Department of Finance
Ministère des Finances

Working Paper
Document de travail

To Measure the Unobservable: A Model of the U.S. NAIRU

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Working Paper 2007-02

February 2007

Résumé

Dans de document de recherche, nous examinons les différents modèles utilisés pour l'estimation du NAIRU (pour Non Accelerating Inflation Rate of Unemployment) dans le but de remplacer notre méthode d'estimation actuelle pour les États-Unis, basée sur un filtre de Hodrick-Prescott, par une méthodologie qui prends explicitement en compte la théorie économique. Nos résultats tendent à démontrer qu'un modèle de forme réduite construit à partir d'une courbe de Phillips, estimé à l'aide d'un modèle d'espace état, et qui incorpore plusieurs variables exogènes, y compris la durée du chômage, produit les meilleurs résultats en terme de simplicité de l'estimation et de la précision des estimés du NAIRU.

Abstract

In this paper we examine different methods of estimating the U.S. NAIRU (Non Accelerating Inflation Rate of Unemployment) in order to replace our current model of the NAIRU – based on a Hodrick-Prescott filter – by a method which explicitly incorporates economic theory. Our results suggest that a reduced-form model using a wage Phillips curve that incorporates different exogenous variables, including the duration of unemployment, and estimated using the state-space methodology, produces the best results in terms of ease of estimation and the precision of the estimates of the NAIRU.

Table of Content

1. Introduction

The roaring 1990s in the U.S. were often characterised as the Goldilocks economy: not too hot, not too cold, but just right (Gordon 1998). During much of the decade, unemployment was steadily decreasing, growth was strong, real wages were growing quickly, yet serious inflationary threats were nowhere to be seen. This apparent change in the historical relationship between inflation and unemployment puzzled most economists and revived interest in the concept of a NAIRU (the Non Accelerating Inflation Rate of Unemployment) that can vary over time.

Ball and Mankiw (2002) offer a thorough review of the possible reasons why the NAIRU would have changed in the past decade or so. In doing so, they identify two main factors which could have led to a change in the NAIRU: demographic changes, leading to the emergence of a more employable work force; and structural changes in the economy. More specifically, the relationship between inflation and unemployment could have been modified by changes in demographics, as baby boomers move through the labour force thus shifting its composition from younger workers (which tend to have higher unemployment rates) to older workers. This theory would explain why the NAIRU increased steadily before 1980 and then began to descend, as the workforce got older and more experienced. Moreover, higher incarceration rates (which generally implies fewer young, relatively uneducated workers) and the greater generosity of disability insurance (which implies that some people who would have had a greater likelihood of being unemployed can now afford not to be part of the labour force) could also have had an impact on the NAIRU by reducing the unemployment rate in recent years. Simultaneously, the emergence of a "new economy" could also have led to a change in the NAIRU. Particularly, greater openness to trade could have led to subdued inflationary pressures as U.S. producers are facing increasing competition from foreign countries. Also, the use of the Internet by job seekers and firms as well as the greater importance of temporary help services in the economy as a whole have led to better job matching and thus could have reduced the structural rate of unemployment.

Whatever the reason for the perceived decline in the NAIRU, the result has been a significant enrichment of the literature on the subject in the last 10 years. In particular, researchers have applied the time-varying parameter regression framework pioneered by Gordon (1997, 1998), and Staiger, Stock and Watson (1997, 2001) for the U.S. to a host of countries such as the U.K. (Driver, Greenslade and Pierse (2003) and Greenslade, Pierse and Saleheen (2003)), OECD countries (Turner *et al.* (2001)), the Euro area (Logeay and Tober (2003)), and New Zealand (Szeto and Guy (2004)).

The aim of this paper is to build on this recent research and revisit our own model of the NAIRU. We currently use a statistical filter to produce estimates of the U.S. NAIRU, and while filters are convenient and easy to use, they fail to provide us with theoretical groundings for our estimates as well as confidence intervals, on which to judge of the accuracy of our estimates. We thus want to select a methodology that combines the filter's ease of use with measurable accuracy (i.e. standard errors and confidence intervals), and which is grounded in economic theory. Ultimately, these estimates of the U.S. NAIRU will be applied in a model of U.S. potential output, which is based on a Cobb-Douglas equation[1]. This measure of potential output is then used in the context of quarterly forecasts as well as for *ad hoc* economic and policy analysis.

We begin our investigation by reviewing the different methods that have been used to estimate the NAIRU in section 2. We analyze a number of different methodologies and functional forms and produce different sets of estimates of the NAIRU. In section 3 we analyze the estimates further in order to select the most appropriate method and functional form for the purpose at hand. Our conclusions, as well as avenues for further research, are presented in section 4.

Overall, our results suggest that a time-varying coefficient model using a wage Phillips curve that allows for feedback from price inflation, and which incorporates the duration of unemployment as a regressor, provides the most precise estimates of the U.S. NAIRU. This model has three main advantages: 1) it is grounded in theory rather than produced by purely empirical methods; 2) it is fairly straightforward to estimate when making use of a state space

[1] See Collins (1998).

model and the Kalman filter; and 3) preliminary results suggest that it is often more precise than most other methods or functional forms found in the literature. Estimates produced using this method indicate that the NAIRU did indeed decrease strongly in the 1990s, falling by almost one percentage point between 1990 and 2000, but also that the period where it decreased the most was between 1980 and 1990, when it fell by almost 1.4 percent points.

2. Modelling the NAIRU

Before turning our attention to modelling per se, we need to define what we mean by the non inflation-accelerating rate of unemployment. Turner *et al.* (2001) distinguish between three concepts of the NAIRU: 1) the short-term NAIRU, defined as the rate of unemployment consistent with stabilizing inflation in the next period; 2) the medium-term NAIRU, or the rate towards which unemployment converges in the absence of temporary supply influences, and once the dynamic adjustment of inflation is completed; and 3) the long-term NAIRU which can be thought of as the equilibrium unemployment rate that corresponds to the steady state value of employment once the NAIRU has adjusted to <u>all</u> supply and policy influences. Of these three concepts, the second one is of particular interest to us as it is the most relevant for purposes of modelling and forecasting and in the context of policy analysis. The short-term NAIRU, while dependant on the NAIRU itself, is highly volatile and is affected by all supply influences (including temporary ones), which makes it of limited use for forecasting purposes. Similarly, the long-term NAIRU, while useful in a theoretical context, is of little use in applied empirical economics as it relates more to an "equilibrium" value, whereas policy-makers and forecasters are interested in the value of NAIRU at a particular point in time. As such, for the remainder of the paper when we refer to the NAIRU, we refer to the medium-term NAIRU.

Turning to modelling, methods to estimate the NAIRU can generally be divided in three broad categories: structural, statistical, or reduced-form[2]. All three categories are discussed below.

[2] This classification is used in Turner *et al.* (2001).

A. Structural models

Structural methods are based on a formal model of the labour market. Although models vary widely, the NAIRU is generally estimated by analytically solving the equations of the model for the equilibrium level of unemployment that is stable subject to built-in constraints. These constraints generally amount to the condition that firms' and workers' decisions with regard to parameters such as wages and profits are compatible. Note that these models are generally not built for the sole purpose of estimating the NAIRU but rather for broader theoretical and analytical purposes. Estimates of the NAIRU are generally a by-product of the equilibrium conditions of the model[3].

A major drawback of structural models is that explanatory variables and the interactions between them need to be fully specified. While this process can be quite laborious in and of itself, it is further complicated by the fact that there currently exists no consensus on which functional form to adopt (Turner *et al.*, 2001). This makes structural models much more difficult to design and estimate than statistical or reduced-form models. What is gained in terms of the analytical richness is more often than not offset by the considerable amount of time and resources needed to design and estimate a full-scale structural model of the labour market. This is why, although structural models can be quite useful in particular circumstances, they remain little-used for the purpose of estimating the NAIRU.

B. Statistical filter models

At the other end of the spectrum of theoretical complexity are "statistical" filter models of the NAIRU. These models implicitly assume that there exists no trade-off between inflation and employment in the long run, and thus that unemployment will on average move around the NAIRU. Unemployment can thus be divided into two components: short-run "cyclical" movements, and the fundamental long-run trend. These methods generally involve the use of some type of statistical filter to extract trend unemployment (or NAIRU) from the "raw"

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[3] Readers interested in further information on structural models of unemployment are referred to Pissarides (2000).

data. The most common type of filter used in econometrics is the Hodrick-Prescott (HP) filter.

Suppose the series y_t (in our case unemployment) is composed of a trend component τ_t and a cyclical component c_t such that $y_t = \tau_t + c_t$ for t = 1, 2, …, T. Hodrick and Prescott (1997) suggest that we can isolate the cyclical component by solving the following problem:

$$\text{(1)} \qquad \underset{\{\tau_t\}}{\text{Min}} \left[\sum_{t=1}^{T} (y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1} (\nabla^2 \tau_{t+1})^2 \right]$$

where ∇ is the difference operator and λ is a penalty parameter that controls the smoothness of the trend estimate. In effect, the HP filter minimises the variance of the original series around the trend subject to a penalty that puts a constraint on the second difference of the series, allowing the HP filter to "curve" over time rather than remain linear like the ordinary least square (OLS) function would. As λ approaches zero, the estimated trend becomes gradually closer to the actual series, whereas as λ approaches ∞, τ_t resembles the linear trend. As such, the choice of the parameter λ will definite how smooth the estimated trend is and how closely it follows the original series. It is customary to use λ =1,600 for quarterly data and 14,400 for monthly data.

Figure 1 presents estimates of the NAIRU using the HP filter for quarterly series and λ =1,600. Quarterly estimates show a downward trend in the NAIRU for most of the 1960s, with a trough in 1967Q4 at 3.9 per cent, and a clear upward trend for most of the 1970s and 1980s. According to our HP filter, the NAIRU peaked at 8.3 per cent in 1983Q1 and gradually decreased to a local minimum of 4.5 per cent in 1999Q4. The NAIRU then trended upward and is estimated to be around 6.0 per cent towards the end of the sample.

According to Figure 1, the NAIRU has climbed back fairly quickly following the trough of the 1990s. This is surprising as one would think the NAIRU would move quite slowly and thus remain at a lower level longer, especially given the exceptional productivity growth of the 1990s. This highlights one of the disadvantages of using the HP filter: the well-known end-of-sample problem. As the second difference term in the HP filter is both forward and backward looking, estimates near the end of the sample are less reliable. Researchers generally try to counter this problem by adding growth rate and level conditioning terms to the HP filter. This allows researchers to discard values that are far from the trend towards the end of the sample, thus providing more accurate estimates of the recent trend in the data. Formally, the modified HP filter, called the multivariate (MV) filter is defined as:

$$(2) \qquad \underset{\{\tau_t\}}{\mathrm{Min}}\left[\sum_{t=1}^{T} w_{1t}(y_t-\tau_t)^2+\sum_{t=2}^{T} w_{2t}\left[g_t-(y_t-\tau_t)^2\right]+\sum_{t=1}^{T} w_{3t}(z_t-\tau_t)^2+\lambda\sum_{t=2}^{T-1}(\nabla^2\tau_{t+1})^2\right]$$

where the w_{it} are weighting parameters, g_t is a growth rate series and z_t a levels series. Figure 3 is a comparison between quarterly estimates of the NAIRU using the HP and MV filters. We see that both series differ only towards the end of the sampling period as the constraints on the filter estimates come in only in last few years to give an estimate of the NAIRU that is closer to what recent empirical research would suggest (in the neighbourhood of 5 per cent). Using the MV filter, the NAIRU is estimated to be 5.2 per cent in 2004Q1, much lower than the 6.0 per cent estimated using the HP filter. The MV filter approach is the one currently used in the Economic Analysis and Forecasting Division to generate estimates of the U.S. NAIRU.

A clear advantage of purely statistical methods is the ease with which estimates can be generated. The HP and MV filters are easily programmed and can be run automatically in little time. However, purely statistical methods suffer numerous shortcomings, aside from the end-of-sample problem mentioned above. The first, and most important drawback of such methods is that they are atheoretical and rest on purely arbitrary assumptions. These estimates make use only of unemployment and do not attempt to explain, model or use information found in other variables such as inflation and supply shocks. In the context of policy work, this is a major disadvantage as it becomes difficult to justify estimates of the NAIRU on the ground of economic theory and to draw relationships between our estimates and other variables such as inflation that are extremely important for economic and public policy. Another drawback is the absence of confidence bands for the HP and MV filters.

