APPENDIX No. 17.

A COMPARISON OF THE DESIGN FOR CERTAIN CHORDS OF THE QUEBEC BRIDGE WITH THOSE FOR SIMILAR MEMBERS OF OTHER GREAT CANTILEVER BRIDGES, ILLUSTRATED WITH OUTLINE DRAWINGS OF THE BRIDGES AND COPIES OF THE SHOP DRAWINGS OF THE CHORDS.

The outlines of six great cantilever bridges are shown on drawings Nos. 31 and 32 and detail plans of the lower chord construction adopted for each bridge, on drawings Nos. 34, 35 and 36.

The position of the chord selected is shown in each case on the outline drawings, except for the Forth bridge; the detail drawing for this bridge is simply a sketch plan showing the general make-up of the main compression members.

In the attached table we introduce for use in comparison, an example giving the dimensions of an ordinary bridge post of the two channel type, the figures being taken from Professor Burr's 'Elasticity and Resistance of the Materials of Engineering.' These dimensions are more or less typical of those latticed columns that have been used in bridge construction with such success during the last twenty-five years; the details of such columns are now designed entirely by practical rules.

It will be noted that the Forth bridge chord is in a class by itself. It is not a latticed section but may be regarded as a solid section built up out of separate plates. No criticism touching the practical success of this design has ever been made, but it is not a class of construction that could be adopted by an American bridge company without making material changes in its shop equipment and methods of handling its business. We have, however, noted in Appendix No. 18 that the work of the Forth bridge designers is worthy of careful study.

The examples taken from American practice may be divided into three groups:—

(1) Chords of the ordinary two channel type which reaches its maximum development in the Monongahela design.

(2) Chords of the four channel type latticed into one column as adopted for the Memphis and Quebec bridges.

(3) Chords of the four channel type, latticed into two columns which are made to act together by means of tie-plate connections.

This type was adopted for the Thebes and Blackwell's Island bridges.

In the following table we give the principal dimensions of the chords shown on the drawings.

TABLE OF CHORD DETAILS PREPARED FROM DRAWINGS NOS. 34, 35 AND 36.

Bridge.	Area of cross section.	Area of latticing in a section (measured at right angles to lattice axis.)	Area of rivets connecting lattices cut by section to outside web (one end only.)	Length of chord = l.		r in plane parallel to latticing.	l ÷ r.	Weight of plain section per lin. ft.	Weight of latticing per lin. ft. of chord.	Depth of chord back to back of web angles.		Width of chord out to out of web plates.		Section of horizontal splice plates.	Section of vertical splice plates.
	Sq. ins.	Sq. ins.	Sq. ins.	Ft.	Ins.	Ins.		Lbs.	Lbs.	Ft.	Ins.	Ft.	Ins.	Sq. ins.	Sq. ins.
Quebec	781	10	4·8	57	0 9/32	19·7	35	2,603	66	4′	6½″	4	9⅜	68	212
Memphis	213	10	4·8	28	2¾	14·8	23	710	48	2	6¼	3	2	46	230
Blackwell's Island	852	25	4·8	31	6⅛	22·0	17	2,840	144	4	0½	5	2⅝	80	187
Thebes	189	11¼	4·8	30	6 5/32	20·1	18	630	78	3	0½	4	4¾	80	70
Monongahela	262	14⅛	7·2	30	6 1/32	25·4	14	873	82	3	0¾	4	1	74	164
"Burr"	35	3¾	1·2	45	0	6·5	83	118	20	1	6	1	0¾	..	....

NOTE.—The horizontal stiffness of the inside ribs of the Quebec bridge chords is less than that of the outside ribs, which is not the case in any of the chords of the other bridges. (See also table in Appendix No. 18.)

It is almost impossible to find any common basis for a comparison of these chords. It must be remembered that latticing is often uniform in size in members on the same bridge doing similar service, but having different loads and cross sections. Thus in the Quebec bridge A 9-L had an area of 781 square inches and A 1-L an area of 301 square inches, yet both members had about the same outside dimensions in cross section, and the same latticing. Therefore as the chords selected for the drawings are not the most heavily stressed chords in the respective bridges, comparison by proportion of lattice to main sections would be unfair. In fact we may say that the drawings given are only typical.

A theoretical comparison between the lattice systems of the different columns might be made by using any one of the various formulas given in Appendix No. 16, but we have already pointed out that no one of these formulas is generally accepted by the profession. There are so many causes of variation in the strength of built up chords of equal area which are not provided for in these formulas that comparison by calculation does not appear to be satisfactory.

Referring to the table it will be noted that the Quebec chord has considerably less horizontal stiffness (see values of $\frac{l}{r}$), less lattice area, less rivet area, and less splice plate area in proportion to the size of the members than any of the earlier bridges. It should be remembered also that the unit stresses for the Quebec bridge were higher than those of the earlier bridges. It will be noted that the earlier designers considerably overran 15 per cent or 20 per cent of splice plate area. This is also true of the Quebec bridge chords, but not to the same extent. Mr. Szlapka states (*see Evidence*) that splice plates having an area of cross section equal to 15 per cent or 20 per cent of the cross section of the member would be satisfactory.

The development of the detail plans of the Blackwell's Island bridge was contemporaneous with that of the Quebec bridge plans; the Quebec designers had not access to the Blackwell's Island plans. In fairness to the Quebec bridge designers, however, it should be pointed out that in the Blackwell's Island bridge the proportions of many of the details are much more nearly in accord with Quebec bridge practice than are those of the earlier bridges, although the principles of the designs are very different.

A consideration of the difference in the designs on drawings Nos. 34, 35 and 36, all of which have been prepared under the direction of engineers of recognized ability and high professional standing, shows that there is as yet no established system of design for large compression members. The individual judgment of the engineer is the determining factor, and this may prove to be erroneous as it did in the case of the Quebec bridge.

The lack of precise knowledge on this subject has been discussed in other appendices.

HENRY HOLGATE,
Chairman.
J. G. G. KERRY,
J. GALBRAITH.

APPENDIX No. 18.

A CRITICAL DISCUSSION OF CERTAIN PARTS OF THE SPECIFICATIONS.

The Quebec bridge was designed to meet the requirements of the specifications approved by the Dominion government in 1898 and amended in 1903. The method adopted by the company to procure tenders was to issue a general specification and to call upon contractors to prepare plans in accordance therewith.

Considering all the conditions pertaining to the undertaking the adoption of this method was not in the best interests of the work. The company was known not to have the capital necessary to immediately proceed with construction, and the preparation of complete preliminary plans would involve a large outlay. The evidence and documents show that the preliminary plans submitted with the tenders were incomplete; this was as might have been expected, as the several contractors who tendered for the work had little assurance that they would get any return for their expenditure of time and money.

Specifications as a rule consist of two distinct portions, one of which relates to design and the other to fabrication, material and execution. In the case of the Quebec bridge, the difficulty of preparing an adequate specification for design was very great. It would have been better to have entrusted the preparation of the plans and specifications to engineers independent of any contracting or manufacturing company, whose previous experience qualified them to handle the work. This course would have avoided duplication of designs involving expensive plans and would have prevented the letting of a contract on incomplete plans formed upon erroneous data; the engineers would have made a proper and sufficient study of the whole project, and in due time competitive tenders upon their plans would have been secured, thus enabling all contractors to tender on a common basis. The privilege of submitting independent plans might have been extended to the bidders. The reason for not following this course is explained by Mr. Hoare in his evidence.

The procedure as outlined above would have been applicable to an enterprise which involved so many new problems and the application of existing knowledge on so large a scale and which demanded the continual exercise of sound judgment.

