate GDP growth volatility. However, our finding of a decline in the volatility of business inventory investment's contribution to GDP growth in the first quarter of 1984 is consistent with the results of MQ for the U.S. data, suggesting that structural changes in inventory management might also have played a role in reducing GDP growth volatility in Canada.

2. Methodology

We use a modified version of the MQ methodology to test for structural breaks in the Canadian real GDP growth volatility. The method comprises of the following two steps:

1. Use Markov-switching processes to estimate the probability of a change in the mean and variance of GDP growth.

2. Test formally for a structural break in these moments using the Andrews-Ploberger test if the Markov-switching processes indicate high probability of such occurrence.

Step 1: The estimation of Markov-switching models

Following MQ, we first incorporate the real GDP growth process in Markov-switching models to estimate the probability of a change in the pattern. In these models, real GDP growth is modelled by an autoregressive (AR) process, and volatility in GDP growth is measured by the variance of the residual term from the AR process, and testing for structural breaks in GDP volatility amounts to testing for breaks in the variance (or equivalently, the standard deviation) of this residual term.

In MQ's studies, they employ a modified version of Hamilton's (1989) autoregressive Markov-switching model that allows the mean and the residual variance to switch across regimes but imposes fixed AR coefficients. Specifically, they use the AR process of

$$y_t - \mu_{s(t)} = \phi(y_{t-1} - \mu_{s(t-1)}) + \varepsilon_{s(t)} \qquad (2.1)$$

where y is real GDP growth, μ_s is the state-dependant mean of GDP growth, ϕ is the AR coefficient, and ε_s is the state-dependant error term. Note that although both the mean and the variance in equation (2.1) are state dependent, the AR coefficient is fixed.[1]

[1] In McConnell and Quiros's 1997 paper, both the mean and variance follow the same states; in their 2000 paper, the mean and variance follow separate states.

One of Hansen's (1992) criticisms of the Hamilton model is that imposing constant AR coefficients *a priori* may bias estimation results. Following Hansen's suggestion, we have modified MQ's estimation techniques by allowing the coefficients of the AR process to vary across regimes in addition to the mean and the residual variance. By using a more general model in which no restrictions are placed on the mean, variance, and the AR coefficients, we have allowed for the possibility that output growth could follow completely different processes in different regimes.

Step 2: Testing for structural breaks using the Andrews-Ploberger test (1994)

Testing for structural breaks with unknown break dates had long been a difficult problem in empirical work because conventional test statistics are not applicable under these circumstances. The reason is that when the break date is unknown, the parameter of the break point appears only under the alternative (that is, when there is a break) but not under the null of no break and hence it is an unidentified nuisance parameter under the null. This implies that the conventional Lagrange multiplier (LM), likelihood ratio (LR), and Wald (W) tests do not have standard asymptotic distribution in these non-standard problems. Andrews and Ploberger (1994) resolve this problem by proposing optimal tests with an average exponential form (Exp-LM, Exp-W, and Exp-LR) within the likelihood function framework.[2] These exponential tests have the greatest asymptotic power against the local alternatives in the class of all tests of asymptotic significance level α.

Andrews and Ploberger suggest two simple limiting forms for empirical use. The two limiting forms for the average exponential LM test statistics[3] can be expressed as:

[2] See Liu, Y., "Testing for Structural Change with Unknown Break Point: Andrews-Ploberger Optimal Tests", Economic Analysis and Forecasting Division, Department of Finance, March 2001, for details.

[3] The limiting exponential W and LR statistics, though not shown here, are defined analogously.

$$aveLM_T = \frac{1}{T_2 - T_1 + 1} \sum_{t=T_1}^{T_2} LM_T(\tfrac{t}{T}) \; ;$$

$$expLM_T = \ln(\frac{1}{T_2 - T_1 + 1} \sum_{t=T_1}^{T_2} \exp(\frac{1}{2} LM_T(\tfrac{t}{T})))$$

where $T_1 = [T \cdot \pi_0]$, $T_2 = [T \cdot (1-\pi_0)]$, and $\pi_0 \in (0,1)$[4]. T is the sample size, and $LM_T(\tfrac{t}{T})$ is the conventional Lagrange multiplier test for parameter instability when the break point t is known. The "*average LM*" statistics $aveLM_T$ is designed for testing the alternatives that are very close to the null hypothesis, while the $expLM_T$ is for testing against more distant alternatives.

Note that when applying the likelihood function-based Andrews-Ploberger test, one may face a problem that in small sample (even in large sample for some cases such as testing for structural break in the variance) the Hessian matrix (second order partial derivative of log likelihood function) that is used to calculate $LM_T(\pi)$ or $W_T(\pi)$ is not necessarily negative definite for some values of π so that the resulting $LM_T(\pi)$ or $W_T(\pi)$ can be negative. This can result in very small average test statistics for $aveLM_T$ or $aveW_T$ even if there is a strong break in the series, or in some cases even negative values for the average test statistics $aveLM_T$ or $aveW_T$. In order to avoid this problem, we have replaced the negative Hessian matrix (- $D^2 l_T(\theta,\pi)$) in our study by the outer product of the information matrix ($\sum_{t=1}^{T} \frac{\partial \log(f_t(\theta,\pi)}{\partial \theta} \cdot \frac{\partial \log(f_t(\theta,\pi)}{\partial \theta'}$). This modification does not change the asymptotic properties of the Andrews-Ploberger tests since the information matrix equality holds under the null.[5]

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[4] Andrews and Ploberger (1994) suggest setting $\pi_0 = 0.02$, while Andrews (1993) suggests setting $\pi_0 = 0.15$ for their "sup-" tests. In our analysis, we set $\pi_0 = 0.15$ to be consistent with McConnell and Quiros (1997, 2000).