As such, we have no way of assessing the statistical precision of the results (Turner *et al.* (2001)).

C. Reduced-form models

i) Time-invariant estimates of the NAIRU

Whereas statistical methods are atheoretical, reduced-form approaches explicitly incorporate economic theory. However, contrary to structural models, which are also built around economic theory, reduced-form models do not require that the entire structure of the system be specified *a priori*. In other words, reduced-form models allow the data to "talk". The simplest estimate of NAIRU in the reduced-form context is derived directly from the expectations-augmented Phillips curve:

$$\pi_t = \pi_{t-1} - \alpha(U_t - U^N) + \nu_t \tag{3}$$

Equation (3) outlines a simple relationship between inflation and unemployment where π_t, inflation in time *t*, is determined by inflation expectations (modelled here as backward-looking), the unemployment gap - the difference between U_t, unemployment in time *t*, and U^N, the NAIRU - as well as exogenous supply shocks (the error term ν_t).

Rearranging (3), we can write:

$$\Delta\pi_t = \alpha U^N - \alpha U_t + \nu_t \tag{4}$$

Assuming that the NAIRU is constant and that unemployment is uncorrelated with the exogenous supply shocks, we can estimate (4) by regressing the change in inflation on a constant and unemployment using ordinary least squares. The estimate of the NAIRU would then be the ratio of the constant to the absolute value of the coefficient of unemployment (Ball and Mankiw, 2002). Estimating (4) using annual data for 1960 to 2004, we get a constant term of 3.6 and a coefficient U of 0.55, yielding a NAIRU of 6.5 per cent.

The functional form outlined in equation (4) has been developed extensively in the literature. Among other modifications, researchers have added lags of unemployment and inflation, variables to model inflation expectations as well as explicit supply shocks. An example of this type of model is the one used by the Congressional Budget Office (CBO 1994). The equation used by the CBO is of the form:

$$\Delta\pi_t = \alpha_0 + \alpha_1 \Gamma\pi_{t-1} + \alpha_2(L)U_t + \alpha_3 FE_{t-1} + \alpha_4 PG + \alpha_5 NIXON + \varepsilon_t \tag{5}$$

where (*L*) is the lag operator, Γ is a polynomial distributed lag (PDL) of the past values of inflation[4], FE is a one-period lag of the difference in the growth rates of the fixed-weighted price index for personal consumption expenditures (PCE) and the fixed-weighted price index for PCE less food and energy (used as a proxy for price shocks), PG is the deviation of labour productivity and its trend, and NIXON is a set of dummy variables used to control for the introduction and subsequent termination of price control measures during the Nixon presidency.

To get an estimate of the NAIRU, the CBO runs this regression using the unemployment rate for married men as a benchmark, and then solves for the rate of unemployment that would keep inflation constant. It then constructs NAIRUs for each demographic group by inserting the NAIRU for married men into each equation. The overall NAIRU is computed as a weighted average of the NAIRUs for each segment of the labour force. Recent estimates of the CBO's NAIRU put it at 5.2 per cent (CBO 2002). Other organisations that use a similar framework to obtain estimates of the NAIRU include Macroeconomic Advisers, who estimate the NAIRU to be about 5.4 per cent in 2004[5].

4 A PDL restricts the coefficients of the lags of inflation to lie on a polynomial. In this case, the lags of inflation are restricted to sum to 1.

5 See Macroeconomic Advisers (2003) and (2004).

ii) A time-varying NAIRU using price equations

While the assumption of a constant NAIRU simplifies the estimation greatly, it is quite unrealistic. In essence, estimating the NAIRU from equation (5) implies that the natural rate of unemployment was the same for a particular demographic group in 1960 as in 2004 and thus rejects the notion of long-term shifts in the determinants of the NAIRU. For this reason, a large and constantly growing body of literature attempts to estimate a time-varying NAIRU (TV-NAIRU) using reduced-from equations of the Phillips curve.

A common specification is dubbed the "triangle" model of inflation, as defined by Gordon (1982), where inflation depends on three basic determinants: inertia, demand and supply.

(6) $$\Delta \pi_t = \alpha(L)\Delta \pi_{t-1} + \beta(L) D_t + \delta(L) z_t + \varepsilon_t$$

Inertia is conveyed by the lags of inflation, D is an indicator of excess demand, and z_t represents supply shocks. Using the unemployment gap (the difference between unemployment and the natural rate of unemployment) as an indicator of excess demand in the above framework and adding an equation that dictates the dynamics of the NAIRU as it varies over time yields the following system of equations:

(7) $$\Delta \pi_t = \alpha(L)\Delta \pi_{t-1} + \beta(U_t - U_t^N) + \delta(L) z_t + \varepsilon_t$$

(8) $$U_t^N = U_{t-1}^N + \eta_t$$

This is a stochastic time-varying parameter regression model where the residuals of (8) have a mean of zero and standard deviation σ_η. If $\sigma_\eta = 0$, the NAIRU is constant, but if σ_η is greater than zero, the NAIRU varies by a given amount every period. This system of equations can be estimated using a state-space representation and the Kalman filter algorithm of Kalman (1960) and Kalman and Bucy (1961) [6]. This powerful algorithm

[6] A description of the Kalman filter technique is found in technical annex 1.

enables the simultaneous estimation of the time-varying NAIRU and the Phillips curve. In contrast with the purely statistical method, the simultaneous estimation of the TV-NAIRU and the Phillips curve grounds our estimation procedure in economic theory rather than on purely arbitrary statistical models. This feature has made the stochastic time-varying parameter regression model the most commonly used functional form for researchers seeking to produce estimates of NAIRU that can be related intuitively to economic theory.

To estimate equations (7) and (8), we regress the change in inflation on lags of inflation, the unemployment gap, the change in relative import prices, a measure of shocks to food prices, the change in real oil prices, the change in the deviation of productivity from its trend, and a variable to control for price restrictions put in place during the Nixon presidency. While the specification of the first two variables on the right side of the equation is straightforward, the other variables - which are part of vector z_t in equation (7) - need to be defined.

The change in relative import prices is meant as a control variable for price shocks coming from abroad and is calculated as the change in the ratio of the GDP price index for imports to the GDP price index for exports. Similarly, to account for price shocks due to fluctuations in food prices, we include a variable defined as the difference between the growth rate (annualized) of the price index for personal consumption expenditures and the growth rate in the price index for food. Fluctuations in real oil prices are captured simply through the addition of the quarterly change in real oil prices to the regression. Finally, we also include a variable designed to capture variations in productivity, calculated as the difference between the growth rate of productivity minus the trend in productivity obtained by an MV filter[7]. We estimate equations (7) and (8) using three different prices measures - CPI inflation, the implicit deflator of GDP, and the implicit deflator for personal consumption expenditures – and use four lags of the dependant variable[8]. The lag structure for each variable in the vector of exogenous shocks is selected using the univariate properties of each series.

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7 Note that, like Gordon (1997), we found that a variable capturing variations in real exchange rates was not statistically significant and hence we rejected it.

8 Some researchers use up to 24 lags of inflation in order to account for inertia in inflation, i.e. the tendency of inflation to be persistent over time. We experimented with various lags of inflation and settled for 4 lags, or the equivalent of one year. Although evidence suggests that inertia in inflation is likely more persistent than modelled here, our results do not differ significantly when using 4,8,12 or 24 lags of inflation.

The Kalman filter algorithm requires that we provide a value for the starting point of the state series as well as a value for its variance. The starting value of each respective state series is selected using a weighted arithmetic averaging procedure. As in Szeto and Guy (2004), the signal-to-noise ratio is selected using a procedure devised by Stock and Watson (1998) that provides an estimate of the variance of the state equation. Note that standard errors of the state equation commonly found in the literature usually vary between 0.01 and 0.04.

Estimates of the NAIRU using the different inflation series are plotted in Figure 4. Visual inspection reveals that the NAIRUs obtained using the time varying regression technique are more stable than those obtained by the HP or MV filters. However, the general trend remains the same: the NAIRU increases gradually during the 1960s and 1970s, peaks around 1980 and then gradually decreases for most of the 1980s and 1990s. Note however, that while estimates using the CPI and the PCE deflator hover around 5 per cent towards the end of the sample, the NAIRU obtained using the GDP deflator trends lower than the other two. It begins to diverge in the early 1970s and is considerably lower towards the end of the sample. Although a difference of this magnitude is somewhat surprising and difficult to explain with certainty, the generally lower value of inflation as measures by the GDP deflator over the sample is likely a factor in the behaviour of the NAIRU estimates.

From our regression results — found in Table 1— we first notice that the unemployment gap is negatively correlated with changes in inflation, suggesting that, all things equal, employment below the NAIRU will tend to put upward pressure on inflation, as predicted by economic theory. Although we do not constrain the coefficients on the lags of inflation to sum to one, we see that the sum is nonetheless fairly close to unity, suggesting a good fit[9]. The exogenous variables are generally statistically significant in all equations, with the exception of change in relative import prices for CPI and oil prices for the GDP deflator. Note however, that the sum of the coefficients on the prices variables (import, food and oil

[9] The coefficients on lagged inflation are typically constrained to sum to one to ensure inflation stability at a natural rate. However, experiments using our dataset proved that the results do not differ significantly whether or not we impose this restriction. In addition, not doing so facilitates the convergence of our model.

prices), as well as the coefficients on the variable that captures the change in productivity deviation tend to be of the "wrong" sign. We would expect changes in import, food and oil prices to have a positive effect on inflation, and hence enter the model with a positive sign. As well, deviations from the trend in productivity should, all things being equal, be associated with decreases in inflation. The fact that for many of the price equations estimated here these signs are inverted is puzzling and may indicate a lack of stability in the relationship. Overall, the coefficients on the different variables are quite similar whether we use the CPI, the GDP deflator or the PCE deflator, and the adjusted R^2 of 0.80 for the CPI and 0.85 for the other two dependent variables indicate a good fit for all three regressions.

Nevertheless, point estimates of the NAIRU vary considerably between each specification, especially as we move farther into our sample period. Estimates are quite close for the first two decades but diverge by as much as 0.8 per cent by 1980Q1, with the CPI equation suggesting a natural rate of unemployment of 6.6 per cent versus 5.8 per cent for the GDP deflator and 6.1 per cent for the PCE deflator. This trend remains visible in 1990Q1 where the CPI NAIRU is at the upper end of the range of estimates and the GDP deflator at the lower end. However, the gap between our series begins to attenuate towards the end of the sample and estimates for 2000Q1 vary from 4.5 per cent (GDP deflator) to 5.2 per cent (CPI).

iii) A time-varying NAIRU using wage equations

So far, our estimates of the NAIRU have focused on the Phillips curve expressed in terms of price inflation. Although the price Phillips curve is the most often used in the literature, we can also derive a Phillips curve using wages. As noted by Gordon (1998), a direct indicator of the role of wages in the inflation process is given by the change in labour's share of national income:

(9) $$\Delta s_s = \Delta \varpi_t - \Delta \theta_t - \Delta \pi_t$$

where s_t is the change in labour's share of national income, ϖ_t represents wages, θ_t is productivity, and π_t is price inflation. If we assume that wage expectations are backward looking, we can model expected wages as $\theta^e_{t+1} = \theta^*_t$, where θ^*_t is trend productivity growth. Thus, by analogy to equation (7), we can write a wage Phillips curve as:

$$(10) \qquad (\Delta\varpi_t - \theta^*_t) = \alpha(L)(\Delta\varpi_{t-1} - \theta^*_{t-1}) + \beta(U_t - U^N_t) + \gamma(L)z_t + \varepsilon_t$$

We assume that the unobserved NAIRU behaves as in equation (8), and estimate (10) with the same time-varying parameter regression technique used for the NAIRU based on the price Phillips curve. For better comparability, we retain the same exogenous variables and the same lag structure. We estimate the system with three different wage variables: 1) the change in compensation per hour from the Bureau of Labour Statistics' (BLS) Current Employment Statistics program; 2) the growth rate of total compensation paid to employees from the Bureau of Economic Analysis' (BEA) National Accounts Table 1.10, and 3) the growth rate of wages and salary paid to employees, also from the BEA's Table 1.10. While compensation per hour is commonly used as a dependant variable in the literature, the other two variables are not - researchers generally use total compensation as well as wages and salary series from the Employment Costs Index, compiled by the BLS. However, since these series are only available from 1982 on, we judged it preferable to use the total compensation and wages and salary from the BEA rather than truncate our sample.