An error of judgment made by the Quebec Bridge Company was the selecting of an engineer who did not possess the necessary special knowledge and experience to prepare the specification (see Appendix No. 7). It is true that this specification was considered to be only tentative, drawn up for the purpose of procuring preliminary tenders, but its history and importance cannot be overlooked. (See Appendix No. 6.) It became the basis of the contracts between the Quebec Bridge Company and its contractors, was approved by the government engineers, and was an essential part of the subsidy agreement whereby the Dominion government undertook to pay the Quebec Bridge Company on certain conditions, one million dollars (Exhibit 12).

The specification itself (Exhibit No. 21), herein called the 1898 specification, was for the most part a copy of a specification issued by the Department of Railways and Canals in 1896; there is nothing in its wording to indicate that the Quebec bridge was an exceptional structure and without precedent or that the propriety of applying to this structure other than the usual clauses in bridge specifications was carefully considered.

We do not think that any engineer would be justified in writing a specification without consulting freely those specifications most used in practice; specifications are in fact the statement of the provisions that engineers have in the past been forced to make in order to secure satisfactory results, and each succeeding revision is the outcome of experience. Such compilations are necessary and cannot be dispensed with, but this fact does not justify an engineer whose special experience has not fitted him to judge of the importance of vital clauses, in revising and rearranging them. The danger in so doing lies in the fact that a clause necessary and useful in one specification may not be applicable under other conditions, and opinions on such matters are valuable only from men of special qualifications. Errors arising from the compilation of specifications by experienced men are by no means uncommon. Mr. Cooper recognized this and so revised the specifications of 1898.

In regular bridge practice the specification is of importance particularly because an American bridge works is a factory for turning out structural steel fabricated in accordance with plans prepared in the drawing office attached to the works. This drawing office is a part of the factory, and in it, as throughout, efficiency is obtained by standardizing and duplication; the drawing office staff consists of a number of well trained computers and draughtsmen whose duty it is to prepare the shop drawings for the work and who are under the control and direction of a designing engineer. Details are designed in accordance with the specifications furnished by the purchaser, except under circumstances when shop equipment requires some deviation to be made to secure facility of manufacture. It is not a part of the duty of the drawing office staff to question the wisdom of the requirements of the specification, nor could the progress of work throughout the factory be satisfactorily maintained if it should attempt to do so.

The evidence shows that the Phœnix Bridge Company followed this usual practice in the preparation of the Quebec bridge designs.

In 1903 it became necessary to design the main spans of the bridge and the 1898 specification was amended by Mr. Cooper, it having been understood ever since 1900 that it would be amended and altered. The history relating to the adoption of these amendments is given in Appendices Nos. 3 and 6.

Mr. Cooper did not recognize these amendments as complete and final, and considered that he had the power to deal with each problem of design as it arose, and he exercised this power when he thought it necessary. The designing of the main span was left to Mr. Szlapka, Mr. Cooper having approved the specifications and no one questioned any decision that these engineers made. The work was done under the immediate direction of Mr. Szlapka.

Before discussing the specification, it will be well to contrast some of the main features of the Quebec bridge with those of other cantilever bridges and the following table is inserted for this purpose:—

INFORMATION CONCERNING GREAT CANTILEVER BRIDGES.

INFORMATION CONCERNING

Name.	Date of Construction.	Designer.	Contractor for Superstructure.	Span.	Width C. to C. Trusses.	Live Load per Lin. Foot.
				Feet.	Feet.	Lbs.
Forth	1882-1889	Baker & Fowler	Wm. Arrol & Co.	1,710	Varying, lower chord 31½ at ends to 120 at piers.	Double track Ry. 2,240 lbs. per track.
Memphis	1886-1892	Geo. S. Morrison	Union Bridge Co.	790	30	Single track Ry. 4,000 lbs. per track.
Monongahela	1902-1903	Boller & Hodge	American Bridge Co.	812	32	Double track Ry. 4,500 lbs. per track.
Thebes	1902-1905	Noble & Modjeski.	American Bridge Co.	671	32	Double track Ry. 5,000 lbs. per track, less 20 p.c.
Blackwell's Island	1901-1908	Dept. Bridges New York City.	Pennsylvania Steel Co.	1,182	60	Roadway and trolley ordinary 8,000 lbs. congested 16,000 lbs.
Quebec	1900	Phœnix Bridge Co.	Phœnix Bridge Co.	1,800	67	Double track Ry., roadway and trolley 4,000 lbs. per track. For extreme conditions mult. 1½.

GREAT CANTILEVER BRIDGES.

Ultimate Strength for Steel Unit 1,000 lbs.	Approx. Weight of Steel in Short Tons per Lin. Ft.	Price of Structural Steel per Lb. Cts.	Working Stresses—Lbs. per sq. inch.	Allowed Shear on Rivets—Lbs. per sq. inch.
Compression, 76·83. Tension, 67·74.	10½	6·50	Max. stresses, Compression, 17,000. Tension.......16,350	About 12,000
Compression, 69·78½. Tension, 66·75.	3½	5·88	Compression, 14,000, if l < 16d,—deduct 750 lbs. for each additional unit over 16 in $\frac{l}{d}$; tension for dead load, 20,000; tension live load, 10,000.	7,500
Compression, 60·70. Tension, 63·75.	4½	4·3	Compression dead load, 21,000 where $\frac{l}{r}$ < 40. Tension dead load, 22,000. Take one-half in each case for live load.	10,000
62·72.	5	*5¼	Compression dead load, 21,000 if $\frac{l}{d}$ < 16. Tension, 20,000. Take ½ in each case for live load.	7,500
Compression, 60±4. Tension, 66±4. Nickel steel eyebars, 85.	13½	*5⅝	Compression, ordinary, $20,000-\frac{l}{r}$; congested, $24,000-100\frac{l}{r}$. Tension, ordinary, 20,000; congested, 24,000. Tension for nickel steel, ordinary, 30,000; congested, 30,000.	Ordinary, 13,000 Congested, 16,000
Compression, 60·70. Tension, 62·70.	13	5·60	Compression, ordinary, $12,000\ (1+\frac{Min.}{Max.})$; extreme, 24,000; both for $\frac{l}{r}$ < 50. Tension, ordinary, $12,000\ (1+\frac{Min.}{Max.}$ Extreme, 24,000.	¾ working stress =18,000 extreme.

* Not official.

It is not possible to set forth all the facts in a table with sufficient minuteness to justify the making of complete comparisons, as many qualifying clauses and special conditions are necessarily omitted. Three items of interest may be noted in the table:—

(1) Only the Forth bridge is at all comparable with the Quebec bridge in regard to span.

(2) Only the Blackwell's Island bridge is comparable with the Quebec bridge in regard to unit stresses both for main members and for details.

(3) All the bridges included in the table were designed by independent engineers except the Quebec bridge.

In this connection we must express the opinion that it is difficult for the employees of a large manufacturing concern to give the design of a bridge of unique features the concentrated attention that it requires.

With regard to precedent only the Forth and Blackwell's Island bridges involved anything like the same total stresses as the Quebec bridge. The design and construction of the Blackwell's Island bridge was contemporaneous with the Quebec bridge.