[5] We are grateful to Professor Don Andrews for his confirmation on this.

Andrews and Ploberger (1994) consider two different types of structural breaks: pure and partial. For the likelihood function $f(Z;\theta)$, where θ is a parameter vector consisting of coefficients and residual variance, "pure" structural break refers to the case where the entire parameter vector θ is subjected to changes under the alternative hypothesis. "Partial" structural break refers to the case where only part of the components of θ is subjected to changes under the alternative hypothesis. In our analysis, we apply the likelihood function-based Andrews-Ploberger test to an AR(1) process to test for a "pure" structural break. Specially, we test the null hypothesis of

$$H_0 : \alpha = \beta = \gamma = 0$$

in the model of

$$y_t = a + \phi \cdot y_{t-1} + \sigma \cdot \varepsilon_t, \qquad \text{for } t \leq T_0;$$

$$y_t = (a + \alpha) + (\phi + \beta) \cdot y_{t-1} + (\sigma + \gamma) \cdot \varepsilon_t, \quad \text{for } t > T_0;$$

$$\varepsilon_t \sim i.i.d.\ N(0,1),$$

where a is the mean, ϕ is the AR coefficient, and ε is the error term. One feature of the Andrews-Ploberger test is that it can detect only one structural break at a time, that is, all parameters under consideration change at the same time. This is often not true in reality. In order to check for multiple breaks and to find the sources of the change, namely, which parameter has changed and at what time point, we sequentially apply the Andrews-Ploberger tests and use maximum likelihood estimation (MLE) to detect the change points. The procedure is as follows. Using the full sample, we first apply the Andrews-Ploberger test for a "pure" structural break, that is, to check whether there is a break when all the parameters of the AR process and the variance of the residual change at the same time. Once we have detected a break, the next step is to determine the break date. The Andrews-Ploberger test by itself contains no information of when the break date occurs. This is obtained by applying the MLE to the AR process (assuming breaks are present in the regression parameters and the residual variance), and the break date is chosen to coincide with the observation where the likelihood function is maximized.[6] After the break date is identified, we split the sample into two sub-samples at this break date. At the known break date, we can test if each parameter is stable across the two sub-samples by using the traditional LM or LR test. We then repeat this procedure (apply the Andrews-Ploberger tests, estimate the break point, and followed by the traditional LM test for parameter stability) for each sub-sample, and sub-sub-sample, and so on, until we do not detect anymore breaks.[7] This procedure hence allows us to check for possible multiple breaks in the GDP growth series. In all the sequential tests, we are more interested in the presence of a break in the standard deviation of the residual, which would suggest the presence of a structural break in the volatility of real GDP growth, than a break in the AR parameters.

3. Some comments on the methodology of McConnell and Quiros[8]

Although both MQ and this analysis use the Andrews-Ploberger test to detect structural break, our approach is substantially different. We apply the likelihood function-based Andrews-Ploberger tests ($aveLM_T$ and $expLM_T$) in our analysis instead of the GMM-based tests used by MQ. More important, our testing and estimation procedures are very different. Several reasons lead us to use a different approach.

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[6] Bai, Lumsdaine and Stock (1998) prove the (pseudo) MLE of the break date in multivariate time series is consistent. Since MQ apply GMM based LM-type Andrews-Ploberger test for possible break, they choose the break date to coincide with the observation where the GMM-based test statistics function $LM_T(\frac{t}{T})$ is minimized.

[7] Bai (1997) shows that this sequential algorithm with least square estimation yields consistent estimators of break dates.

[8] These comments also apply to Debs (2001) since his methodology follows that of MQ's.

3.1 Likelihood function-based tests versus GMM-based tests

Assuming $i.i.d.$ (under the null) normal error term, MQ use the GMM-based Andrews-Ploberger test for investigating structural breaks in the model of

Model 1: $y_t = \mu + \phi\, y_{t-1} + \varepsilon_t$,

where $\varepsilon_t \sim i.i.d.\ N(0, \sigma_1^{\,2})$, for $t \le T_0$;

and $\varepsilon_t \sim i.i.d.\ N(0, \sigma_2^{\,2})$, for $t > T_0$.

Given the independent normal error distribution specification in Model 1, it would have been more efficient to use the MLE than the GMM. Moreover, MQ do not directly use the moment condition based on $E(\varepsilon_t^{\,2}) = \sigma^2$. Instead, they propose moment conditions based on the property of the standard deviation $E(\left|\sqrt{\tfrac{\pi}{2}}\varepsilon_t\right|) = \sigma$ and the orthogonal condition between the regressor and the error term, and conduct the Andrews-Ploberger test based on this GMM set-up. However, Andrews and Ploberger (1994) present their optimal tests only within the likelihood-function framework, and they do not extend their proof of techniques to the GMM framework. Sowell (1996) extends Andrew and Ploberger’s test to the GMM framework but with the requirement of smoothness condition, that is, the moments are continuously partially differentiable in the parameters. MQ's moment condition of standard deviation does not satisfy this smoothness requirement. Therefore, given the independent normal distribution assumption, using the likelihood function-based Andrews-Ploberger test statistics is more appropriate and efficient than using the GMM-based test statistics.