Regression results from the wage Phillips curve (found in Table 2) are generally comparable to those generated using a price Phillips curve. The coefficients of the unemployment gap variable are of the expected sign, with the exception of wages paid where it is positive but quite small. However, in this case the coefficients on the relative import prices and the change in productivity deviation are of the expected sign for all three dependant variables. Again, most exogenous variables enter the regression in a statistically significant manner, with the notable exception of oil prices, which are statistically significant only in the wages paid specification. Notice also that the adjusted R^2s are significantly lower than for the price Phillips curve, indicating a looser fit.

The estimates of the NAIRU — plotted in figure 5 — follow a similar pattern as those generated using a price Phillips curve. NAIRU estimates from the growth rates of compensation paid and wages paid tend to trend higher than those produced using price measures of compensation per hour, reaching 7.2 per cent in the late 1970s, but gradually coming back to 5.3 per cent towards to end of the sample. Just like NAIRU estimates generated using the GDP deflator were lower than the CPI or the PCE deflator almost everywhere, the compensation per hour series tends to produce lower and more stable estimates than the other two wage series. With regard to point estimates of the NAIRU, we see that estimates early in the sample period are quite similar, both between the different measures of wages, but also between price and wage Phillips curves. However, while the NAIRU estimated using compensation per hour follows a fairly stable path, estimates obtained with compensation paid and wages paid diverge significantly in 1970Q1 and 1980Q1. In 2000Q1, the estimated NAIRU using compensation per hour was 5.0 per cent, compared to 5.4 per cent for both total compensation paid and wages paid.

iv) Combining price and wage equations

Separate estimation of the NAIRU by price and wage Phillips curves implies that there is no feedback between the two measures of inflation. However, in reality movements in prices have an influence on wage inflation and vice versa. In fact, Phillips' original intuition, which led to the development of what is now called the Phillips curve, stemmed from his observation that lower unemployment meant firms had to offer higher wages, which led them to increase prices to compensate for those higher salaries. Hence a more realistic way of estimating the NAIRU would be to allow for feedback from prices to wages and from wages to prices. One way to do so is to follow Staiger, Stock and Watson (2001) and model price and wage inflation as a cointegrated system.

This method stems from the observation made by Staiger, Stock and Watson that price inflation and nominal wage inflation adjusted for productivity growth share a common stochastic trend that disappears when we use real wages adjusted for productivity growth. In other words, price inflation and nominal wages minus productivity growth are integrated of order 1, but real wages less productivity growth is I(0). Note that this framework is quite similar to the structural form used by Gordon (1998) and Eller and Gordon (2002), which allows for feedback from one price measure to the other. Formally, we estimate the following equations:

(11) $$\Delta\pi_t = \alpha_\pi(L)\Delta\pi_{t-1} + \alpha_{\pi\varpi}(\Delta\varpi_{t-1} - \theta^*_{t-1} - \Delta\pi_{t-1}) + \beta_\pi(U_t - U^N_t) + \delta_\pi(L)z_t + \varepsilon_{\pi t}$$

(12) $$\begin{aligned}(\Delta\varpi_t - \theta^*_t) &= \alpha_\varpi(L)(\Delta\varpi_{t-1} - \theta^*_{t-1}) + \alpha_{\varpi\pi}(\Delta\varpi_{t-1} - \theta^*_{t-1} - \Delta\pi_{t-1}) + \beta_\varpi(U_t - U^N_t)\\ &+ \delta_\varpi(L)z_t + \varepsilon_{\varpi t}\end{aligned}$$

where $(\varpi_{t-1} - \theta^*_{t-1} - \pi_{t-1})$ can be thought of as the cointegrating term of the system[10]. We estimated these equations for our three price variables as well as for the three wage variables. As is suggested by Staiger, Stock and Watson (2001) and Gordon (1998), we include lags of the cointegrating term to account for persistence in the feedback between price and wage measures. All other variables – and their lag structure – are unchanged from the original price and wage equations. We also assume that the time-varying parameter U^N behaves according to equation (8).

Figure 6 plots the NAIRU estimates from the original price and wage equations and the corresponding NAIRUs obtained through the cointegrated system. Summary regression results are found in Table 3. We observe that the NAIRUs from the equations including the cointegrating terms tend to follow the general shape of the NAIRU estimates obtained through the original price and wage equations. However, the inclusion of the cointegration relationship produces higher estimates of the NAIRU in all the different price and wage specifications and accentuates the rise in the NAIRU in the 1970s and early 1980s. Statistically speaking, regression estimates are fairly similar. However, while the inclusion of the cointegrating terms does not have a material effect on the goodness of the fit of our price equations, it significantly improves the fit of the wage equations. In the case of compensation per hour, the adjusted R-squared more than doubles. This is evidence that feedback from movements in price inflation adds a significant amount of information to the

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[10] The term $(\varpi_{t-1} - \theta^*_{t-1} - \pi_{t-1})$ is derived from the wage Phillips curve augmented with lags of price inflation: $(\Delta\varpi_t - \theta^*_t) = \alpha_1(L)(\Delta\varpi_{t-1} - \theta^*_{t-1}) + \alpha_2(L)\pi_{t-1} + \beta(U_t - U^N_t) + \delta(L)z_t + \varepsilon_t$. Rearranging, we can obtain $(\Delta\varpi_t - \theta^*_t) = [\alpha_1 + \alpha_2](L)(\Delta\varpi_{t-1} - \theta^*_{t-1}) - \alpha_2(L)(\Delta\varpi_{t-1} - \theta^*_{t-1} - \pi_{t-1}) + \beta(U_t - U^N_t) + \delta(L)z_t + \varepsilon_t$.

wage Phillips curves, which — all things being equal — improves the estimates of the NAIRU.

v) Using the duration of unemployment

The final model to be estimated in this section of the paper stems mostly from the work of Fedorov (2003), who argues that uncertainty about the NAIRU can be reduced if one takes into account significant information about labour markets found in variables other than the unemployment rate[11]. Fedorov's insight is grounded in the search and efficiency-wage theories of labour markets, in which what really matters for wage determination is the exit rate from unemployment, which determines the duration of unemployment, rather than the unemployment rate itself. In other words, what matters for unemployed workers – and what will affect their reservation wage – is the number of unemployed workers there are in relation to the number of workers being hired by firms in the economy, or the exit rate from unemployment. As such, Fedorov observes that the search theory of labour markets suggests that even with a low entry rate into unemployment, if firms are not hiring and unemployment duration is long, unemployed workers will likely lower their reservation wages because they fear a long spell of joblessness, hence putting downward pressure on wages and therefore on inflation. This is what we usually observe in recessions, and would imply that more of observed unemployment is cyclical or temporary rather than structural or permanent. Conversely, for the same unemployment rate, if the entry rate is higher and unemployment durations are shorter, unemployed workers are less likely to lower their reservation wages as they will expect to find another job quite soon. Similarly, from the point of view of the efficiency-wage theory of labour markets, the length of time an agent is left unemployed determines the size of the "punishment" it receives if he or she is caught shirking. As such, the difference between the efficiency and the market-clearing wage (necessary to discourage workers from shirking) will be smaller if unemployment duration

[11] Note that we also experimented with a number of different exogenous variables such as union participation (frequently used in European studies), the minimum wage (deflated using CPI), as well as various demographic variables. Results, however, were inconclusive and as such have not been included in the text.

(the punishment for shirking) is high. The opposite is true for low levels of unemployment duration.

Thus, if we suppose that there exists an "equilibrium" duration of unemployment at which inflation is stable (that is there is an equivalent number of workers entering and leaving the unemployed pool), changes in the unemployment rate will affect inflation only if they are associated with corresponding changes in unemployment duration. In Fedorov's words:

> "Thus, we are able to distinguish between cyclical and permanent changes in the unemployment rate by observing what happens with unemployment duration. If an increase or decrease in the unemployment rate is accompanied by an increase or decrease in unemployment duration, it is interpreted as cyclical or transitory. Otherwise, it is considered permanent; that is, it coincides with an increase or decrease in the NAIRU".

This suggests that the information contained in the duration of unemployment could potentially improve our estimates of the NAIRU. Note, however, that the relationship between duration and the NAIRU outlined here is not consistent with all theories of wage determination. In particular, in a world in which "insiders" (those in employment) have significant wage bargaining power (perhaps because of unions) and "outsiders" (the unemployed), higher duration of unemployment does not necessarily imply less pressure on wages. This is because insiders, who are the ones playing a determinant role in the wage-setting process, can largely ignore outsiders (including the unemployed) and thus may be able to push wages higher despite high levels of the duration of unemployment. However, these models are perhaps more useful in explaining the persistence of unemployment in economies with high union density and strong employment protection legislation, such as many Western European countries, than the U.S. economy.

Going back to the problem at hand, we see that Fedorov's intuition about the importance of the information contained in the duration of unemployment can be easily integrated to our framework by adding the average duration of unemployment (with its lags) as an explanatory variable into our cointegrated wage equation. This would allow changes in wage inflation to be determined both by price inflation (through our cointegrating term) and by the duration of unemployment (adjusted for trend productivity growth). Following this reasoning, equation (12) becomes:

$$(13)\quad \begin{aligned}&(\Delta\varpi_t - \theta_t^*) = \alpha_\varpi(L)(\Delta\varpi_{t-1} - \theta_{t-1}^*) + \alpha_{\varpi\pi}(L)(\Delta\varpi_{t-1} - \theta_{t-1}^* - \Delta\pi_{t-1}) + \xi(L)(\log D_t)\\&+\beta_\varpi(U_t - U_t^N) + \delta_\varpi(L)z_t + \varepsilon_{\varpi\pi t}\end{aligned}$$

Estimates of the NAIRU from equation (13) are plotted in Figure 7 and the regression output is found in Table 4[12]. From our results, it appears that Fedorov's insight is correct as equation (13) produces the best fit of all the reduced-form equations we have estimated so far (as well as the highest log likelihood). There is a negative relationship between the unemployment gap and inflation (as expected) as well as on the measure of unemployment duration, which is statistically significant. The negative sign on duration is consistent with our theoretical framework as it implies that the higher the duration of unemployment is, the lower employees reservations wages will be, thus putting downward pressure on wage inflation. The coefficients on the exogenous variables are of the expected sign, with the exception of oil prices where the estimation implies a negative relationship between oil prices and wage inflation. However, because oil prices are not statistically significant in the model, it is easier to disregard this counter-intuitive result. All other exogenous variables are jointly significant at 5 per cent.

The estimates of the NAIRU generated are generally similar to those produced using other functional forms. The NAIRU rises through the 1960s and 1970s and peaks around 1980 to gradually decrease until the end of the sample. Notice, however, that estimates produced by the duration model have somewhat more variability than most other functional forms (despite having a roughly similar signal-to-noise ratio). Estimates suggest that the NAIRU was around 5.1 per cent in 1960 - lower than most other functional forms reviewed so far – peaked at a little over 7 per cent in 1980 and then decreased throughout the 1980s and 1990s to settle around 5 per cent in 2000 and thereafter.

12 To avoid fruitless repetition, we present results for only one wage variable, compensation per hour. This choice is based on the much better fit of this variable in the cointegrated wage Phillips curve framework (Table 3) and the fact that it is the wage variable most commonly used in the literature on the NAIRU.

In sum, evidence clearly points to a significant decrease in the NAIRU in the 1980s and 1990s, as most economists have suspected. While the NAIRU gained a total of 1.6 per cent from 1960 to 1980, it then decreased by 2.1 per cent in the following twenty years. Note, however, that the largest five-year decrease in the NAIRU did not occur in the 1990s, but in the five years immediately following its peak (1980-1985), where it decreased by 0.75 percentage points. Nevertheless, the next five years (1985-1990) also saw an impressive decrease in the NAIRU (-0.51 percentage points), as did the first and second half of the 1990s, where the NAIRU decreased by a total of 0.81 percentage points. Finally, the pace of decrease in the NAIRU appears to have slowed considerably towards the end of the sample as it edged down by a mere 0.04 percentage points in the four years between 2000 and 2004.