The Forth bridge was built on a system not suited to the established American methods of bridge construction, so that its distinctive features of design, construction and erection were not followed. It is proper to add that the achievements of the Forth bridge engineers deserve much closer study than appears to have been given to them on this continent. Messrs. Baker and Fowler succeeded in erecting a structure which weighs considerably less per lineal foot than the Quebec bridge and which is designed to carry about one-half the rolling load and several times the wind load specified for the Quebec bridge. The main compression chords of the two bridges are of practically equal area, but the material in the Forth bridge is of a considerably higher ultimate strength than that used in the Quebec bridge, the unit stresses are less and the design of the cross section of the chords is such that they should be able to carry a greater unit stress with safety. On great bridges these are factors to be observed and it is to be regretted that the stress sheets and full engineering studies in connection with the Forth bridge have not been published.

It is evident that the designers of the Quebec bridge were compelled to work from experience gained on much smaller bridges.

In discussing the specification we deal not only with the clauses immediately connected with the downfall, but with others that were not in our judgment calculated to ensure a safe and satisfactory structure.

The specification is here understood to mean the 1898 specification as amended in writing by Mr. Cooper.

As a document the specification is unsatisfactory, some of the clauses having been amended by Mr. Cooper, some set aside in favour of his well known and generally accepted standard specification and some remaining in force with a context that altered their meaning. No general or complete revision of the specification embodying Mr. Cooper's amendments was ever compiled.

As a matter of fact although the 1898 specification was retained as the official specification and much of the work done in accordance with it, we believe that Mr. Cooper depended upon his own inspection of the plans under the revised specifications to secure satisfactory details. His opinions upon most debatable questions of design were well known to the staff of the Phœnix Bridge Company, which had previously designed and built many structures under his direction and was accustomed to his methods. It is on record that the Phœnix Bridge Company requested Mr. Cooper to set aside the 1898 specifications altogether and to substitute for them his own standard specifications.

A complete bridge specification must set forth the character of the material that is to be used, the loadings that are to be carried, the stresses to be permitted in the members and provisions concerning details, fabrication and erection to be observed; in fact everything essential to the proper carrying out of the work as intended.

The 1898 specification for material was used without alteration except in one particular, Mr. Cooper having raised the minimum limit for the ultimate strength of eyebar material from 60,000 lbs. to 62,000. The metal specified was the ordinary grade of structural steel.

It will be noted by reference to the table that the Quebec bridge specification called for material of slightly lower ultimate strength than that used in any of the other bridges while the bridge itself had the longest span of all. The need of a better material than structural steel for the construction of long span bridges is generally recognized, because the decrease of total weight and consequently of cost in a large truss with increase of permissible unit stress is very rapid. In the Quebec bridge the dead load stresses constituted roughly two-thirds of the stress on the main members.

The designers of the other two great bridges introduced special grades of steel so that high unit stresses could be safely used. The Forth bridges engineers were not permitted to load their metal to more than one-fourth its tensile strength, and for compression, used a steel of about 25 per cent stronger than that supplied for Quebec. Nickel steel with a permissible unit stress 50 per cent higher than allowed on material in the same bridge and similar to that used at Quebec was introduced into bridge practice by the Blackwell's Island bridge engineers. The use of this alloy as a structural material was investigated and favourably reported upon in 1903 by a special commission of which Mr. Cooper was a member.

It was Mr. Cooper's opinion that it was wiser to use the ordinary grade of metal for the Quebec bridge and to load it to the highest working stresses that were considered practically safe.

ELASTIC LIMIT.

We do not know whether Mr. Cooper in his amendments intended the term 'elastic limit' to mean the elastic limit of a test specimen or of a full sized member. There is also some uncertainty as to the true meaning of the term 'elastic limit,' which is unfortunate as the maximum working stresses specified are made to depend upon this characteristic of the material.

The 'elastic limit' accepted by bridge designers as a controlling factor in their work cannot be determined by the method prescribed by the 1898 specification, and yet this method (the drop of the beam) was used. Both Mr. Cooper in his standard specifications and the engineers for the Blackwell's Island bridge provide for a much closer determination of this characteristic. In reality the determination is a delicate and time consuming process for a research laboratory and impossible under the conditions existing in a rolling-mill; to such an extent is this true that it is not called for in the carefully prepared specification issued by the American Railway Engineering and Maintenance of Way Association in 1906. The principle apparently followed in the latter specification is that mill tests are sufficient for mill purposes and that the true elastic limit can be most safely obtained by proportion from the ultimate strength. The assumption generally made is that the true elastic limit for structural steel is about 50 per cent of the ultimate strength.

The material actually supplied for the bridge was regularly tested and a comparison between its probable elastic limit and the 32,000 lbs. per sq. in. apparently expected by Mr. Cooper, is possible. The full size eyebar tests, a record of which will be found in Exhibit 86, show that the metal in service shape had a safe ultimate strength not in excess of 55,000 lbs. per sq. in. and a reported elastic limit of 28,000 lbs. per sq. in. These tests were made on long bars in the Phœnix Iron Company's large testing machine and the results might be reduced by calibration of the machine and closer observation of the elastic limit. It will be noticed that the proposed extreme working stresses (24,000 lbs. per sq. in for the Quebec bridge were nearly equal to the elastic limit of the eyebars.

The elastic limit in compression was assumed in accordance with the usual practice to be the same as that in tension. An examination of the voluminous test records (Ex. No. 28) shows that an ultimate strength in excess of 60,000 lbs. per sq. in. was not regularly secured, so that, accepting the 50 per cent relation mentioned above, the elastic limit in compression becomes 30,000 lbs. per sq. in. It should be noted that these tests were made on specimens of about one-half of one sq. in. sectional areas. The compression members were built up of wide thin plates riveted together into webs. We know of no test that has ever been made to establish the relation between the strength and elastic limit of such plates and those of small test specimens, nor do we know what effect the punching, riveting and painting have on the material in the webs as compared with the solid plate. It was noted at the wreck that the paint between the plates of members that had been fabricated for over three years was still fluid. From the analysis of full-sized tension tests we think it possible that the elastic limit of the plates in the compression members was not much above 27,000 lbs. per sq. in. instead of 32,000 lbs. as apparently assumed.

UNIT STRESSES.

The maximum unit stresses that Mr. Cooper proposed to use were about 21,000 lbs. per sq. in. under ordinary loading and 24,000 lbs. per sq. in. under extreme conditions. He considered that the extreme conditions as specified would never occur.

By reference to the table it will be seen that the specified stresses for the Quebec bridge under working conditions are in advance of currnt practice and we believe that they are without precedent in the history of bridge engineering. Under extreme conditions the Quebec bridge stresses are in general harmony with those permitted in the Blackwell's Island bridge.

We have already indicated that the dimensions of the Quebec bridge were such that the use of the highest safe unit stresses was justifiable and good engineering practice. If we were sure that the loads were correctly estimated, that the stresses acted in the bridge exactly in accordance with the assumptions and that the elastic limit of the built-up members was not less than 32,000 lbs. per sq. in, 24,000 lbs. per sq. in would not be an unsafe stress for structural steel, provided that the material is regular in quality and the details satisfactorily worked out to suit such a stress.

Mr. Cooper provided for the effect of live load by the use of the so-called fatigue or $\frac{\text{min}}{\text{max}}$ formula. This method which was formerly much used has more recently been abandoned in general practice and is not adopted by Mr. Cooper in his standard specifications. In the hands of an experienced engineer this method will be made to produce much the same results as the more modern impact formulas. We do not know why this formula was used in this case, except that it was adopted by Mr. Hoare in 1898 from the 1896 specifications of the Department of Railways and Canals and was probably retained in 1903 for convenience.

Mr. Cooper adopted the ordinary straight line formula for compression members making the dead load unit stress equal to $(24{,}000 - 100\frac{l}{r})$ lbs. per sq. in. We have already indicated in Appendix No. 13 that this formula is purely empirical and does not agree particularly well with the recorded tests upon large columns. It is the most generally accepted formula of practice, but we do not believe that the engineering profession has at present a satisfactory knowledge of the action of large steel columns.