3.2 Choice of model

MQ apply the Andrews-Ploberger test to test for the presence of a break in the standard deviation in Model (1) while keeping the mean μ and the AR coefficient ϕ constant. Their results indicate a structural break occurring in 1984Q1. MQ then conduct the Chow test to investigate whether the detected break in the standard deviation is caused by a possible break in the regression coefficients.

We prefer to choose a different test procedure from that of MQ's. The issue here is to test whether a possible break in the regression coefficients has resulted in the break in the standard deviation (which MQ detected while imposing constant regression coefficients). We prefer to address this issue by testing

$$H_0:\ b_1 = b_2 \quad \text{versus} \quad H_1:\ b_1 \neq b_2$$

using the general model of

Model 2:
$$y_t = x_t' b_1 + \varepsilon_t,\ E(\varepsilon_t^{\,2}) = \sigma_1^{\,2}, \quad \text{for } t \le T_0;$$
$$y_t = x_t' b_2 + \varepsilon_t,\ E(\varepsilon_t^{\,2}) = \sigma_2^{\,2}, \quad \text{for } t > T_0;$$

but not the model of

Model 3:
$$y_t = x_t' b_1 + \varepsilon_t, \quad \text{for } t \le T_0;$$
$$y_t = x_t' b_2 + \varepsilon_t, \quad \text{for } t > T_0;$$
$$E(\varepsilon_t^{\,2}) = \sigma^2, \qquad \text{for all t;}$$

which is the base model used by MQ to conduct Chow test, nor the model of

Model 4:
$$y_t = x_t' b_1 + \varepsilon_t, \quad \text{for } t \le T_0;$$
$$y_t = x_t' b_2 + \varepsilon_t, \quad \text{for } t > T_0;$$
$$E(\varepsilon_t \mid x_t) = 0, \qquad \text{for all t;}$$

which is used by MQ to conduct the GMM-based Andrews-Ploberger test.

We prefer Model (2) to Model (3) because the Chow test is designed to test for parameter stability in the linear regression Model (3) with known break point T_0 and constant residual variance over the full sample. A structural break in the residual variance makes the Chow test statistics no longer F-distributed and hence conclusions drawn from the Chow test may be incorrect. Therefore, a break in the residual variance, as suggested in MQ's empirical findings, invalidates the standard distribution of the Chow test and weakens their conclusion. In this case, Wald, LM, or LR test would be more appropriate.

We prefer Model (2) to Model (4) based on the following reasons. MQ conduct GMM-based Andrews-Ploberger tests based on Model (4) with unknown change point T_0 but impose no restrictions on the residual variance. This is not the best way to investigate whether a structural break in the residual variance detected with fixed regression coefficients is caused by a possible break in regression coefficients. In their study and ours as well, the structural instability under consideration (namely the alternative hypothesis to be tested) is a one-time structural change rather than gradually time-varying changes. In other words, the variance is either constant in the full sample if there is no break (under the null) or the variance differs across sub-samples but is constant in each sub-sample if there is a break (under the alternative). This is a restriction we need to impose on the residual variance when we are testing for structural stability of volatility. From MQ's test procedure and their results, one can only conclude that there is no structural break in the regression coefficients when the residual variance is unrestricted (namely it is allowed to have heteroskedasticity and to change gradually over time). It remains unknown whether there is a break in the regression coefficients when the residual variance is kept constant in each sub-sample. Therefore, MQ's test may not have provided adequate answers to the issue they want to address. We suggest that the proper test is one that is based on Model (2) with restriction on the residual variance rather than on Model (4).

3.3 Test procedures

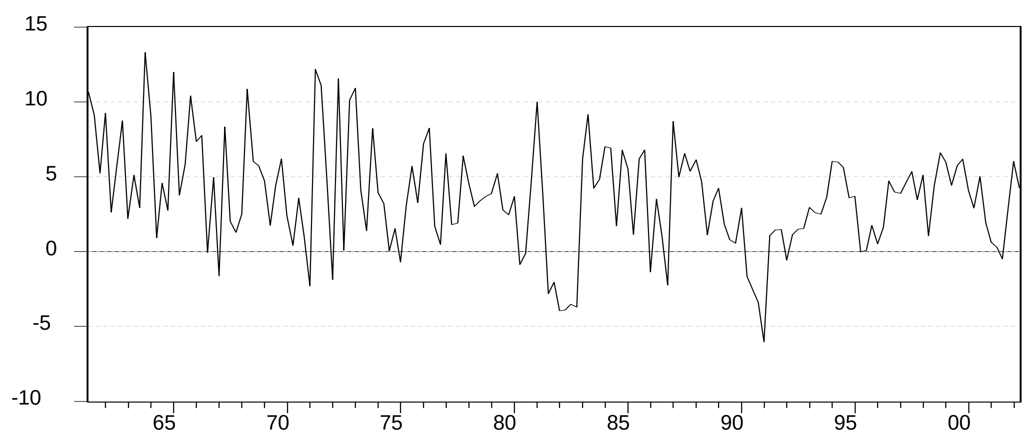
MQ's test procedure is a partial structural break test involving two sequential steps: Test for a break in the variance first while keeping the regression coefficients fixed; if there is a break in the variance, then test for a possible break in the coefficients. This procedure, however, is insufficient to identify the breaks in the variance if there is also a break in the coefficients. Suppose one detects a break in the variance first and subsequently also finds evidence of a break in the coefficients. There can be two possible explanations to these findings: either the true process has no break in the variance and the detected break in the variance is purely caused by a break in the regression coefficients, or the true process has breaks in both coefficients and variance. To investigate these possibilities requires applying "pure" structural break tests (as we discussed earlier) in the test procedure.