3. Model selection

Now that we have surveyed a number of ways to model the NAIRU, we need to narrow our options and examine which method is more appropriate for the purpose at hand. We have already disqualified structural models on the grounds of parsimony and ease of estimation. We can also disqualify reduced-form models where the NAIRU is deemed to be a constant, as we find it difficult to accept the notion that the NAIRU does not change over time. The structure of the economy, the behaviour of firms and workers, as well as the constraints they face, have changed substantially since the 1960s, and we cannot find justification in the theory to accept that, despite these changes, the NAIRU remains the same today as in the 1950s.

This narrows our options to statistical and time-varying reduced-form models. More precisely, we have the choice between 15 different models: two multivariate filters (one with a price inflation variable and one with a wage inflation variable[13]), 12 different time-varying models of price and wage inflation (differing by whether or not they allow for feedback and the measure of price or wage inflation used), and one cointegrated wage equation that includes the duration of unemployment. In order to choose between all these competing

[13] Again, for brevity, we present only the MV filter for CPI inflation and compensation per hour.

models, we use 3 different procedures to evaluate the precision and robustness of our results.

A) Model selection: criteria

i) Forecast accuracy

First, since we derive the NAIRU from Phillips curves, it follows intuitively that a "good" model of the NAIRU should explain movements in inflation fairly well. Consequently, a test of the accuracy and robustness of the different models would be to evaluate how well they predict their respective inflation variable. In the case of the state space models, this is fairly straightforward and only necessitates rewriting our equations as a system and plugging in the estimated coefficients. In the case of the multivariate filter, we adopt the approach used by Szeto and Guy (2004) and first compute the unemployment gap generated by the filter (i.e. $U_t - U_t^N$) and then estimate the wage or price Phillips curve by OLS using the estimated unemployment gap as a regressor. The forecasts are evaluated using the root mean squared error (RMSE):

$$\text{RMSE} = \sqrt{\frac{1}{h+1}\sum_{t=s}^{s+h}(\hat{y}_t - y_t)^2} \tag{15}$$

where $\hat{y}_t$ is the forecasted value and y_t is the actual value. Results of the forecasting experiment are found in Table 5[14].

[14] In addition to the RMSEs, we also computed the modified Diebold-Mariano test statistic that includes the adjustment suggested by Harvey, Leybourne, and Newbold (1998). The test statistic - which follows a Student-*t* distribution and is abbreviated as HLN-DM – serves to indicate whether or not there is a statically significant difference in forecast accuracy between the forecasts generated using the multivariate filters and the ones produced using reduced-form models. On the basis of the HLN-DM statistics, we could not find any evidence of a statistical difference between the filters and the reduced-form models. However, while we computed the HLN-DM test statistic for reasons of completeness and comparability with other studies in the literature on forecasting, we put little faith in the ability of the test to confidently detect differences in forecast accuracy. As is documented by Bram and Ludvigson (1998), Clark (1999), and more recently in Luger (2004), the HLN-DM statistic is plagued by numerous problems. In this case, the lack of power of the test is made worse by the fact

We notice that, although the RMSEs are quite small for all the different competing specifications, both multivariate filters have larger RMSEs than the reduced-form models. This tends to confirm that we can gain accuracy by using reduced-form models with a theoretical foundation rather than atheoretical statistical models. Of all the reduced-form specifications, it appears the price Phillips with feedback (GDP deflator) is the most accurate with an RMSE of 0.014, followed closely by the simple price Phillips curve (GDP deflator) and the price Phillips with feedback (PCE deflator) with a root mean square error of 0.016. The least accurate forecasts of inflation were produced using the variables compensation paid and wages paid in the context of a simple wage Phillips curve. The addition of a cointegrating term to the wage Phillips curve improves forecast accuracy, but only slightly[15].

ii) Goodness of fit

In addition to forecast accuracy, Table 5 also reports the value of the Akaike Information Criteria (AIC). The AIC is often used to discriminate between different nested or non-nested models as it relatively easy to compute, provide we have the log-likelihood function, and offers a straightforward and comparable measure of goodness of fit. Formally, we can say that the lower the AIC, the closer the estimated model is to the true data-generating process. In the context of a regression the AIC can be expressed as:

(16) $$\frac{-2\ell}{\mathrm{T}}+\frac{2k}{\mathrm{T}}$$

where k is the number of regressor and ℓ is the log-likelihood function defined as:

(17) $$\ell=-\frac{\mathrm{T}}{2}(1+\log(2\pi)+\log(\frac{\hat{\varepsilon}'\hat{\varepsilon}}{\mathrm{T}})$$

that the differences between our RMSEs are small. As such, we do not report the HLN-DM test statistic in Table 5.

15 Similar results were obtained in an out-of-sample forecasting experiment where we estimate the different equations from the first period in the sample to the fourth quarter of 2000 and then compare forecasts over the period ranging from 2000Q1 to 2004Q2.

From Table 5, we see that in the class of univariate models (i.e. that do not allow for feedback between wages and prices), those based on a price Phillips curve appear to better fit the underlying data generating process than those based on wage indicators. This is consistent with estimation results presented in Tables 1 and 2 but goes against results from our forecasting experiment where price Phillips curve models generally outperformed wage-based models. Also, with the exception the model using CPI, price Phillips curves that allow for feedback tend to have a better fit, as measured by the AIC, than other model classes, whether wage or price based or cointegrated or not. Nevertheless, again it appears that the addition of the duration of unemployment to the wage Phillips curve improves the fit as this model outperforms all others on the basis of the AIC criteria.

iii) Accuracy

An additional criterion, and in many ways a more important one for the purpose at hand, is the degree of precision of the actual estimates of the NAIRU generated using the different specifications (measured by the estimates' standard errors). It is well documented that NAIRU estimates typically have large standard errors (see, for example, Staiger, Stock, and Watson (1996)). Furthermore, errors in NAIRU estimates stem not only from errors inherent to the statistical method used (errors we can measure), but also from uncertainty with regard to the "real" distribution of the NAIRU (which unfortunately is impossible to quantify). In any case, the lower the standard error around our estimates of the NAIRU, the more confidence we can have in our estimates and the better they are for the purpose of estimating and forecasting the output gap.

The last column of Table 5 reports the standard error of the last data point (2004Q1) for the different estimates of the NAIRU for each reduced-form method. We report the standard error around the last data point because it is the most interesting to us in the context of forecasting and also because standard errors around the state variable in state space models tend to grow larger as $t \rightarrow T$. The standard errors vary from 1.46 for the univariate wage Phillips curve, to 0.48 for the cointegrated wage Phillips curve with the duration of unemployment. While this difference can appear small, one needs to remember that a 95 per cent confidence interval around an estimated value of the NAIRU is $\pm$ twice the standard error.

Hence, what emerges from Table 5 is that taking the duration of unemployment into account significantly improves the precision of the NAIRU estimates. Even when we consider the second most precise estimate, generated using a univariate wage Phillips curve with wages paid as the dependant variable, the difference in the standard errors increases the error band by 0.78 below and above the estimated NAIRU. More precisely, according to the cointegrated wage Phillips curve with the duration of unemployment the NAIRU stood at 5.0 per cent in the first quarter of 2004 with a 95 per cent per cent confidence interval that ranges from 4.1 per cent to 5.9 per cent. The second most precise functional form generates an estimate of the NAIRU for the first quarter of 2004 of 4.9 per cent with a 95 per cent confidence interval ranging from 3.2 per cent to 6.6 per cent. Therefore, while there is significant uncertainty surrounding NAIRU estimates even when using the cointegrated wage Phillips curve with the duration of unemployment, it is clearly the most precise of all 13 reduced-form specifications estimated here. This is especially visible when we plot each set of estimates and their error bands (Figure 8). We believe that our estimates of the NAIRU are improved by the addition of the duration of unemployment mainly because doing so incorporates information about long-run movements in unemployment that is not contained in the unemployment rate and other variables commonly used. If changes in the unemployment rate affect inflation only to the extent that they are accompanied by changes in the duration of unemployment — as argued by Fedorov (2003) — it follows that information contained in the duration of unemployment will be critical to the purpose of estimating the NAIRU.

Furthermore, the confidence intervals surrounding our estimates of the NAIRU using the cointegrated wage Phillips curve with the duration of unemployment are smaller than those typically found in the literature. For example, Staiger, Stock, and Watson (1996) find that a typical value of the constant NAIRU in 1990 was 6.2 per cent with a confidence interval ranging from 5.1 per cent to 7.7 per cent. Staiger, Stock and Watson (2001) find a NAIRU of about 4.9 per cent in 2000 with a 95 per cent confidence interval ranging from 3.8 per cent to 6.0 per cent using a time-varying specification similar to equation (11). Similarly, Greenslade, Pierse, and Saleheen (2003) estimate the U.S. NAIRU to be 4.5 per cent in 1999 with a 95 per cent confidence interval ranging from –1 per cent to about 10 per cent, and Turner et al. (2001) find a NAIRU of 5.2 per cent in 1999Q1 with a confidence interval of 4.0 per cent to 6.4 per cent using a time-varying specification similar to equation (7). These comparisons are highlighted in Table 6.

Overall, estimates produced using our new preferred model do not differ significantly from those produced using our current method (Figure 9). Naturally, estimates obtained through the reduced-form model exhibit much less variation than those obtained through a statistical filter, in this case the MV filter. Like the MV filter estimates, the reduced-form estimates trend up throughout the 1960s and 1970, but whereas they peak in 1980, the MV filter estimates peak in 1983. And although the end-point estimates for 2004Q3 are quite similar – 5.2 per cent with our current method and 5.0 per cent using the reduced-form model – it is only because the MV estimates were arbitrarily forced down by applying judgment on the original series produced with the HP filter (see Figure 3).

4. Concluding remarks

In this paper, we estimated a wide array of models of the U.S. NAIRU, ranging from simple univariate filters to sophisticated time-varying parameter models. We find that the most accurate estimates of the NAIRU are obtained using a time-varying model based on a wage Phillips curve that allows for feedback between wage and price inflation and that incorporates the duration of unemployment as a regressor. This method produces estimates of the U.S. NAIRU that are consistent with the recent literature on the NAIRU and represent an improvement on our current multivariate filter approach. These estimates are also generally more precise than those of most models of the U.S. NAIRU found in the literature.

Using this model we find that the U.S. NAIRU followed what resembles an inverted U shape between 1960 and 2004, rising steadily until its peak in 1980 and then decreasing at a fast pace for the next 20 years, settling at around 5.0 per cent after 2000. Moreover, we find that oil prices are not statistically significant in the model suggesting that price pressures coming from the recent increase in oil prices should not have an effect on the medium-term NAIRU. Looking forward, future research is likely to focus on advanced modelling methods for unobserved variables. An area that looks promising a priori is the broad category of non-linear models of unobserved variables. As mentioned by Szeto and Guy (2004), there is significant evidence that the unemployment rate adjusts in a non-linear manner. In particular, the use of standard logistic smooth transition models (LSTAR), as done by Skalin and Teräsvirta (2002) and Bärdsen et al. (2003), is likely to provide new information on the behaviour of unemployment, which could be used in modelling the NAIRU.