There is a wide field for experiment which must be worked over before engineers can claim to have a sufficient knowledge of steel to design both safely and economically, and perhaps the most serious criticism of the structural engineers of the present day is that they have permitted this field to remain undeveloped for twenty-five years' dur-during which time they have adopted a new metal for their work and new shapes and sections.

We think that in popular engineering opinion the ultimate strength of steel columns is largely over-estimated, the diagram on drawing No. 20 indicating that for the Quebec chords it was not safe to expect an ultimate strength in excess of 32,000 lbs. per sq. in, so that under the extreme conditions specified the margin of safety would only have been one-third.

This is a point on which current engineering practice is open to direct criticism. The older engineers, upon the results of whose experiments the profession is now depending, did not think of loading metal in compression to the unit stress used in tension because they recognized that the ultimate unit strength of members in compression was far less than that of members in tension.

The later school of engineers seems to have adopted the principle that the action of bridge members under stresses in excess of the elastic limit is a matter of indifference as they will never be so stressed. The action within the elastic limit being practically the same under both conditions, they adopt the same working stresses in tension and in compression. Their practice has been attended with complete success, but this may be attributed to the fact that the material has ordinarily not been stressed to much above half the elastic limit.

Under the Quebec bridge conditions, where high working stresses were imperative, the wisdom of the practice of loading in compression as heavily as in tension becomes questionable. We believe that in no great public structure should stresses be permitted in excess of one-half of the ultimate strength of any compression member, no matter how high the elastic limit may be.

It will be noted that Mr. Cooper in specifying the stresses for the lower chords of the Quebec bridge omitted the term in the column formula containing the ratio $\frac{l}{r}$. In this practice he is supported by the engineers of the Monongahela and Thebes bridges, who made a similar provision, but reduced the maximum stress to that allowed by the usual formula for a column with $\frac{l}{r}$ equal about 40.

The failure of the Quebec chords does not prove that Mr. Cooper was theoretically incorrect and cannot be directly connected with this clause in the specification. The specification, however, permitted stresses in advance of any previous practice and the proportioning of columns to safely carry such stresses is yet to be learned.

We have already pointed out the seriousness of the error made in the estimation of the dead load which resulted in computed stresses nearly 10 per cent higher (see Evidence) than had been expected. A comparison of these computed stresses with the elastic limit of the material as estimated from the test records will show how narrow a margin of safety was provided in the actual design.

We are not prepared in the present status of the art of bridge-building to approve the unit stresses stated in the amended specification.

RIVET STRESSES.

It will be noted from the table that the rivet stresses used were much in excess of previous practice. These seem to have been adopted almost by an oversight. The 1898 specification contained a clause usual in low stress specifications, permitting the rivets to be worked to three quarters of the allowed stress in the member. This clause was not cancelled by the 1903 amendments and under extreme conditions permitted a stress in rivet shear of 18,000 lbs. per sq. in. The tests made in 1904 under the direction of the American Railway Engineering and Maintenance of Way Association have established the fact that a riveted connection begins to work under a stress in rivet shear between 12,000 and 15,000 lbs. per sq. in. and that deformation in even a simple connection is marked when a stress of 25,000 lbs. per sq. in, is reached. These results have been confirmed both in tension and compression by the tests made for the commissioners (see Appendix No. 15). It is therefore clear that the Quebec specification permitted the use of stresses in details which were outside the limits of established practice and are now know to be unsafe. Knowledge of the action of the rivets in riveted connections is very incomplete.

BUILT UP COLUMNS.

In our findings we have stated that the bridge failed through weakness of the lower chords and particularly in the latticing of those chords. In Appendix No. 16 will be found a discussion of lattice design and of the data that Mr. Szlapka had to guide him in his work. The main outline of the latticing in the Quebec bridge was sketched as early as 1898. There was practically nothing in the specification that was of any service to the designers in this connection and they violated none of its provisions in the design. There are some clauses dealing with latticing, but they were copied from small bridge practice and were wholly inadequate for the Quebec structure. The main criticism that can be made of the designers was that they had the means of checking their theories by use of the testing machine and that they did not do this nor did they thoroughly study the possibilities of lattice formulas.

LOADINGS.

In 1903 Mr. Cooper revised the loadings, increasing the specified train loads and decreasing the wind pressures. While Mr. Cooper undoubtedly made an improvement on the 1898 specification in this respect, he does not seem to have taken full advantage of the improved financial situation due to the decision of the government to guarantee the Quebec Bridge Company's securities. This is explained by Mr. Cooper in his evidence in which there is no reference to the changed financial conditions. (See Appendix No. 5.) Mr. Cooper apparently did not realize the great change in the traffic conditions that would probably follow the opening of the National Trancontinental Railway nor the demands for transportation resulting from the rapid development of Canada. His specified train loading is not greater than that used regularly in Canadian practice and is lighter than that subsequently adopted for the National Transcontinental Railway, and sufficient provision was not made for probable increases of live load.

Considering together the high unit stresses permitted and the loads specified, the specification was not for a bridge well suited to the purposes it would have been called upon to serve.

DEAD LOAD.

The specification requires that the dead load used for calculating the stresses shall not be less than the actual weight of the structure when completed. The evidence shows that the designers failed to comply with this requirement. The effect of their error is shown on drawing No. 4 (see also exhibits 98, 100 and 101). In view of the high unit stresses specified this error was serious enough to have required the condemnation of the bridge even if it had not failed from errors in the design of the compression chords.

The obvious intention of the clause was to compel the designers to check their assumed dead loads by actual calculations from their detailed drawings as soon as these were developed, and it carried with it an obligation on the consulting engineer not to approve any drawings until he was satisfied that the assumed weights were ample. It is not customary in practice to be exacting about the observation of this clause because the weight of an ordinary span for a given loading can be very closely estimated; but no excuse can be offered for applying the precendents of practice to a structure that was entirely outside the range of experience. No evidence has been given to show that any effort was made either in the Phœnix Bridge Company's office or by the consulting engineer to check the assumed weights at the earliest possible date and the error was passed without notice until a large portion of the bridge had been actually built in the shops and the members weighed. It is in evidence, and we have already stated, that the scale weights were within 1 per cent of the weights as finally computed from the drawings. The consequences of this error were considered by Mr. Szlapka and Mr. Cooper before erection was resumed in 1906 and they state that it was their opinion that the error was not fatal to the safety of the bridge, and the work of erection was proceeded with.

ERECTION.

No special provision is made in the specification for an oversight of the methods of erection by the Quebec Bridge and Railway Company's engineer, or for his approval of the general system of erection, or of the means adopted to solve the various problems arising in connection with it. There is no evidence to show that anyone outside the Phœnix Bridge Company attempted to deal with this practical problem. Mr. Cooper states that the erection plans and devices were not subject to his approval although he was advised of them unofficially and general progress on erection was regularly reported to him.

It was apparently intended, as is the usual practice, to leave all such arrangements in the hands of the contractor, making him provide all necessary plant and holding him responsible for everything that might happen.

The erection staff of a large construction company is best qualified by experience to design erection plant. We are of opinion, however, that the erection difficulties to be met with on a structure like the Quebec bridge are so serious and the necessary risks to be run during erection are so great that if the employment of a bridge engineer is necessary at all, it is especially necessary in this connection. In fact the responsible engineer on such a project should direct the work in all its branches and the contractor is entitled to look to him as a trained specialist for instructions and assistance at all times and especially in emergencies.