4. Empirical findings of structural breaks in Canadian real GDP growth volatility

4.1 Testing for the probability of a regime change using Markov-switching processes

A simple visual inspection of Canadian real GDP growth over history (see Chart 1) suggests that a break may have occurred in the volatility of the series sometime in the late 1980s or early 1990s.

Chart 1
Canadian Real GDP Growth Rates
Annualized Quarterly Rates (per cent)



Our first step to test this hypothesis is to estimate Canadian real GDP growth over the 1962Q2 to 2002Q2 period with a general form AR(1) Markov-switching model of[9]:

$$y_t - \mu_{s(t)} = \phi_{s(t)}(y_{t-1} - \mu_{s(t-1)}) + \varepsilon_{s(t)} \ , \qquad (4.1)$$

where $\mu_{s(t)} = \mu_0 + \mu_1 \cdot s(t)$;

$$\phi_{s(t)} = \phi_0 + \phi_1 \cdot s(t) \ ;$$

$$\varepsilon_{s(t)} = \varepsilon_t \cdot \sigma_{s(t)} \quad \text{with} \quad \varepsilon_t \sim \text{i.i.d. } N(0,1), \quad \sigma_{s(t)} = \sigma_0 + \sigma_1 \cdot s(t);$$

and $s(t) = 0, 1$ follows a two-state first order Markov chain with transition probabilities of:

$$P(s(t) = 0 \mid s(t-1) = 0) = p,$$

$$P(s(t) = 1 \mid s(t-1) = 1) = q.$$

Our specification of equation (4.1) allows the mean (μ), the variance (σ^2), and the AR coefficient (ϕ) to change with the regime ($s(t)$).

Chart 2 compares the estimated probabilities of a change in regime in real GDP growth between our general model and a fixed AR-coefficient model.

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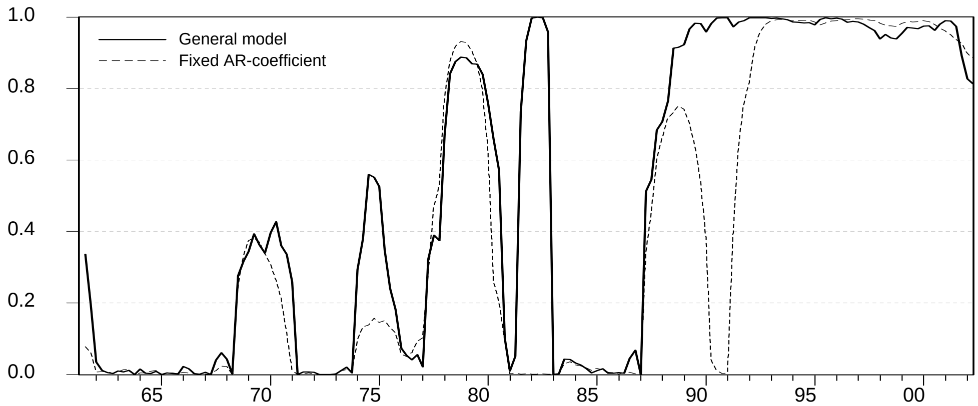
[9] The LR test of AR(4) Markov-switching versus AR(1) Markov-switching model does not reject the AR(1) specification (the p-value is 0.24). The estimated probabilities of regime changes with AR(4) Markov-switching are also similar to those with AR(1) Markov-switching model. Thus, we report the results for AR(1) only.

Chart 2
Probabilities of Change in Regimes for
Canadian GDP Growth



Two observations emerge. First, the general model shows a high probability (1.0) of a regime change in 1982. With the fixed AR-coefficient model, the probability of a regime change in 1982 is zero. Second, the general model indicates another regime change in 1987 and remains in that regime thereafter. Although the fixed AR-coefficient model also suggests a regime change in 1987, the probability falls to zero in 1990 before reversing itself in 1991 and staying at that level thereafter.

The result of the likelihood ratio test (see Table 1) strongly rejects the null hypothesis of constant AR coefficient $H_0 : \phi_1 = 0$ in favour of our general model with regime-dependent AR-coefficients for the Canadian GDP growth process. Hence, for the rest of this paper, empirical results from Markov-switching models refer to those using the general model only.

Table 1: LR Test of Constant AR Coefficient for Canadian GDP Growth

	Model of equation (4.1)	Fixed AR coefficient
Log likelihood	-185.744	-192.871
P-value $H_0 : \phi_1 = 0$	0.0002	

To further confirm our findings that there have been changes in the regime in real GDP growth, we apply the Hansen (1992, 1996) test to test the Markov-switching model against a linear AR model.[10] We use the Hansen test because the classical LM, LR, and Wald tests are not applicable in this case. The reason is that the appearance of transition probabilities under the alternative hypothesis (the Markov-switching model) but not under the null (the linear AR model) invalidates the standard asymptotic properties of these three classical tests.

Specifically, we use the Hansen test to test the general form AR(1) Markov-switching model of equation (4.1) against the linear AR(1) model of:

$$y_t - \mu_0 = \phi_0 (y_{t-1} - \mu_0) + \sigma_0 \varepsilon_t \ , \qquad (4.2)$$

Thus, the null hypothesis is:

$H_0 : \phi_1 = \mu_1 = \sigma_1 = 0$ in the model of equation (4.1).

The result of the Hansen test strongly rejects the linear AR specification in favour of our Markov-switching model for the real Canadian GDP growth process (see Table 2).