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6. Figures

Figure 1: U.S. NAIRU Obtained by HP filter

Quarterly data, $\lambda = 1600$

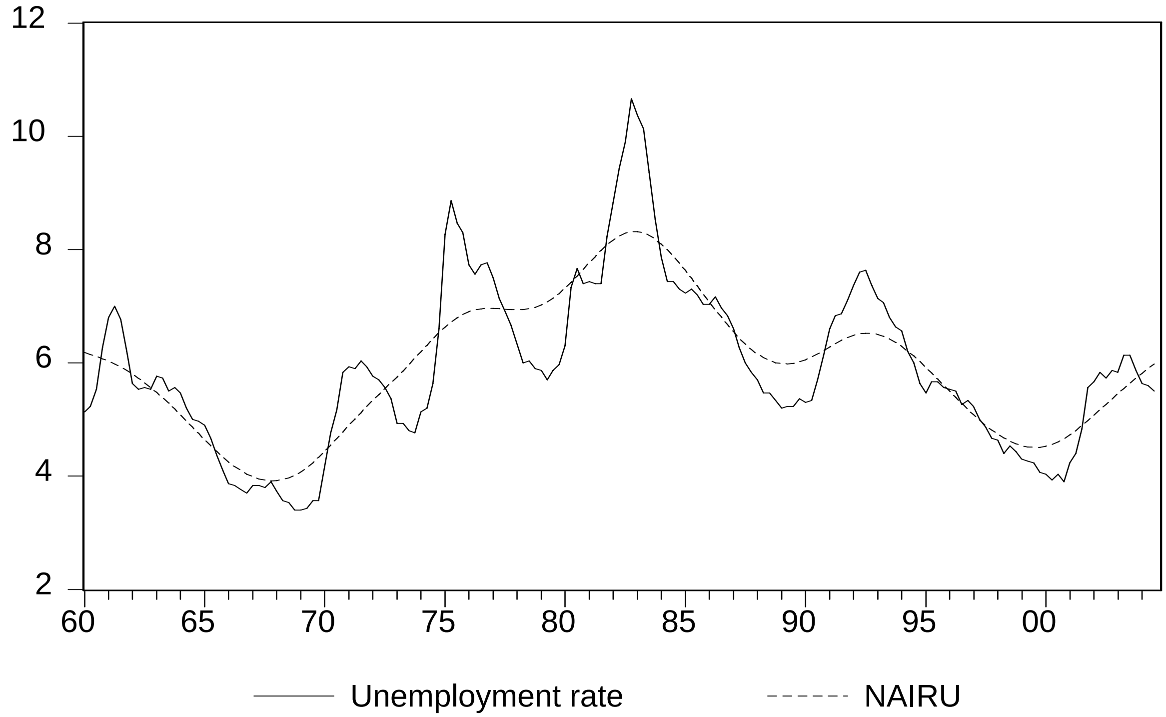
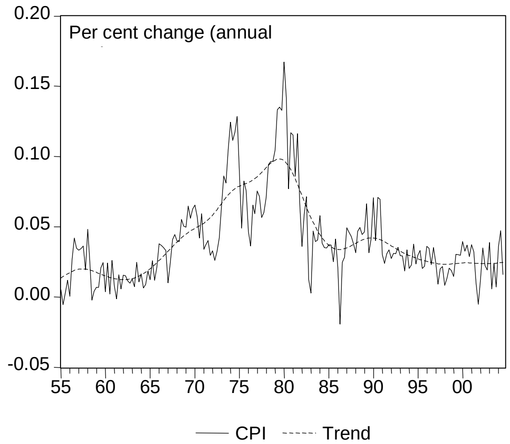
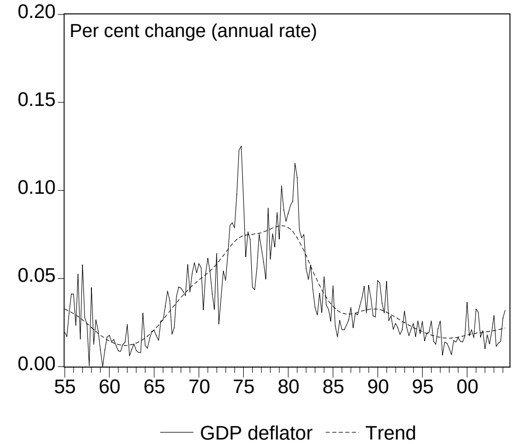


Figure 2: Time series with trend values[16]

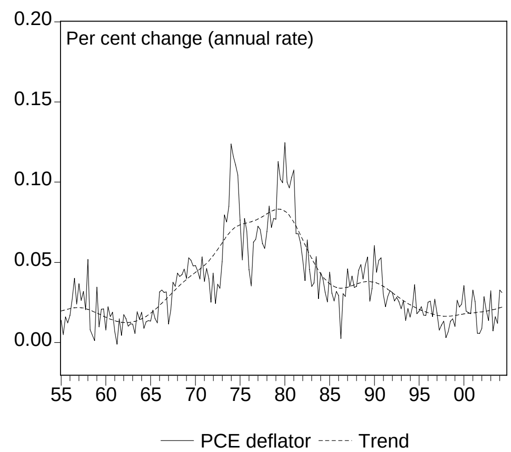
a. CPI



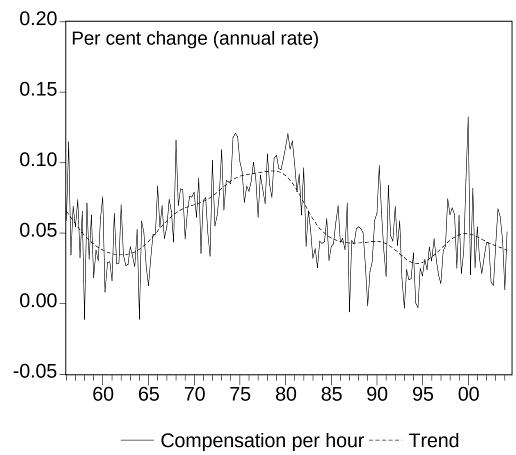
b. GDP deflator



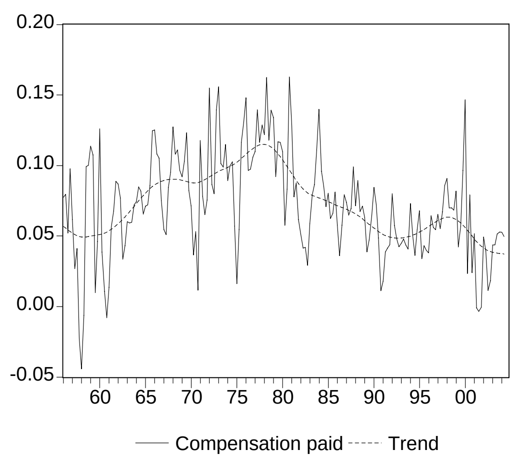
c. PCE deflator



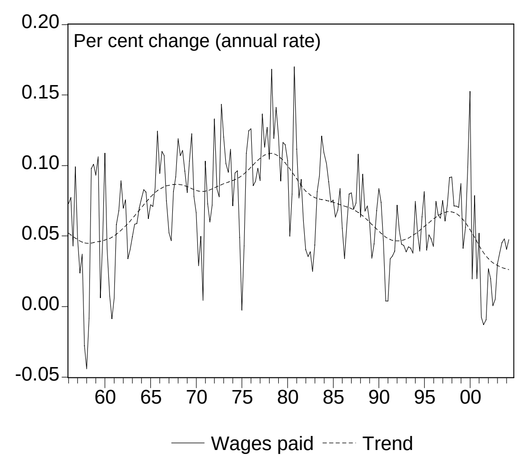
d. Compensation per hour



e. Compensation paid



f. Wages paid



[16] Trend determined using the HP filter.