The specification throughout shows that the whole subject was not considered with sufficient care not only from a technical standpoint but from the practical or business standpoint as well. Inconsistencies are of frequent occurrence; ambiguity and lack of precise definition pervade the whole, and we desire to direct particular attention in this connection to the important clauses 4, 5, 6, which read as follows:—

(4) After the stress sheets have been approved and before the construction of any part of the structure shall be proceeded with, complete working drawings shall be furnished, showing all details of construction, which shall conform to the general design, shapes and dimensions shown on the stress sheets and to the conditions of this specification. The drawings shall be approved by the engineer before the work of construction is proceeded with.

DRAWINGS.

(5) After the final detail drawings referred to have been approved by the engineer, the contractor is to prepare his shop drawings from the detail drawings, complying carefully therewith, and making no changes without the written consent of the engineer. Working drawings are to be sent in triplicate for the approval of the engineer, who will retain two sets and return the third after making thereon any corrections required, after which the required number of corrected sets will be sent by the contractor to the engineer without delay. The approval of the said working drawings will not relieve the contractor from the responsibility of any errors thereon.

(6) The requisite number of copies of general and detail drawings for all purposes shall be furnished by the contractor upon orders of the engineer.

HENRY HOLGATE,
Chairman.
J. G. G. KERRY,
J. GALBRAITH.

APPENDIX No. 19.

MISCELLANEOUS—QUEBEC BRIDGE INQUIRY.

WEATHER CONDITIONS.

The temperatures and wind velocities for some weeks preceding the accident are shown on drawing No. 37. It will be noted that there were no exceptional conditions in either case, both temperature and wind being moderate and usual. The wind blowing at the time of the accident was so light that wind pressure has not been included in calculating the stresses existing at that time. The drawing shows a wind velocity late in the day on August 29, of about 25 miles per hour, which would theoretically produce the almost negligible pressure of about 2 lbs. per sq. ft., on the truss surface exposed. The form of the truss is such that a correct analysis of the wind forces is most difficult to make and it was considered that less error would result from the neglecting of these forces than from an effort to determine them accurately.

A list of the maximum wind velocity recorded at the Quebec observatory is given on drawing No. 37. This list indicates that the pressure of 25 lbs. per sq. ft. assumed in the 1898 specifications was sufficient for the site, a wind velocity of nearly 90 miles per hour being necessary to produce such a pressure.

The following record of deflections, which is filed as Exhibit No. 55 is of interest as furnishing data for predicting the movements of cantilever arms under wind.

OBSERVATIONS ON THE DEFLECTION OF THE CANTILEVER ARM UNDER HEAVY WINDS.

November 12, 1906.—Front leg of large traveller at P-1. Panel 2 of cantilever arm partly erected. East wind 55 miles an hour. Deflection taken on middle of first transverse strut above deck between posts P-1.

Deflection observed—2½ inches.

November 16, 1906.—Front leg of large traveller at P-1. Panel 2 of cantilever arm almost completed. East wind, 65 miles an hour. Deflection taken at same point.

Deflection observed—3½ inches.

February 3, 1907.—Front leg of large traveller at T O cantilever arm erected complete. West wind, 45 miles an hour. Deflection taken at same point.

Deflection observed—2 inches.

HENRY HOLGATE,
Chairman.
~~J. G. G. KERRY,~~
J. GALBRAITH.

REPORT

ON

DESIGN OF QUEBEC BRIDGE.

By C. C. SCHNEIDER.

PENNSYLVANIA BUILDING,
PHILADELPHIA, PA., January, 1908.

SIR,—By telegram of September 9, the writer was appointed by you, on behalf of the Dominion government, with the approval of the Honourable the Minister of Railways and Canals, for the following purposes:—

'To inquire into and pass upon the sufficiency of the present design of the Quebec bridge, which collapsed on the 29th of August, 1907; to thoroughly examine the plans of the superstructure and members thereof, &c.; to look thoroughly into all matters in connection witth the proposed reconstruction of the said bridge, and to state whether, in his opinion, the present design is sufficient.'

After receiving your verbal instructions, the writer visited the site of the Quebec bridge in order to examine the collapsed structure; and immediately commenced to collect such information as might aid him in his work, and proceeded with the examination of the plans, which he received from your department, September 17, 1907.

Not being limited in the scope of his investigations, he understands his duty to be to report on the following questions:—

First.—The sufficiency of the present plans of the Quebec bridge, as to their conformity to the specifications as approved by the government.

Second.—The advisability of modifications in the present plans, should they be found inadequate, using as far as practicable the fabricated material now on hand.

Third.—The advisability of discarding the present plans of the Quebec bridge, and recommendations as to a new design.

The writer has thoroughly investigated the subject submitted to him, and now has the honour to submit the following report:—

The present design of the Quebec bridge is a cantilever of 1,800-feet span between centres of piers, with a suspended span of 675 feet, two cantilever arms each 562 feet 6 inches long, and two anchor arms each 500 feet long; making a total length of 2,800 feet, not including the approach spans, which will not be considered in this report. The transverse distance between centres of trusses is 67 feet. The bridge is to carry two steam railway tracks and a roadway on each side 17 feet wide in the clear, suitable for ordinary highway traffic, with one electric railway track on each roadway.

The writer has computed the strains resulting from the loads given in the specifications as revised by Mr. Theodore Cooper, March 2, 1904, a copy of which is attached to this report in Appendix A.

In comparing the results of his computations with the strain diagrams submitted by the Phœnix Bridge Company, he has come to the following conclusions:—

Floor system.—The sections required in the floor-beams and stringers conform to those required by the specifications.

Trusses.—The strains in the trusses resulting from the live load agree with those computed by the writer. The strains from the dead load as computed by the writer, however, are greater than those shown on the diagram submitted by the Phœnix Bridge Company, for the reason that the actual weight of the steel superstructure is in excess of that estimated previous to its construction.

Bracing.—The strains and sections of the various members composing the lateral and sway bracing of the trusses and the bracing of the floor system as well as their details and connections are in accordance with the requirements of the specifications.

Appendix B accompanying this report gives the writer's computations of the strains in the main members of the trusses. The strains resulting from the dead load are based on the actual weight of the structure taken from the shipping weights of the steel work and distributed in accordance with the positions of the various members, thus representing the conditions which would exist in the finished structure. These loads as concentrated on the various panel points of the trusses are shown on diagram included in Appendix B. The table also contains the sectional areas of the members as shown in the shop drawings, the unit strains required by the specifications and the unit strains as they would occur in the completed structure, based on the actual weight of the members; also strains occurring during erection under conditions existing August 29, 1907.

The tables in Appendix B have been computed in accordance with the writer's interpretations of the specifications, which are:

That the value of $\frac{\text{max.}}{\text{min.}}$ by which the permissible unit strains are determined is derived from the dead and live loads only; but that in proportioning the members these unit strains shall be used for the sum of the strains from dead, live and snow loads.

That as the specifications require that 'only ½ of the maximum wind force need be considered in proportioning the chords,' and nothing is mentioned in reference to the web system, this also applies to wind strains in web members.

That in the formulæ under the head of 'Combined and reversed strains,' L_1 denotes the live load strain of opposite sign from that of the dead load; that the expression '$D - L_1$' is the arithmetical difference between these strains; and that '$D + L + L_1$' is the arithmetical sum of these strains.