Table 2: P-value of the Hansen Test for
Markov-Switching Model (4.1) versus Linear AR Model (4.2)

Bandwidth*	M = 0	M = 1	M = 2	M = 3	M = 4
P-value	0.000	0.001	0.001	0.004	0.005

*The bandwidth parameter M in the Hansen test is the length of serial correlation in the variance estimation. We follow Hansen's suggestion to use different bandwidths to check the sensitivity of the Hansen test statistics.

The findings from the Markov-switching models point to the possibility of several breaks in the Canadian real GDP growth process over the last four

[10] See Liu, Y., “Hansen’s Test for Regime-Switching Models”, Economic Analysis and Forecasting Division, Department of Finance, March 2001, for details.

decades. Note that changes in the mean, variance, or AR coefficients could have caused these breaks. However, Markov models by themselves are insufficient to conclude the presence of structural breaks in the GDP volatility since they produce only the joint probabilities of switching regimes in the mean, variance, and AR coefficients. We therefore require formal tests to confirm the presence of structural breaks.

4.2 Testing for structural breaks using the Andrews-Ploberger test

We apply the likelihood function-based Andrews-Ploberger test to test for structural breaks in Canadian real GDP growth using the AR (1) process of[11]:

$$y_t - a = \phi_1 (y_{t-1} - a) + \sigma \cdot \varepsilon_t \ , \qquad \text{for } t \le T_0;$$

$$y_t - (a+\alpha) = (\phi_1 + \beta_1)(y_{t-1} - (a+\alpha)) + (\sigma + \gamma) \cdot \varepsilon_t \ , \quad \text{for } t > T_0;$$

$$\varepsilon_t \sim \ i.i.d. \ \ N(0,1) \, .$$

We first apply the Andrews-Ploberger test for a "pure" structural break and test the null hypothesis of

$$H_0 : \alpha = \beta_1 = \gamma = 0 \, .$$

Using the full sample from 1961Q3 to 2002Q2, our test results show a strong break in the first quarter of 1987. Applying the Andrews-Ploberger test sequentially, we also find a break occurring in 1973Q4 in the sub-sample from 1961Q3 to 1987Q1 but no break in the sub-sample from 1987Q2 to 2002Q2. We then use LM and LR tests to find which parameter has changed across the known break points. These tests further show that the break in 1973Q4 is caused by regression coefficients but not in the volatility. However, the break in 1987Q1 is caused by a strong break in the volatility, accompanied also by breaks in the

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[11] The LR tests of an AR (4) against AR (1), and an AR (2) against AR (1), conducted with break and without break (linear AR), clearly accept the AR (1) specifications for the full sample and for the two sub-samples when split at 1987Q1, the detected break date.

regression coefficients. Hence, we can conclude that between 1961Q3 and 2002Q2, there is only one structural break in the volatility of real GDP growth occurring in 1987Q1. Table 3 summaries these findings and Table 4 provides the relevant critical values.

Table 3: Structural breaks in the Canadian GDP Growth process

Andrews-Ploberger tests				
Sample	*aveLM*	*expLM*		Break date
1961Q3 to 2002Q2 (full sample)	25.57	23.67		1987Q1
1961Q3 to 1987Q1 (sub-sample 1)	8.11	5.07		1973Q4
1987Q2 to 2002Q2 (sub-sample 2)	4.08	2.90		No break
P-values of LM test at known break points				
Break date	$\alpha = \beta_1 = 0$ (all coefficients)	$\alpha = 0$ (mean)	$\beta_1 = 0$ (AR coefficients)	$\gamma = 0$ (standard deviation)
1987Q1 (full sample)	0.0004	0.06 (no break)	0.0004	0.000 (break)
1973Q4 (sub-sample 1)	0.003	0.01	0.01	0.06 (no break) (0.16 if LR test used)

Table 4: Critical Values of the Andrews-Ploberger Test*

	10%	5%	1%
aveLM	5.10	6.07	8.21
$\exp LM$	3.49	4.22	5.77

*The critical values used here are from Andrews and Ploberger (1994)

The results of Table 3 are robust to the type of AR process used. Despite the fact that the tests of AR (4) against AR (1) and AR (2) against AR (1) do not reject AR (1), we still conduct the same test procedures using higher-order AR processes. Although not reported here, results using AR (2), AR (3), AR (4), and AR (5) processes all confirm that there is only one break in the growth volatility at 1987Q1. This robustness shows that the break is not likely caused by model misspecification. Table 5 summarizes the estimation results.

Table 5: Changes in the Canadian Real GDP Growth Process (percent)

Sample	Full sample: 1961Q3 – 2002Q2	Sub-sample: 1961Q3 – 1987Q1
Break date	1987Q1	1973Q4
Standard deviation:		
Pre-break	3.87	No break
Post-break	1.90	No break
Mean:		
Pre-break	no break	5.19
Post-break	no break	3.03
AR coefficients:		
Pre-break	0.18	-0.12
Post-break	0.66	0.36

Our results are different from those of McConnell and Quiros' (1997) and Debs' (2001). MQ estimate the break point occurring in the second quarter of 1991 while Debs estimates a similar break point occurring in the first quarter of 1991. Differences in the data type used (constant-1992 dollar Laspeyres data used by MQ and Debs versus Fisher chained data used in this study) could not have accounted for the discrepancies in the break dates. Using constant-1992 dollar Laspeyres data, Liu and Painchaud (2002) have also identified the first quarter of 1987 as the break date. We can thus conclude that differences in the estimation procedures are major reasons for the discrepancies in the estimated break dates.