Figure 3: Comparison between U.S. NAIRU obtained by HP and MV filter

Quarterly data, $\lambda = 1600$

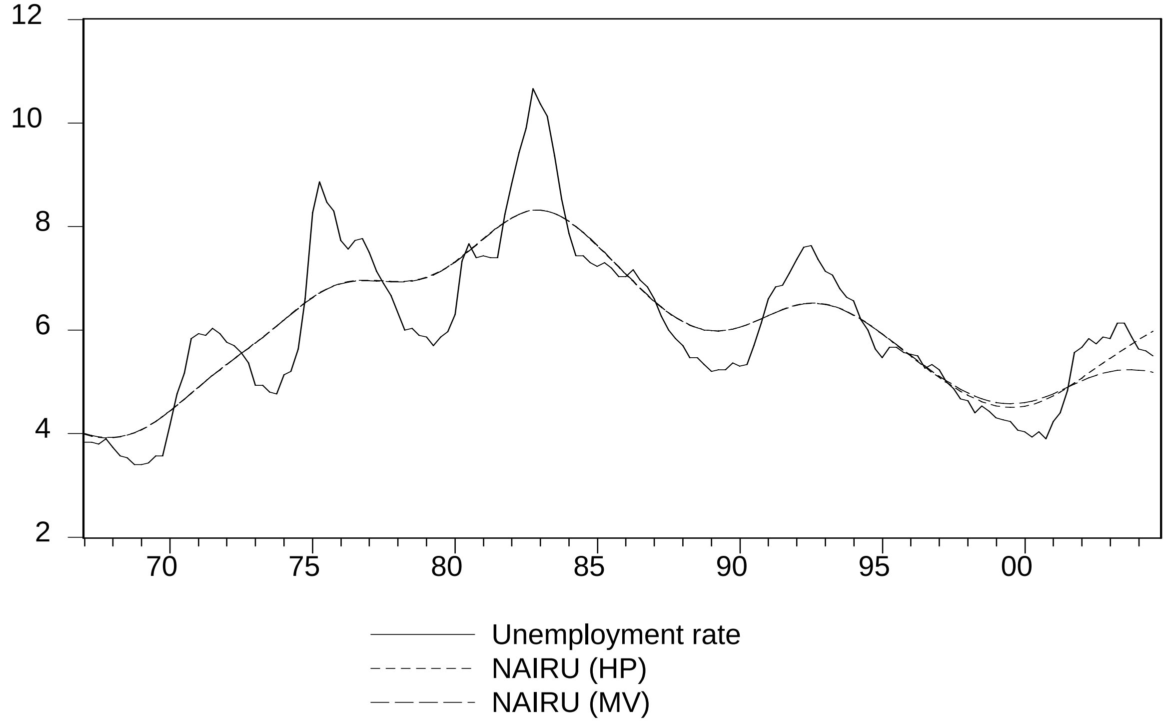


Figure 4: Estimated NAIRU – Price Phillips curve

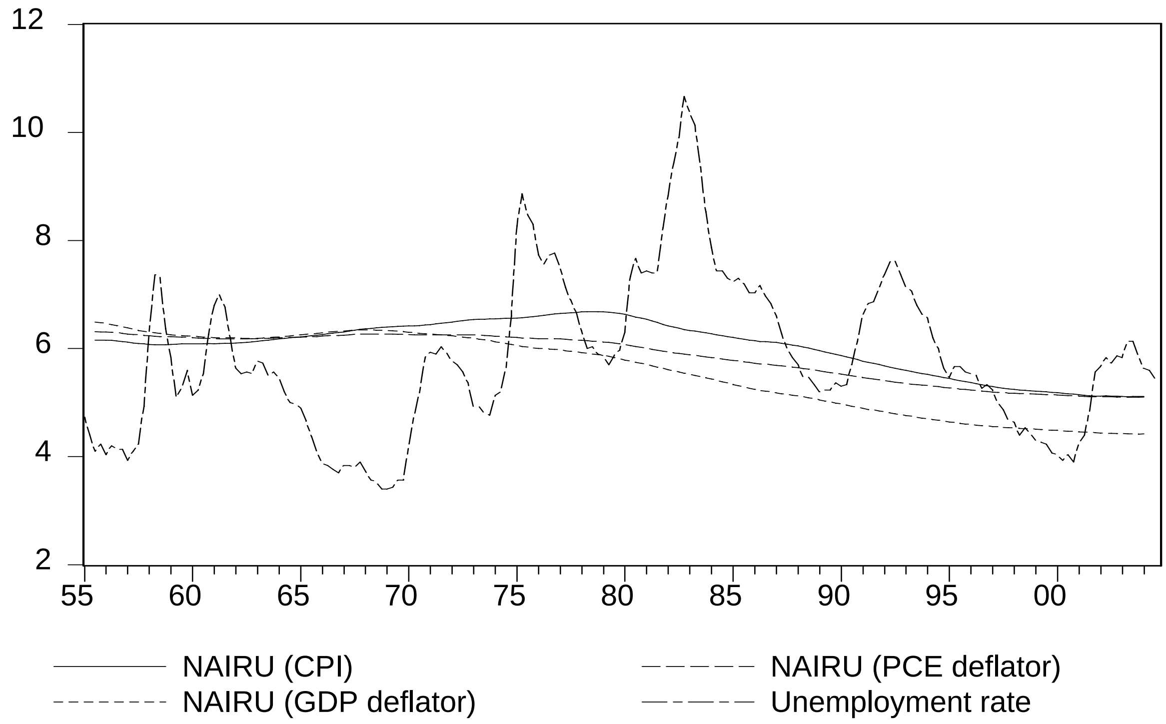


Figure 5: Estimated NAIRU – Wage Phillips curve

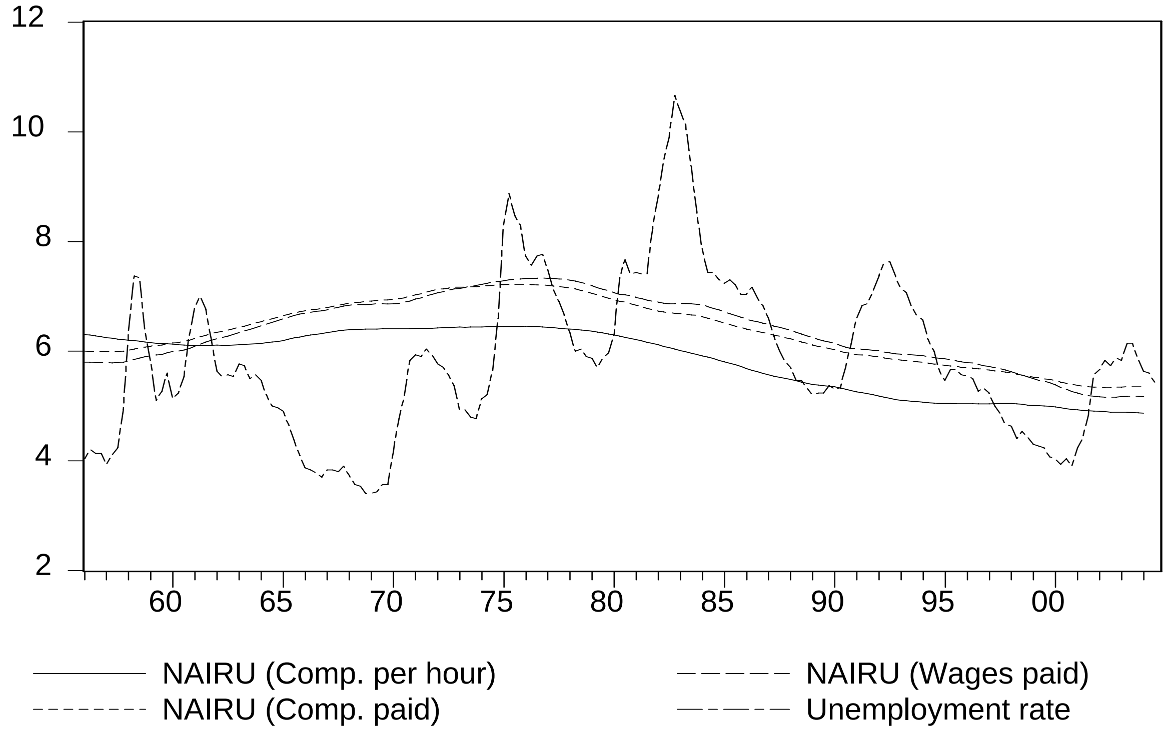
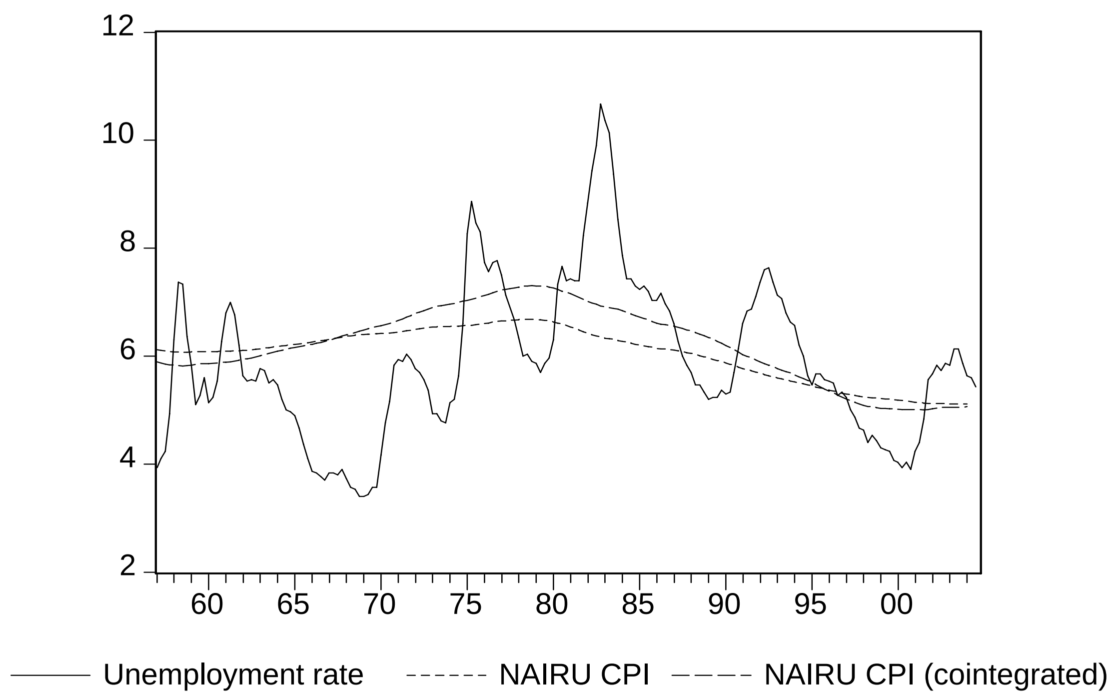
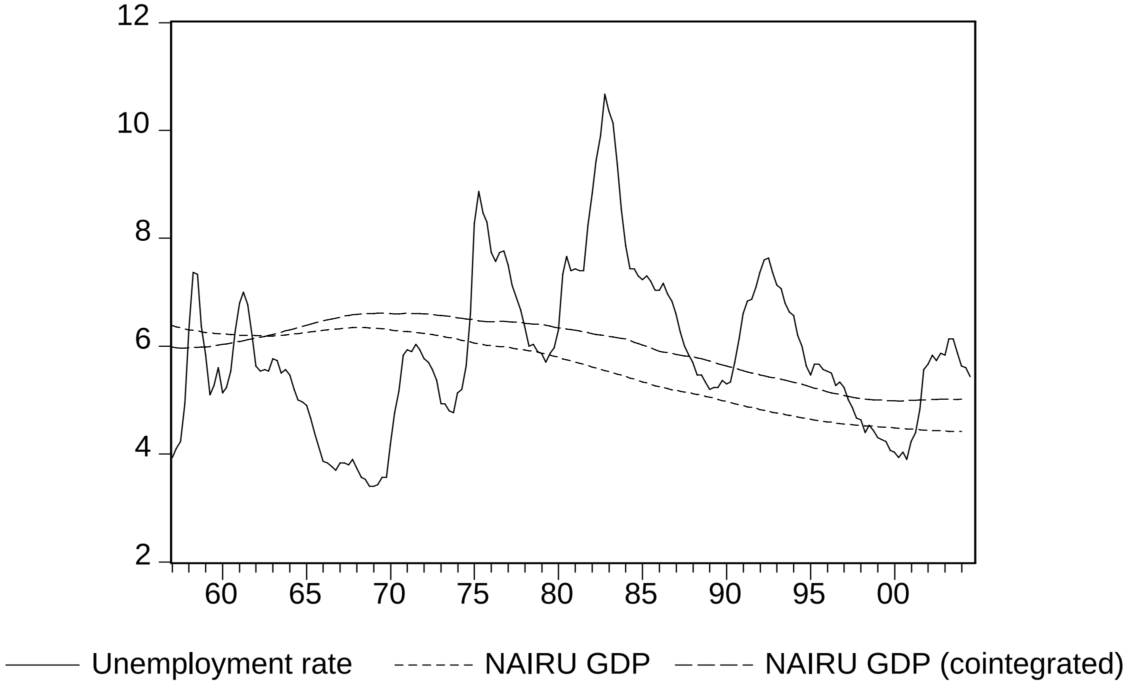


Figure 6: NAIRU estimates – original price and wage equations vs. cointegrated system