By examination of this table, it will be noticed that the actual unit strains in most of the members of the trusses exceed the limits of the specifications. In the upper chords of the cantilever arm (excepting in the panels from U_2 to U_6, which were proportioned for the erection strains), from 10 to 18 per cent; and in the lower chords (with the exception of the panels from L_0 to L_4, which were also proportioned for the erection strains), from 7·5 to 24 per cent. In the upper and lower chords of the anchor arms, the unit strains in all panels exceed these limits from 11 to 20 per cent. The unit strains in the chords of the suspended span also exceed the limits of the specifications: the upper chords 16 to 18 per cent; the lower chords from 7½ to 9½ per cent. While the strains in some web members come within the limits, in some cases they are in excess as much as 21 per cent, and in one case 57 per cent. The trusses, therefore, as designed, do not conform in this respect to the requirements of the specifications approved by the government.

However, there are other points affecting the strength of the structure, not covered by the requirements of the specifications, to which the writer begs to call your attention. These refer more particularly to certain details which appear to have been left to the judgment of the designer.

The writer considers the details the most important parts of the design of a permanent structure, even more so than the general proportions of its members.

Most of the details and connections have received careful and conscientious consideration, and are generally in proportion to the members which they connect and in accordance with the standards of good practice. However, there is a deficiency in many of the compression members, as their connections—such as the latticing—are not sufficient to make the parts composing them act as a unit. The most pronounced defect in this respect exists in the lower chord members of the cantilever and anchor arms. These members consist of four separate ribs, not particularly well developed as compression members, and their connections to each other are not of sufficient strength to make them act as a unit.

As discussions on this subject have of late appeared in print, asserting that a scientific method of proportioning the latticing of compression members is not known, the writer takes exception to these statements, and claims that the strains in lattice bars can be computed with enough accuracy to make them sufficient to develop the full strength of the member.

A discussion on the theory and strength of compression members, including an analysis of the strains in lattice bars, will be found in Appendix C accompanying this report.

DISCUSSION OF PERMISSIBLE UNIT STRAINS.

As the present design of the trusses of the Quebec bridge does not conform in all respects to the requirements of the approved specifications, the question arises: Are the trusses as designed strong enough to carry the specified loads without considering the specifications?

In order to decide that question it is necessary to consider the maximum unit strains which might be permitted in the members of the trusses as coming within the limits of safety. If we knew all the strains occurring in a member of a structure, and if the material and workmanship were perfect, we could allow strains up to the true elastic limit of the material. These ideal conditions of material and workmanship, however, cannot be realized in practice, and in addition to the computed direct strains on which the proportions of the members are based, there are secondary strains produced by the bending from their own weight and deformation of the trusses under load. Allowance must, therefore, be made for these contingencies in determining on unit strains which may be considered within the limits of safety.

The specifications provide for two kinds of live loads for the trusses:—

First. A live load consisting of a train on each track. The strains produced by this load, together with the dead load and specified snow load, are limited to a certain unit strain per square inch.

Second. A provision for future increase of 50 per cent in the live load. For the strains produced by this extreme live load, together with the dead and specified snow loads combined with the wind force, a higher unit strain is specified.

The first case will be called hereafter the working load, and the second case the extreme load. The strains produced by the working load, which is by no means excessive, should leave a reasonable margin for safety. The strains produced by the extreme loads should remain within the elastic limit of the material.

Tension Members:

Eyebars.—The elastic limit in full-sized annealed eyebars cannot be depended upon to be more than 28,000 pounds per square inch. A direct tension of 24,000 pounds per square inch, together with secondary strains caused by the friction on the pins during deformation, and the uncertainty of a uniform distribution of the strains over all the bars, may increase the strain to at least 27,000 pounds per square inch, which is just within the elastic limit, with practically no margin for safety.

A strain of 21,000 pounds per square inch in direct tension combined with the secondary strains, &c., may produce an extreme fibre strain of about 24,000 pounds per square inch, or ⅞ of the elastic limit of the eyebars. The unit strains to be allowed on eyebars for direct tension, therefore, should not exceed 24,000 pounds per square inch for the extreme load.

Compression Members:—

In accordance with the accepted theory of compression members, the fibre strain near the center of a column increases in proportion of the length to the least radius of gyration, and, therefore, an allowance must be made for the buckling caused by the tendency to bend.

The usual practice in bridges of ordinary span is to consider the gross section of the compression members in computing their strength. This is generally done in connection with the conservative unit strains of about half of the elastic limit, thus giving a considerable margin for safety; but in the case of the Quebec bridge, where the unit strains are unusually high, approaching the elastic limit, the net areas of the members should be used in estimating the safe limit. Some of the compression members consist of sections which are composed of angles and a number of plates riveted together. The rivet holes reduce the sectional area, and, while these holes are filled up with rivets, they do not fill the holes so perfectly as to make them take the place of the material punched out of the rivet holes. In some of the lower chord members, the net section is about 86 per cent of the gross section, and the elastic limit, which is estimated to be 32,000 pounds per square inch, is thereby reduced to about 27,500 pounds per square inch of gross area. If we, therefore, assume the maximum permissible unit strain on the gross section for the specified extreme loading as 24,000 pounds per square inch, and the secondary strains as only 3,000 pounds per square inch, or approximately 12½ per cent of the direct strain, the total fibre strain per square inch would be 24,000 + 3,000 = 27,000 pounds. This strain nearly reaches the elastic limit of 27,500 pounds per square inch with scarcely any margin for safety.

The maximum permissible strain of 24,000 pounds per square inch for the direct compression caused by the extreme load would have to be reduced in accordance with the accepted formulæ for compression members, making it $24,000 - 100\ ^{l}/_{r}$; where l = length, and r = least radius of gyration of member.

For the working load there should be the same margin for safety as in tension members. As stated before, the elastic limit in compression members, owing to the reduction of their sections by the rivet holes, may be reduced to 27,500 pounds per square inch of gross section. Deducting 3,000 pounds per square inch for secondary strains would leave 24,500 pounds per square inch on the gross section as the maximum strain in direct compression within the elastic limit. Allowing ⅞ of this strain the same as for tension members, we have 21,000 pounds per square inch as permissible strain for direct compression, which should be reduced by the usual formulæ, making it $21,000 - 90\ ^{l}/_{r}$. These limiting strains should be applied to all compression members. The writer does not advocate these high unit strains, but only desires to fix a limit within which the strains may be considered safe, and which could be used in comparison with the tables in Appendix B.

The extreme unit strains within which in the writer's judgment the structure may be considered to be able to sustain the loads provided for in the specifications are:—

First. For the dead and live loads combined with the snow load: For tension, 21,000 pounds per square inch of net section; for compression, $21,000 - 90\ ^{l}/_{r}$ per square inch of gross section.

Second. For the extreme provision of 1¼ times the live load, dead and snow loads, combined with ¾ of the wind strains: For tension, 24,000 pounds per square inch of net section; for compression, $24,000 - 100\ ^{l}/_{r}$ per square inch of gross section.

The table included in Appendix B gives these unit strains for different ratios of $^{l}/_{r}$.

By applying the above unit strains to the trusses of the cantilever and anchor arms in the present design of the Quebec bridge, we find the following discrepancies:

CANTILEVER AND ANCHOR ARMS.

Upper Chords.

The upper chords are composed of eyebars for which the maximum permissible strain as stated above should not exceed 21,000 pounds per square inch for the working load, and 24,000 pounds per square inch for the extreme load.

The tables in Appendix B show that the strains in all panels, excepting those from U_2 to U_6 of the cantilever arm, are in excess of these limits for either case of loading.

Lower Chords.

The lower chord in itself is not pin-connected, but is composed of a number of sections butting against each other and connected with splice plates. If the lower chords of the cantilever and anchor arms were strictly pin-connected, that is, bearing against the pin only, the strains would act in the axis of the member without any other bending movements from the dead load than those caused by the friction of the pin in the pin hole, as they would be able to rotate around the pins and thus adjust themselves during erection.