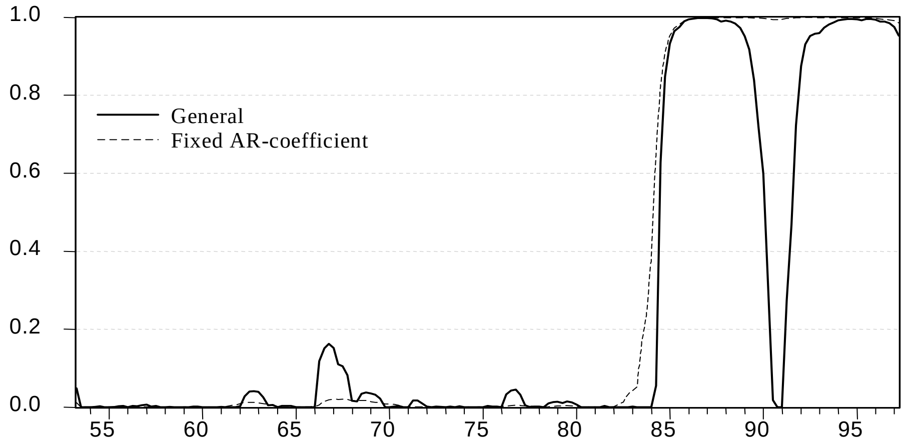
To a large extent, the similarities between MQ's and Debs' results owe to the fact that they use similar estimation procedures. Both MQ and Debs impose constant parameters in the AR processes when they test and estimate the break date in the volatility. As we pointed out earlier, this method is unable to distinguish whether the detected break is caused by a break in the AR parameters or it is truly a break in the volatility. Our results show that the perceived break in GDP growth volatility in 1991Q1 is caused by the break in the AR parameters only. In fact, Debs (2001) mentions in a footnote in his study that the break date in the volatility of GDP growth would be 1987Q1 (the same as our result) if one allows for different AR parameters before and after 1973Q4. Debs, however, does not pursue this finding but instead proceeds to impose fixed AR parameters in the GDP growth process. This restriction is the key factor that leads Debs to conclude that the break date for GDP growth volatility has occurred in 1991Q1.

5. Re-examining U.S. real GDP growth volatility

Before proceeding to identify the sources of the break in the Canadian output growth volatility, we apply our testing and estimating procedures to verify the structural break in the volatility of U.S. real GDP growth. We are interested in whether using our methods would also yield different results for the U.S. data. For ease of comparison, we use the same AR(1) processes (in both the Markov-switching model and Andrew-Ploberger tests) and the same sample period (from 1953Q2 to 1997Q2) as those of McConnell and Quiros (1997, 2000).

We first re-estimate the U.S. real GDP growth process using the general Markov-switching AR(1) process of equation (4.1). The estimated probabilities of regime changes (see Chart 3) show substantial differences between the general model and the fixed AR-coefficient model from the 1980s onward. The probabilities of regime changes in both models show a clear jump around 1984 (as reported by MQ), but our general model also shows another possible change in the regime occurring at around 1991.

Chart 3
Probabilities of Change in Regimes for
U.S. GDP Growth



We use the Andrew-Ploberger test to check the statistical significance of these breaks. For the full sample, test results (see Table 6) show a strong break occurring in 1984Q1 as reported by MQ. Our results, however, also uncover a weak break occurring in 1991Q4 for the sub-sample from 1984Q2 to 1997Q2. This finding is consistent with the above result of our general-form Markov-switching model but is not detected by MQ.

Table 6: Andrews-Ploberger Tests of the U.S. GDP Growth Process

Sample	Andrews-Ploberger test statistics		Break date
	aveLM	*ExpLM*	
1953Q2 to 1997Q2 (full sample)	14.70	21.66	1984Q1
1953Q2 to 1984Q1 (sub-sample 1)	1.45	0.92	No break
1984Q2 to 1997Q2 (sub-sample 2)	7.88	4.58	1991Q4
	Critical Values*		
	10%	5%	1%
aveLM	5.10	6.07	8.21
ExpLM	3.49	4.22	5.77

*The critical values used here are from Andrews and Ploberger (1994)

Further LM tests at these known break points (see Table 7) show that the break at 1984Q1 is caused by the break in the standard deviation while the break at 1991Q4 is caused by the break in regression coefficients. Therefore, we conclude that there is only one break in the volatility of the U.S. GDP growth process at 1984Q1. This confirms MQ's finding. However, we have also identified that the AR parameters are locally significantly different between sub-samples although the differences are not significant over the full sample.

Table 7: P-values of LM Test at Known Break Points

	$H_0: \alpha = 0$	$H_0: \beta = 0$	$H_0: \gamma = 0$
1984Q1 (full sample 1953Q3-1997Q2)	0.97	0.81	0.000
1991Q4 (sub-sample 1984Q2-1997Q2)	0.001	0.003	0.34

6. The source of the break in Canadian GDP growth volatility

Our next step is to identify, if possible, the source of the break in the Canadian output growth volatility by investigating the volatility of the components' contribution to GDP growth. We proxy the contribution to GDP growth of each component by an AR process and apply the same testing procedures as we used earlier to identify possible breaks in the variance of the residual terms. If a component's contribution to GDP growth volatility has the same break date and changes in the same direction as that of aggregate GDP growth volatility, we can then conclude that that particular component is responsible for the break in the aggregate GDP growth volatility.