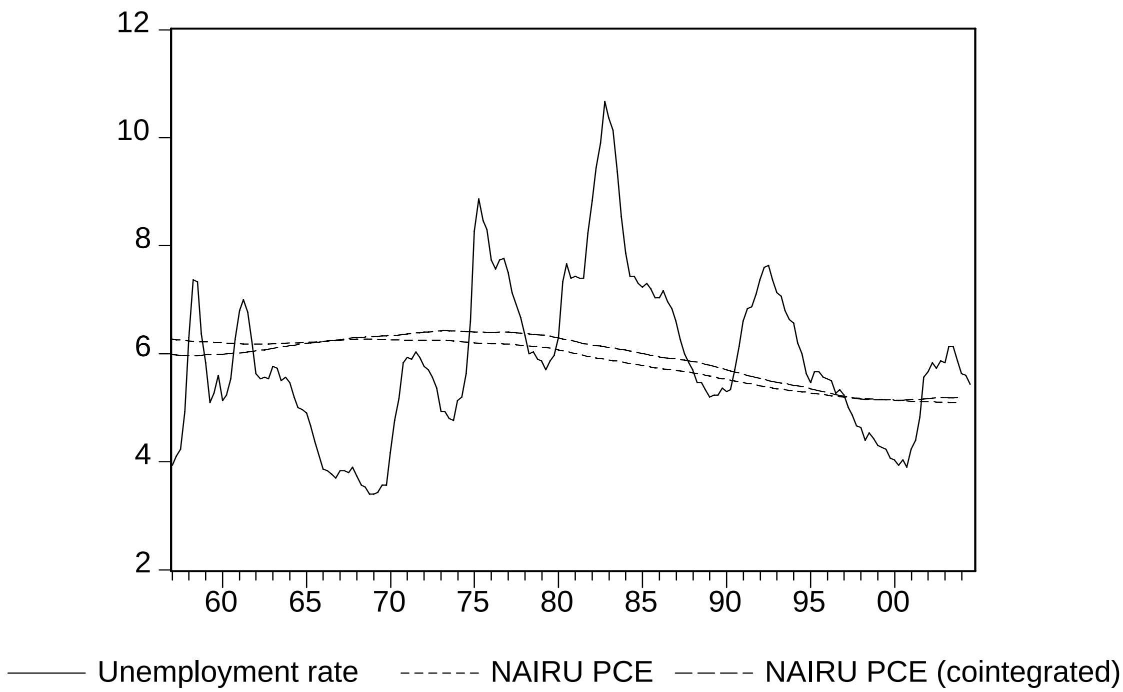
a. CPI



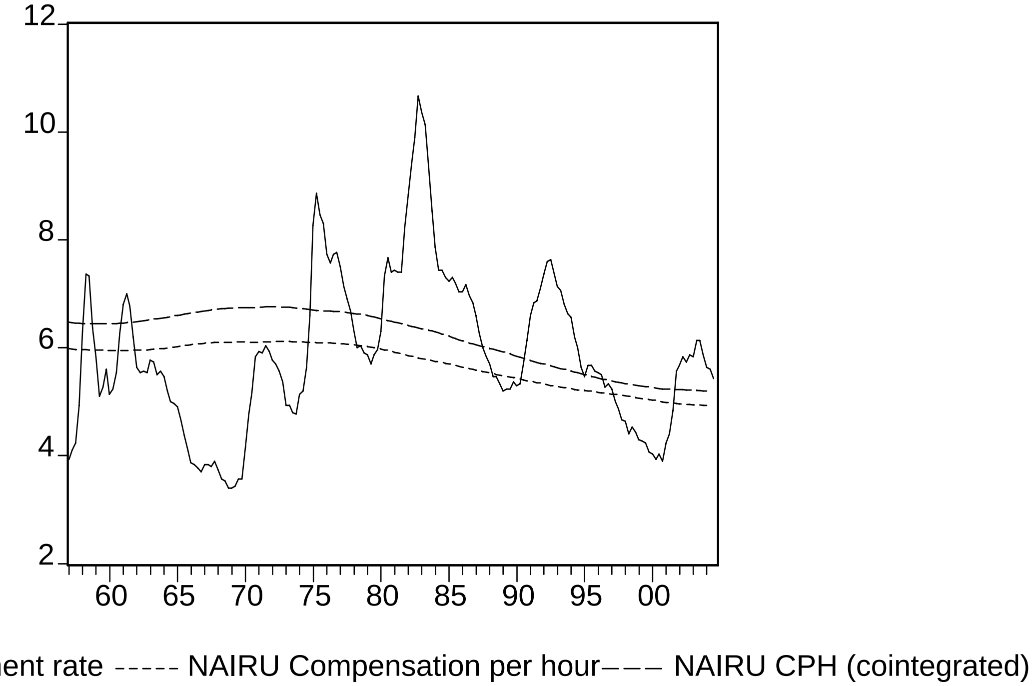
b. GDP deflator



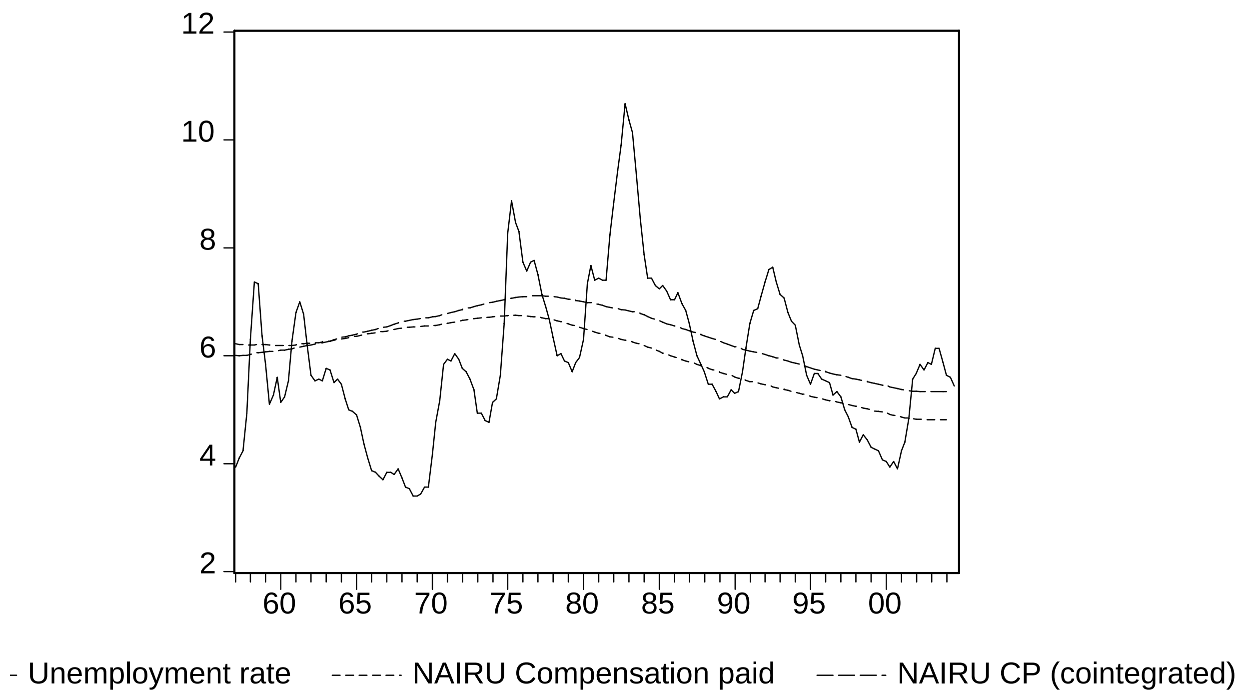
c. PCE deflator



d. Compensation per hour



e. Compensation paid



f. Wage paid

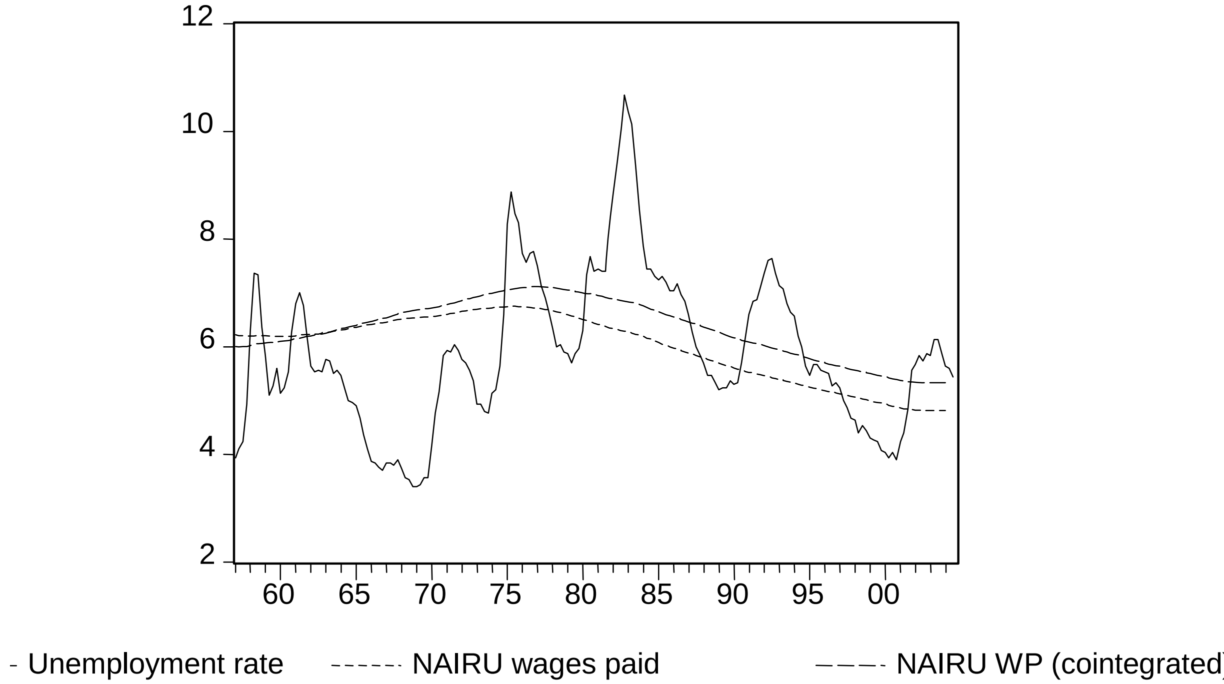


Figure 7: NAIRU estimates – cointegrated wage Phillips curve including the duration of unemployment

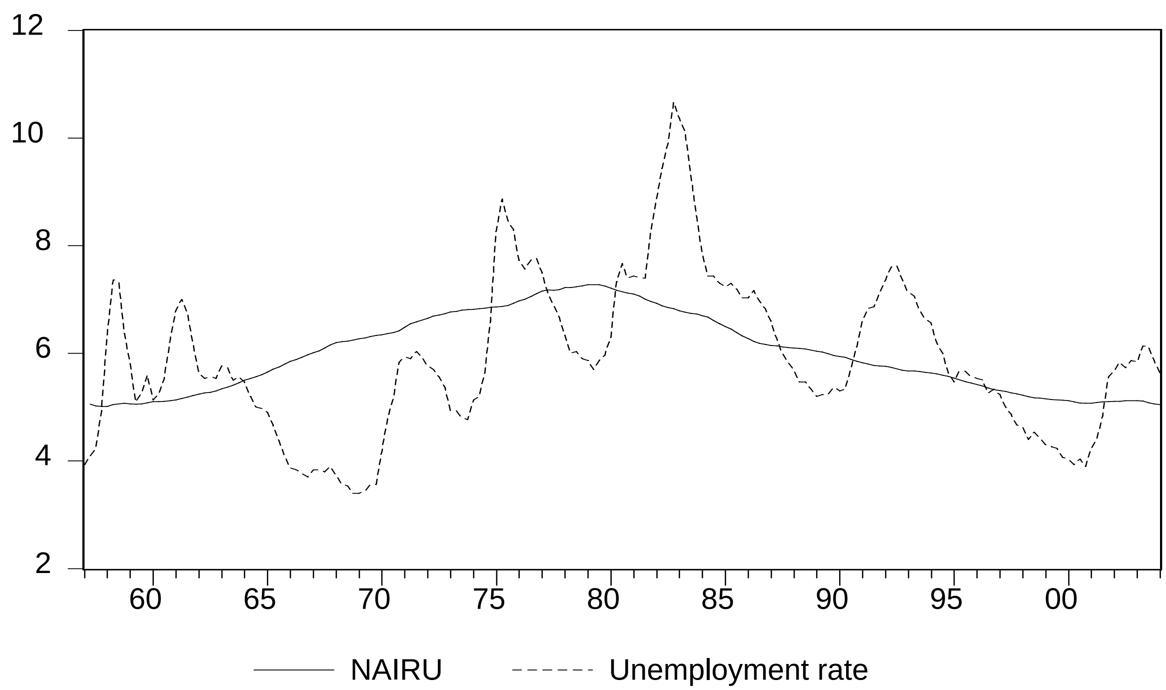
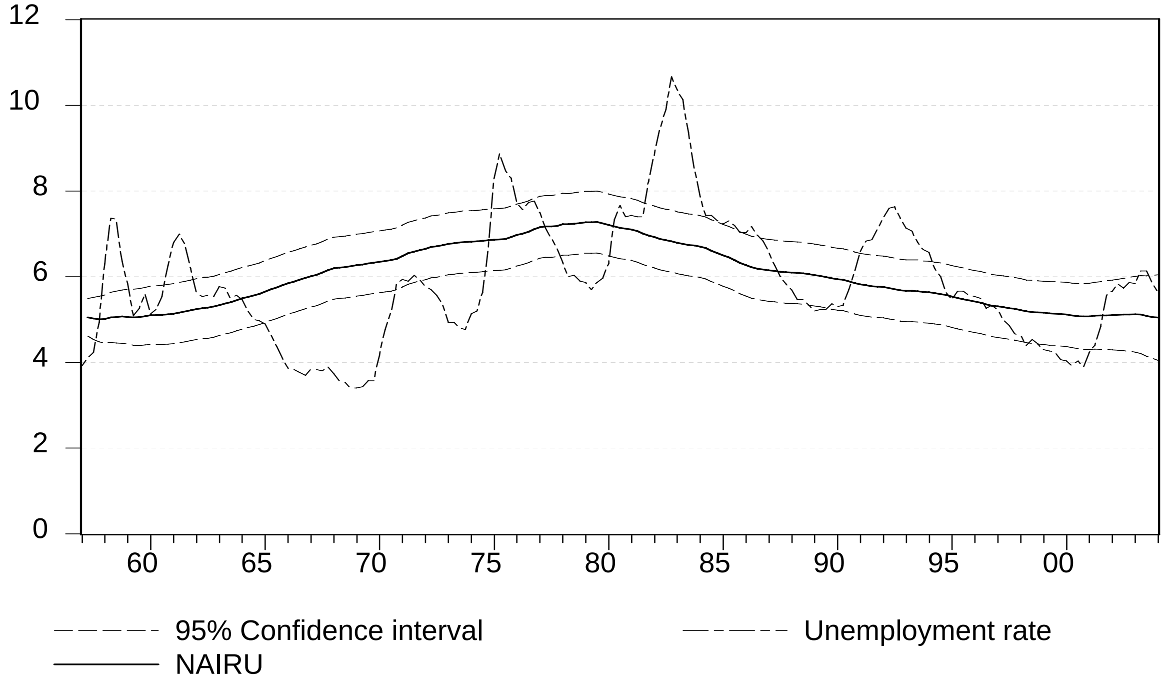


Figure 8: NAIRU estimates and 95% confidence interval

a. Cointegrated wage equation w/ duration



b. Univariate wage equation (wages paid)