If the lower chords were continuous members and fully spliced, and the web members rigidly connected to them similar to those of the Firth of Forth bridge or the suspended span, the strains produced by the deformation would become an important factor, but could be approximately calculated and provided for in the sections. Since, however, the lower chord members of the Quebec bridge are butt-jointed, they are neither continuous nor pin-connected, and it is impossible to make the whole section bear uniformly under the various conditions of loading.

With accurate workmanship and proper method of erection, the joints of the chord members may come to a full and even bearing for one condition of loading, and in this condition the strains would be transmitted from one section to another in the direction of their axis and distributed over their entire cross-section. For all other conditions of loading, the strains are transmitted eccentrically, thus producing secondary strains in addition to the direct strains and those produced by the initial eccentricity inherent in all compression members. These secondary strains will be found in Appendix D accompanying this report.

By comparing the strains in the tables in Appendix B with the limits fixed by the writer, we find that all the lower chord members are deficient (with the exception of L_0 to L_1 of the cantilever arm) and would not be strong enough to safely carry the specified loads provided for in the specifications, even if they had been properly braced with lattice bars of sufficient strength; and that the inadequate latticing shown on the drawings would still further reduce their strength.

Web System:—

The web system of the trusses of the cantilever and anchor arms is composed of tension and compression members. The main posts are pin-connected to the upper and lower chords, while the web members among themselves are only partly pin-connected; that is, the diagonals, with the exception of the one nearest the center post, are eyebars and pin-connected at both ends.

Some of the sub-diagonals and floor-beam suspenders are compression and others tension members. The connections of the sub-diagonals are riveted at both ends. The floor-beam suspenders are pin-connected to the lower chord, but have riveted connections at their intersections with the main diagonals and sub-diagonals.

From the tables in Appendix B it is evident that the strains in the posts of the cantilever and anchor arms are excessive (with the exception of L_8-U_8), also in about one-half of the diagonals. The strains in the center posts are also excessive. The strains in the floor-beam suspenders, and in the sub-diagonals come practically within the safe limits.

Suspended Span:—

The trusses of the suspended span are practically riveted structures with the chords fully spliced and the web members rigidly connected, excepting the main or tension diagonals, which consist of eyebars pin-connected at their ends.

The weakest parts of the suspended span are the upper chords (see Appendix B), their unit strains being from 44 to 48 per cent in excess of the safe limits fixed by the writer. The strains in the lower chords and web members, excepting U_0-C_1 and C_1-L_2, are practically within those limits.

Sufficiency of Specifications:—

In considering the sufficiency of the specifications, the question arises: Would the trusses of the Quebec bridge have been safe if they had been designed to comply with the requirements of the specifications and the details had been in proportion to the strength of the members?

By referring to the tables in Appendix B, we find that the permissible unit strains limited by the specifications for the two kinds of loading, that is, the working load and the extreme load, are close to, or within the limits of those determined by the writer in all members of the trusses of the cantilever and anchor arms, except in the lower chords and in the posts over piers, for which strains are permitted beyond these limits.

In connection with this subject, the writer believes it to be within the scope of his investigations to report upon the specifications for the Quebec bridge.

The purpose of these specifications has evidently been to keep all the strains, even for the extreme loading, well within the elastic limit of the material. That this has not been realized in all the members of the structure is evident from a study of the tables in Appendix B. The writer has already given his reasons for recommending limiting unit strains, and has shown that the specifications permit too high unit strains for the posts over the piers and for the lower chord of the cantilever and anchor arms. The writer also considers the use of a formula for the permissible strains based on the minimum and maximum strains in each member, as given in the specifications of the Quebec bridge, to be unsuitable for practical purposes, as it is not supported by facts established by recent experiments, and causes unnecessary complications in the computation of the strength of the members; giving besides anomalous results.

The well-established theory of the elastic line is based on strains within the elastic limit. As a single strain above the elastic limit produces a permanent set and destroys the property of uniform elongation in the metal, its effect is not different from the effect of repeated strains, the single strain having practically destroyed the usefulness of the material. The elastic limit, therefore, is actually the ultimate strength for all practical purposes.

The static effect of a live load is the same as that of a dead load, depending upon the amount and distribution of the load only. The dynamic effect of a live load, commonly called impact, however, depends upon the conditions under which the live load is applied. The conditions which affect the impact on a railway bridge are the conditions of the track, the dynamic action produced by the deflection of the bridge, the action of insufficiently balanced drivers, the reciprocal motion and vibration of the machinery and the velocity of the train.

As the static and dynamic effects of a live load depend each upon such entirely different conditions, it seems rational to consider each separately in order to arrive at a more scientific solution of the problem of determining the safe working strains in railway bridges. As the internal strain of a member in a structure is proportional to its elongation or reduction in length, it is evident that it makes no difference, as far as the resistance of the material is concerned, whether this strain is produced by the weight of the structure, by the static effect of a superimposed load, or by the dynamic effect of a moving load. If, therefore, the impact is added to the live load, reducing its effect to that of a static load, a uniform permissible strain may be used, thus avoiding complications and making the strength of the details and connections in proportion to that of the main members, as the impact applies to all parts.

RECOMMENDATIONS AS TO THE RECONSTRUCTION OF THE QUEBEC BRIDGE.

As it is evident from the writer's investigations that the trusses of the present design are not of sufficient strength to carry the loads provided for in the specifications, the question arises:

Can the fabricated members of the remaining half of the Quebec bridge, or a portion thereof, be utilized in the reconstruction of the bridge?

This might be accomplished in two different ways:

First.—By using the remaining portion of the floor system and reinforcing the remaining members of the trusses; rebuilding only that portion which has been wrecked.

The members composing the floor system and the lateral bracing of the remaining half of the bridge might be utilized in the reconstructed bridge. However, to make the bridge strong enough to carry the specified loads with a reasonable margin of safety, the sections of most of the members of the trusses would have to be increased. An examination of the detail plans of the members of the trusses from the standpoint of a manufacturer of structural steel work has convinced the writer that this is impracticable.

The weakest parts of the trusses of the anchor and cantilever arms are the lower chord members. Their sectional areas would have to be increased at least 50 per cent in order to reduce the unit strains to safe limits. The only way this could be done would be to cut them apart, drill additional rivet holes and rivet them up again with additional material. During these various manipulations the members would become distorted, and would require the reboring of the pin holes to larger size, and the refacing of the ends. This refacing would shorten the members enough to make them useless. The use of the remaining chord members is, therefore, impracticable. The same applies to most of the other compression members.

The upper chords of the cantilever and anchor arms being composed entirely of eyebars could be reinforced with additional bars, which would require in some panels as much as 20 per cent additional material. This operation would not only require new pins, but also the changing of the upper ends of the posts to which they are attached. The writer, therefore, considers it impracticable to use any of the finished truss members of the remaining half of the bridge.

Second.—By using the present floor system and building new trusses, following the same outlines as in the present design, but proportioning the members and connections for the loads provided for in the specifications.

If the remaining portion of the floor system and bracing, weighing about 8,000,000 pounds, were to be used in the new structure, it would require for the trusses a design similar to the present one, and also, the same distance between the posts to which the floorbeams are attached. This is almost an impossible task, and further as, in the writer's opinion, the present design of the trusses can be improved upon, the new design should be worked out on entirely different lines to avoid many of the complications and objectionable features existing in the present design.