Table 8 reports the estimated break points for the volatility of components' contribution to growth using the full sample from 1961Q2 to 2002Q2. It shows that structural declines have occurred in the contribution growth volatility of personal spending, total government spending, business investment in residential and non-residential construction, and business investment in inventory. None of the breaks, however, occurs at the same time as that of the aggregate output, namely, at 1987Q1. In fact, declines in the volatility of most of these components occur after 1987Q1 except for personal spending on durable goods and business investment in inventory where their breaks occur in 1981Q3 and 1984Q1 respectively. Although further tests on sub-samples show multiple breaks in the volatility of components' contribution to growth, the break dates are far from 1987Q1. Therefore, we choose to report only the results obtained with the full sample.

Table 8: Volatility of Contribution to Real GDP Growth (Standard deviations)

	Break date	Pre-break	Post-break	Difference
Personal consumption spending	1991Q1	1.64	1.03	-0.61
Goods	1991Q1	1.33	0.79	-0.54
Durables	1981Q3	1.24	0.76	-0.48
Semi-durables	1991Q1	0.34	0.20	-0.14
Non-durables	1991Q3	0.74	0.27	-0.46
Services	No break			
Total government spending	1990Q3	1.22	0.62	-0.61
Business investment	No break			
Residential construction	1990Q2	0.96	0.52	-0.44
Non-residential construction	1992Q4	0.76	0.31	-0.45
M&E	No break			
Business inventory investment	1984Q1	3.73	2.00	-1.73
Government inventory investment	1984Q3	0.21	0.05	-0.16
Exports	1971Q3	1.94	3.23	1.30
Imports	1980Q1	2.36	3.21	0.85

The fact that the contribution growth volatility of business inventory investment shows a break in 1984Q1 is important. Although the break date precedes that of GDP, it is similar to that observed by MQ in U.S. data. This suggests that, as in the U.S., inventory-related innovations might have played a role in reducing GDP growth volatility in Canada.

Since imports enter the GDP identity with a negative sign, the contribution growth volatility of imports in Table 8 should be interpreted with care. From Table 8, it is clear that the variance of imports contributes to increase in GDP growth volatility. However, if the size of the covariance between imports and other components of final demand is large, it is possible that the negative impact from the covariance could more than offset the positive impact from imports' variance. In that case, imports could contribute to declines in GDP growth volatility.

Nevertheless, results in Table 8 show that we cannot precisely identify which component is responsible for the structural break in the volatility of real GDP growth. This is very different from the experience with U.S. data where identification of the source is easier and more precise.

There are several reasons why we cannot explicitly identify the source of the break in GDP growth volatility. Note that GDP growth volatility is not just the simple sum of the volatility of the components' contribution to growth. Instead, it is a weighted sum of the volatility of the components' contribution to growth plus the covariance among the weighted growth rates, where the weights are dependent on the AR parameters that characterize each contribution to the growth process.

Specifically, suppose we decompose output into the following components

$$Y = C + G + I + X - M \ ,$$

then the growth rate of output can be decomposed into its component contribution to growth,

$$\frac{\Delta Y_t}{Y_{t-1}} = CG^C(t) + CG^G(t) + CG^I(t) + CG^X(t) - CG^M(t) \,,$$

where CG^q is the contribution to output growth by component q (q =*C*,*G*,*I*,*X*, *M*).

Since the growth rate of output is the sum of its components' growth contributions, the volatility of output would be the sum of the volatility of its component growth contribution plus the covariance terms among these component contributions provided that the volatility of a series is measured by its variance. However, in the aforementioned literature and this paper, volatility is measured by the variance of the residual term of the AR process that characterizes the series rather than the variance of the series itself. In this case, the volatility of the aggregate is a weighted sum of the volatility of the components' contribution to growth plus the covariance among the growth contributions where the covariance is a function of the parameters of the AR processes.

To show this, consider a simple example where an aggregate consists of only two components:

$$Z_t = X_t + Y_t \ ,$$

and X_t follows an AR(1) process of $(1-\alpha L)X_t = \varepsilon_t$ and Y_t follows another AR(1) process of $(1-\beta L)Y_t = v_t$. We can then rewrite Z_t as

$$(1-\alpha L)(1-\beta L)(X_t + Y_t) = (1-\beta L)\varepsilon_t + (1-\alpha L)v_t \ ,$$

or equivalently,

$$(1-\alpha L)(1-\beta L)Z_t = \omega_t \ ,$$

where $\omega_t = (1-\beta L)\varepsilon_t + (1-\alpha L)v_t$.

If ε_t and v_s are uncorrelated for all t, s, then ω_t is a process of moving-average with order 2 (MA(2)), and hence Z_t follows an ARMA(2,2) process (see Hamilton (1994)), which can be fitted empirically quite well by an AR process. The variance of ω_t is

$$Var(\omega_t) = \sigma_{\varepsilon_t}{}^2 + \beta\sigma_{\varepsilon_{t-1}}{}^2 + \rho_{v_t}{}^2 + \alpha^2\rho_{v_{t-1}}{}^2 + CORR_t \ ,$$

where $CORR_t = 2\{\text{cov}(\varepsilon_t, v_t) + \alpha\beta\,\text{cov}(\varepsilon_{t-1}, v_{t-1}) - \alpha\,\text{cov}(\varepsilon_t, v_{t-1}) - \beta\,\text{cov}(\varepsilon_{t-1}, v_t)\}$ is the covariance terms between ε and v , and $\sigma_{\varepsilon_t}{}^2 = E(\varepsilon_t{}^2)$ and $\rho_{v_t}{}^2 = E(v_t{}^2)$ are the volatility of X and Y, respectively.