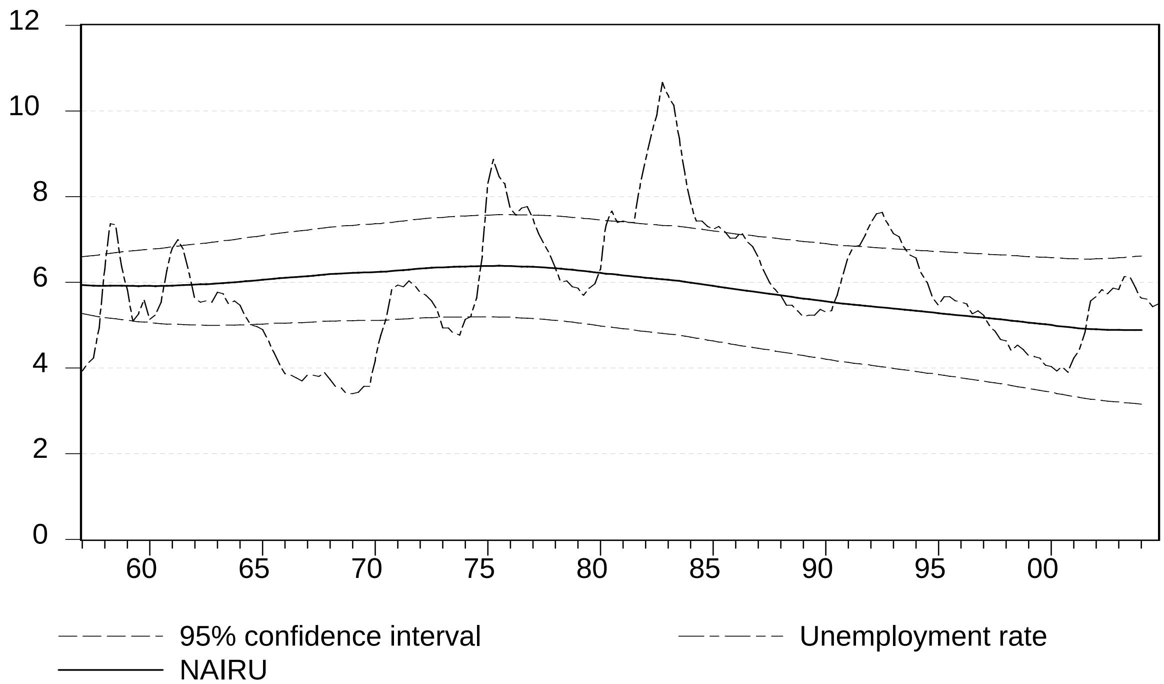
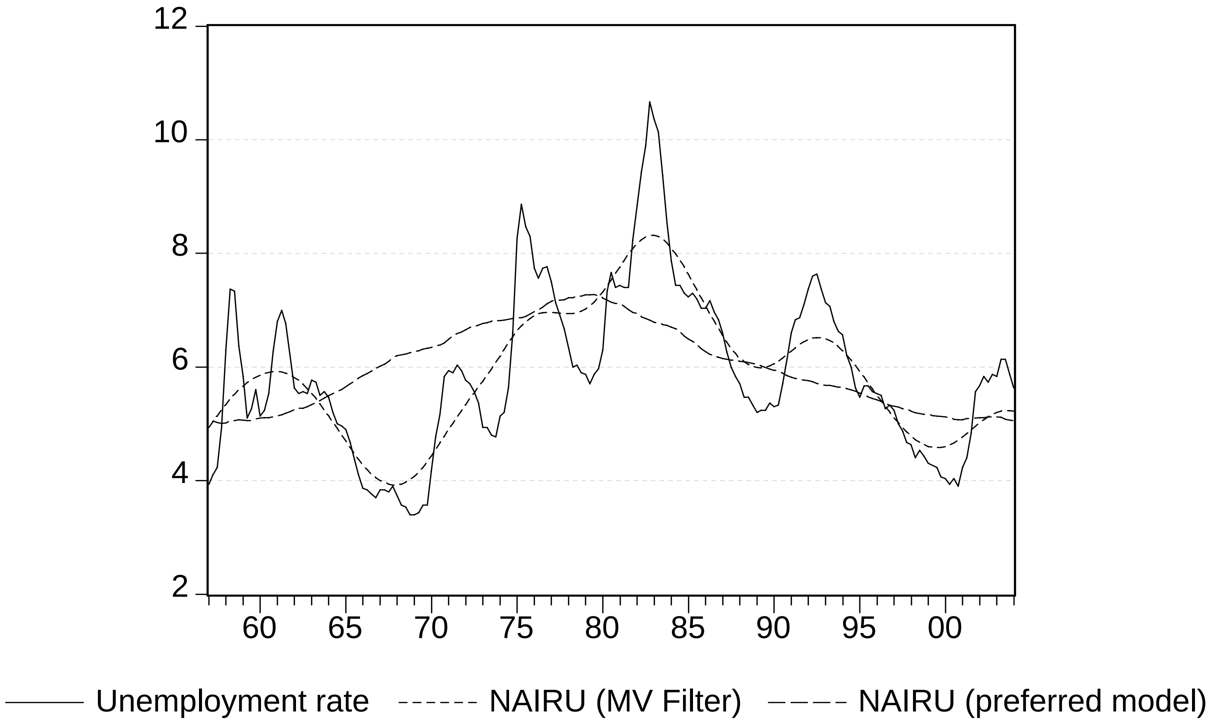


Figure 9: Comparison between MV filter estimates and cointegrated wage Phillips curve including the duration of unemployment



7. Tables

Table 1: Estimation output from price Phillips curve

	Lags	*CPI*	*GDP deflator*	*PCE deflator*
Lagged inflation	1-4	0.918*	0.868*	0.890*
$U-U^N$	-	-0.006*	-0.008*	-0.006*
Change in relative import prices	1-4	-0.193	0.486*	0.248*
Food prices	0-4	-0.254*	-0.161*	-0.151*
Oil prices	0-4	0.001*	0.001	0.002*
Change in productivity deviation	0-4	0.142*	0.068*	0.096*
Nixon controls	-	-0.003	-0.013*	-0.007*
$\overline{R}^2$		0.80	0.85	0.85
Log Likelihood		545.9	553.2	545.1
Point estimates of NAIRU:	1960:01	6.1	6.2	6.2
	1980:01	6.6	5.8	6.1
	2000:01	5.2	4.5	5.1

Note: All coefficients are summed.
* Indicates that coefficients are jointly significant at 5 per cent level

Table 2: Estimation output from wage Phillips curve

	Lags	*Comp. Per hour*	*Comp. Paid*	*Wages paid*
Lagged wage variable	1-4	0.651*	0.801*	0.810*
$U-U^N$	-	-0.003*	-0.002*	0.001*
Change in relative import prices	1-4	0.737*	0.176*	0.349*
Food prices	0-4	-0.209*	-0.161*	-0.222*
Oil prices	0-4	-0.003	-0.088	-0.009*
Change in productivity deviation	0-4	-0.010*	-0.205*	-0.191*
Nixon controls	-	0.005	-0.017	-0.004
$\overline{R}^2$		0.39	0.38	0.39
Log likelihood		418.4	404.1	395.9
Point estimates of NAIRU:	1960:01	6.1	6.1	6.0
	1980:01	6.3	6.9	7.1
	2000:01	5.0	5.4	5.4

* Indicates that coefficients are jointly significant at 5 per cent level

Table 3: Estimation output from price and wage Phillips curve (cointegrated system)

Dependent variable	$\bar{R}^2$	*Log likelihood*	*Point estimate of NAIRU*		
			1960Q1	1980Q1	2000Q1
CPI	0.81	434.0	5.9	7.3	5.0
GDP deflator	0.84	604.6	6.0	6.3	5.0
PCE deflator	0.84	601.7	6.0	6.3	5.1
Compensation per hour	0.91	595.6	6.4	6.5	5.3
Total compensation paid	0.57	437.9	6.1	7.0	5.4
Wages paid	0.56	433.2	6.0	6.8	5.4

Table 4: Estimation output from cointegrated wage Phillips curve including the duration of unemployment

	Lags	*Coefficients*	Change in the level of the NAIRU (%)	
Lagged compensation per hour	1-4	0.778*	1960-1965	0.62
U-U^N	-	-0.004*	1965-1970	0.47
$(\varpi_{t-1} - \theta^*_{t-1} - \pi_{t-1})$	1-4	0.310*	1970-1975	0.51
Lagged duration of unemployment	1-4	-0.006*	1975-1980	0.27
Change in relative import prices	1-4	0.080*	1980-1985	-0.75
Food prices	0-2	0.071*	1985-1990	-0.51
Oil prices	0-4	-0.001	1990-1995	-0.41
Change in productivity deviation	0-2	-0.010*	1995-2000	-0.40
Nixon controls	-	-0.007	2000-2004	-0.04
$\bar{R}^2$		0.92		
Log likelihood		607.8		
Point estimates of NAIRU:	1960:01	5.2		
	1980:01	7.2		
	2000:01	5.1		

* Indicates that coefficients are jointly significant at 5 per cent level

Table 5: Root mean squared errors (RMSE) of inflation forecasts, standard errors (S.E.) of NAIRU estimates, and Akaike Information Criteria (AIC).

Model	**RMSE**	**AIC**	**S.E.**
1) Multivariate Filter			
▪ Price equation	0.049	-	-
▪ Wage equation	0.047	-	-
2) Price Phillips curve			
▪ CPI	0.024	-5.37	1.44
▪ GDP deflator	0.016	-5.45	1.07
▪ PCE Deflator	0.017	-5.33	0.98
3) Wage Phillips curve			
▪ Compensation per hour	0.029	-4.07	1.46
▪ Compensation paid	0.041	-4.01	1.16
▪ Wages paid	0.042	-3.81	0.86
4) Price Phillips curve with feedback			
▪ CPI	0.028	-4.16	1.03
▪ GDP deflator	0.014	-5.95	1.06
▪ PCE Deflator	0.016	-5.92	1.21
5) Wage Phillips curve with feedback			
▪ Compensation per hour	0.016	-5.86	0.96
▪ Compensation paid	0.037	-4.21	1.03
▪ Wages paid	0.038	-4.16	1.18
6) Wage Phillips curve with feedback and duration of unemployment	0.021	-6.06	0.48

Table 6: Comparison of standard errors around different estimates of U.S. NAIRU

Author	**Reference year**	**Estimate**	**Range of estimates**
Stock and Watson (1996)	1990	6.2%	5.1% - 7.7%
Turner et al. (2001)	1999	5.2%	4.0% - 6.4%
Staiger, Stock and Watson (2001)	2000	4.9%	3.8% - 6.0%
Greenslade, Pierse, and Saleheen (2003)	1999	4.5%	-1.0% - 10.0%
Blouin (2005)	2000	5.1%	4.2% - 6.1%

Technical Annex 1: The Kalman Filter Algorithm[17]

Let y_t be a vector of variables observed at time *t*. y_t can be represented in terms of an unobserved vector ζ_t called the state vector and be represented by the following system of equations:

$$y_t = A'x_t + H(z_t)'\xi_t + w_t$$
$$\xi_{t+1} = F\xi_t + v_{t+1}$$

This system is called the state-space representations of $\mathbf{y}_t$, where A, H, and F are coefficient matrices and *x* is a vector of exogenous variables. In addition, we assume that:

$$\sigma_w^2 = R,\ \sigma_v^2 = Q,\ E(w_t, v_t) = 0$$

A popular algorithm used to estimate the parameters A, H, F, R, and Q and make inferences about ζ_t is called the Kalman algorithm or Kalman filter. The Kalman filter calculates linear least squares forecasts of ζ_t in relation to data observed up to time *t* such that:

$$\hat{\xi}_{t+1|t} \equiv \hat{E}(\xi_{t+1} \mid Y_t)$$
$$\text{where } Y_t \equiv (y_t', y_{t-1}', ..., y_1', x_t', x_{t-1}', ..., x_1')'$$

The forecasts are generated recursively (i.e. $\hat{\xi}_{1|0}$, $\hat{\xi}_{2|}$, …, $\hat{\xi}_{T|T-1}$) with mean square error (MSE) matrix:

$$P_{t+1|r} \equiv E\left\lfloor \xi_{t+1} - \xi_{t+1|t})(\xi_{t+1} - \xi_{t+1|t})'\right\rfloor.$$

Assuming that the errors are normally distributed, the distribution of y_t conditional on (x_t, Y_{t-1}) is normal with mean $(A'x_t + H'\hat{\xi}_{t|t-1})$ and variance $(H'P_{t|t-1}H + R)$. Hence the density function of y_t conditional on (x_t, y_{t-1}) is given by:

$$(2\pi)^{-n/2} \mid H'P_{t|t-1}H + R \mid^{-1/2} \exp\left\{-\frac{1}{2}(y_t - A'x_t - H'\hat{\xi}_{t|t-1})'(H'P_{t|t-1}H + R)^{-1}(y_t - A'x_t - H'\hat{\xi}_{t|t-1})\right\}$$

which can be estimated by maximum likelihood to obtain optimal predictors of the state vector and its covariance matrix.

[17] A much more detailed description of the Kalman filter algorithm can be found in Hamilton (1994).