Suppose the volatility of X_t has a break at T_1 such that

$$\sigma_{\varepsilon_t}{}^2 = \sigma_1{}^2 \quad \text{for } t \le T_1 \ ;$$

$$= \sigma_2{}^2 \quad \text{for } t > T_1 \,,$$

and the volatility of Y_t also has a break at $T_2\,(\geq T_1)$ such that

$$\rho_{v_t}{}^2 = \rho_1{}^2 \text{ for } t \leq T_2 \;;$$

$$= \rho_2{}^2 \text{ for } t > T_2 \,,$$

then the volatility of the aggregate Z_t is

$$Var(\omega_t) = (1+\beta^2)\sigma_1{}^2 + (1+\alpha^2)\rho_1{}^2 + CORR_t \quad \text{for } t \leq T_1 \;;$$

$$= (1+\beta^2)\sigma_2{}^2 + (1+\alpha^2)\rho_1{}^2 + CORR_t \quad \text{for } T_1 + 2 \leq t \leq T_2 \;;$$

$$= (1+\beta^2)\sigma_2{}^2 + (1+\alpha^2)\rho_2{}^2 + CORR_t \quad \text{for } t \geq T_2 + 2 \;.$$

For ease of discussion, we have ignored the variance at the change points $T_1 + 1$ and $T_2 + 1$. Based on this simple example, we can see that

(1) If only one component has a break or several components have a common break date, there will be one break in the volatility of Z_t occurring at the component's break date. By implication, if one or several components' contribution to the GDP growth process have the same break date and the volatility changes in the same direction, then the break in the contribution to growth of these components is the source of the break in GDP volatility.

(2) If none of the components has a break ($\sigma_1 = \sigma_2$ and $\rho_1 = \rho_2$), it is still possible to observe a break in the volatility of Z_t if the covariance among its components ($CORR_t$) has a break.[12]

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[12] This may explain why McConnell and Quiros (1997) documented a strong break in the U.S. GDP volatility at 1984Q1, but no break in their preliminary decompositions of the GDP into consumption, government expenditure, investment, exports and imports.

(3) If each component has a break and the break dates are different from each other, there could be multiple breaks in the volatility of Z_t occurring at all of these different components' break dates. However, the effect of breaks in the covariance terms may cancel out the effect of the breaks in the variance. As a result, the aggregate may exhibit no break or the number of breaks is less than the total of its components' break points.

The failure to properly identify the source of the break in the volatility of real Canadian GDP growth may have been caused by the break in the covariance amongst the components' growth contributions. Although we cannot directly test this possibility, we can see that the covariances between the components' contribution to real GDP growth reported in Tables 9 and 10 are very different before and after 1987Q1. There are strong positive correlations between imports and some components of final demand, suggesting that negative impacts from the covariance between imports and some components of final demand might have helped to reduce the growth volatility of GDP.

Table 9: Covariances between contributions to real GDP growth: 1961Q2 to 1987Q1

	Personal expenditures	Business investment	Government spending	Business inventory	Government inventory	Exports	Imports
Personal expenditures	4.827	1.671	0.365	-1.131	-0.032	1.460	2.325
Business investment		3.899	0.012	-0.221	-0.037	0.322	2.437
Government spending			1.646	-0.730	0.021	0.064	-0.008
Business inventory				14.850	0.072	-1.116	3.775
Government Inventory					0.070	0.094	0.198
Exports						8.462	4.624
Imports							9.447

Table 10: Covariances between contributions to real GDP growth: 1987Q2 to 2002Q2

	Personal expenditures	Business investment	Government spending	Business inventory	Government inventory	Exports	Imports
Personal expenditures	2.513	1.022	0.124	-0.505	0.004	0.072	1.093
Business investment		3.241	-0.322	0.716	0.002	0.524	3.024
Government spending			0.605	-0.175	0.001	-0.369	-0.203
Business inventory				5.427	-0.010	-0.446	3.617
Government Inventory					0.005	0.005	-0.003
Exports						9.098	4.830
Imports							9.008

7. Conclusion

In this paper, we investigate structural breaks in the Canadian real GDP growth volatility by using a modified version of McConnell and Quiros's (1997, 2000) methodology that they used to investigate breaks in U.S real GDP growth volatility. Our method involves using a less restrictive form of Markov-switching models than McConnell and Qiuros to estimate the probability of a break occurring in the data. We also use a different testing and estimation procedure of the Andrew-Ploberger test to identify the break. Using our methods, we identify one structural decline in the volatility of the Canadian GDP growth in the first quarter of 1987, instead of 1991 as reported by McConnell and Quiros (1997) and Debs (2001).

We have also applied our method to the U.S. data. Although our findings do not alter the conclusion of McConnell and Quiros that a structural break in U.S. output growth volatility occurred at 1984Q1, we have detected an additional break in the data that is caused by the break in the coefficients of the AR process.

We, however, cannot identify precisely the source of the break in the volatility of the Canadian output growth by analysing the volatility of the components' contributions. One possible reason is the volatility of output growth is determined not just by the volatility of the components' contribution to growth, but also by the covariance of the volatility of the components' contribution to growth. It is possible that structural breaks in the covariance among these component growth contributions might have dominated other effects and make the task of identifying the source more difficult. However, our finding of a break in the volatility of business inventory's contribution to growth in 1984Q1 is consistent with the results of McConnell and Quiros for the U.S., suggesting that the structural change in inventory management might also have played a role in reducing GDP growth volatility in Canada.

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