A third proposition is to adopt an entirely new design, retaining only the length of span in order to use the present main piers, with some modifications. The anchorage piers would have to be partially rebuilt as new anchorages would be required.

Referring to the features which appear to be objectionable in the present design, the writer begs to call your attention to the following:—

The polygonal lower chords of the cantilever and anchor arms are not well adapted for a cantilever bridge on account of the difficulties in fabrication and proper fitting, which make them not only more costly than chords forming a straight line, but also less safe. The polygonal chords of the present design produce a reversal of strains in some web members, which on that account require not only more material than members with strains in one direction only, but also cause unnecessary complications in their details and connections.

The wind forces in a rationally designed bridge should produce strains in the chords, the lateral and sway bracing only. On account of the shape of the chords of the Quebec Bridge, the wind strains affect also the web members of the trusses, producing in these members additional strains, consequently requiring in these members more material and more complicated details.

The writer considers that in a rationally designed structure the strains should be carried in the most direct line to the piers. The more complicated the design and the oftener the strains have to change their direction before reaching their destination, the more assumptions have to be made, which again reduce the degree of accuracy of the results of the computations; therefore, the simpler the design, the safer it will be with the same unit strains.

CONCLUSIONS.

The results of the writer's investigations and his recommendations may be briefly summarized as follows:—

First.—The floor system and bracing are of sufficient strength to safely carry the traffic for which they were intended.

Second.—The trusses, as shown in the design submitted to the writer, do not conform to the requirements of the approved specifications, and are inadequate to carry the traffic or loads specified.

Third.—The latticing of many of the compression members is not in proportion to the sections of the members which they connect.

Fourth.—The trusses of the bridge, even if they had been designed in accordance with the approved specifications, would not be of sufficient strength in all their parts to safely sustain the loads provided for in the specifications.

Fifth.—It is impracticable to use the fabricated material now on hand in the reconstruction of the bridge.

Sixth.—The present design is not well adapted to a structure of the magnitude of the Quebec Bridge and should, therefore, be discarded and a different design adopted for the new bridge, retaining only the length of the spans in order to use the present piers.

Seventh.—The writer considers the present piers strong enough to carry a heavier structure, assuming that the bearing capacity of the foundations is sufficient to sustain the increased pressure.

This report is accompanied by the following Appendices:

A.—Copy of revised specifications.

B.—Tables containing computations of strains in the members of the trusses, a table giving permissible strains for compression members, also diagrams of dead load concentrations and loads and strains during erection, August 29, 1907 (20 prints.)

C.—Review of the literature on the theory of compression members up to the present time.

D.—Investigation of secondary strains in trusses.

Respectfully submitted,

C. C. SCHNEIDER.

M. J. BUTLER, Esq.,
Deputy Minister and Chief Engineer,
Department of Railways and Canals.

APPENDIX A.

QUEBEC BRIDGE SPECIFICATIONS FOR LOADS AND STRAINS FOR CANTILEVER AND SUSPENDED SPANS, BY THEODORE COOPER.

FLOOR SYSTEM.

Railroad Stringers.—To be designed to carry Cooper's E-40 engines with unit strains not exceeding 10,000 lbs. per square inch of net section.

Trolley Stringers.—Loaded with cars weighing 56,000 lbs. on two axles ten feet apart, not to be strained above 13,000 lbs. per square inch of net section. Cars thirty feet over all.

Highway Stringers.—Loaded with 24,000 lbs. on two axles ten feet apart, strains not to exceed 15,000 lbs. per square inch of net section.

Transverse Floor Beams.—With all tracks loaded as above they must not be strained above 15,000 lbs. per square inch of net section, or 12,000 lbs. with both railroad tracks loaded.

The webs of all girders shall be considered as resisting shearing strains only and will not be estimated as doing any flange duty.

TRUSSES.

The maximum strains produced by the following live loads and wind shall be used for proportioning all members of the trusses or towers:—

1st. A continuous train of any length weighing 3,000 lbs. per foot of track, moving in either direction on each track.

2nd. A train nine hundred feet long consisting of two E-33 engines followed by a load of 3,300 lbs. per lin. ft. upon each railroad track and moving in either direction.

3rd. A train load 550 feet long consisting of one E-40 engine followed by 4,000 lbs. per lin. ft. of track, on each track.

4th. For the suspended span a lateral wind force of 700 lbs. per lin. ft. of the top chord and 1,700 lbs. per lin. ft. of the lower chord, one half of which shall be used for lateral and diagonal bracing.

For the cantilever and anchor arms a lateral force of 500 lbs. on the top chord and 1,000 lbs. on the lower chord, per lin. ft. in addition to the wind force on the suspended span, shall be considered.

Only one-third of this maximum wind force need be considered in proportioning the chords. It shall be considered as a live load. Unless this increases the strains due to the live and dead loads only more than 25 per cent the sections need not be increased.

Reversal of strains by the wind acting in opposite directions need not be considered; but where the maximum wind forces reverse the strains in any member the member must be designed to resist each kind of strain.

Allowed Working Strains.—Under the above working loads in combination with the dead loads, the allowed strains in all members of the trusses and towers shall not exceed the following limits:—

Tension Chords and Diagonals.—

$$12{,}000\left(1+\frac{\text{Min}}{\text{Max}}\right)\text{ lbs. per sq. in. of net section.}$$

Compression Chords.—(Where l does not exceed 50 r).

$$12{,}000\left(1+\frac{\text{Min}}{\text{Max}}\right)\text{ per sq. in.}$$

Main Posts.—

$$\left(12{,}000-50\frac{l}{r}\right)\left(1+\frac{\text{Min}}{\text{Max}}\right)\text{ lbs. per sq. in.}$$

TRUSSED FLOORBEAMS.

Tension Struts.—

$$10{,}000\left(1+\frac{\text{Min}}{\text{Max}}\right)\text{ for R. R. loading.}$$

$$12{,}000\left(1+\frac{\text{Min}}{\text{Max}}\right)\text{ for total loading.}$$

Compression Struts.—

$$\left(10{,}000-40\frac{l}{r}\right)\left(1+\frac{\text{Min}}{\text{Max}}\right)\text{ for R. R. loading.}$$

$$\left(12{,}000-50\frac{l}{r}\right)\left(1+\frac{\text{Max}}{\text{Min}}\right)\text{ for total loading.}$$

WIND STRUTS AND LATERALS.

Tension.—20,000 lbs. per sq. in.

Compression.—$20{,}000-90\dfrac{l}{r}$ per sq. in.

For counters and intermediate posts, the live load on the railroad tracks shall be increased 15 per cent.

COMBINED AND REVERSED STRAINS.

The allowed positive and negative strains upon any member subject to any combination of $\pm D, \pm L, \mp L'$ shall be determined by the following formulae:—

$$\text{Allowed} \pm \text{Strain, } 12{,}000\left(1+\frac{D-L}{D+L+L'}\right)$$

$$\text{Allowed} \mp \text{Strain, } 12{,}000\left(\frac{L'}{D+L+L'}\right)$$

PROVISION FOR FUTURE INCREASE OF LIVE LOAD.

In addition to the previous provisions as to the working loads and strains, no member of the trusses or towers shall be strained to exceed three-quarters of the elastic limit under the extreme assumption of an increase in the train loads of 50 per cent above those previously specified. Or, not to exceed 24,000 for the chords and main diagonals, or $24{,}000-100\dfrac{l}{r}$ for the posts.

The material to be medium steel of the best quality and made by the open hearth process.

All details, proportion of parts, workmanship, &c., to be in accordance with the best accepted practice.

Corrected to date, March 2, 1904.

Appenda. June 13, 1905.

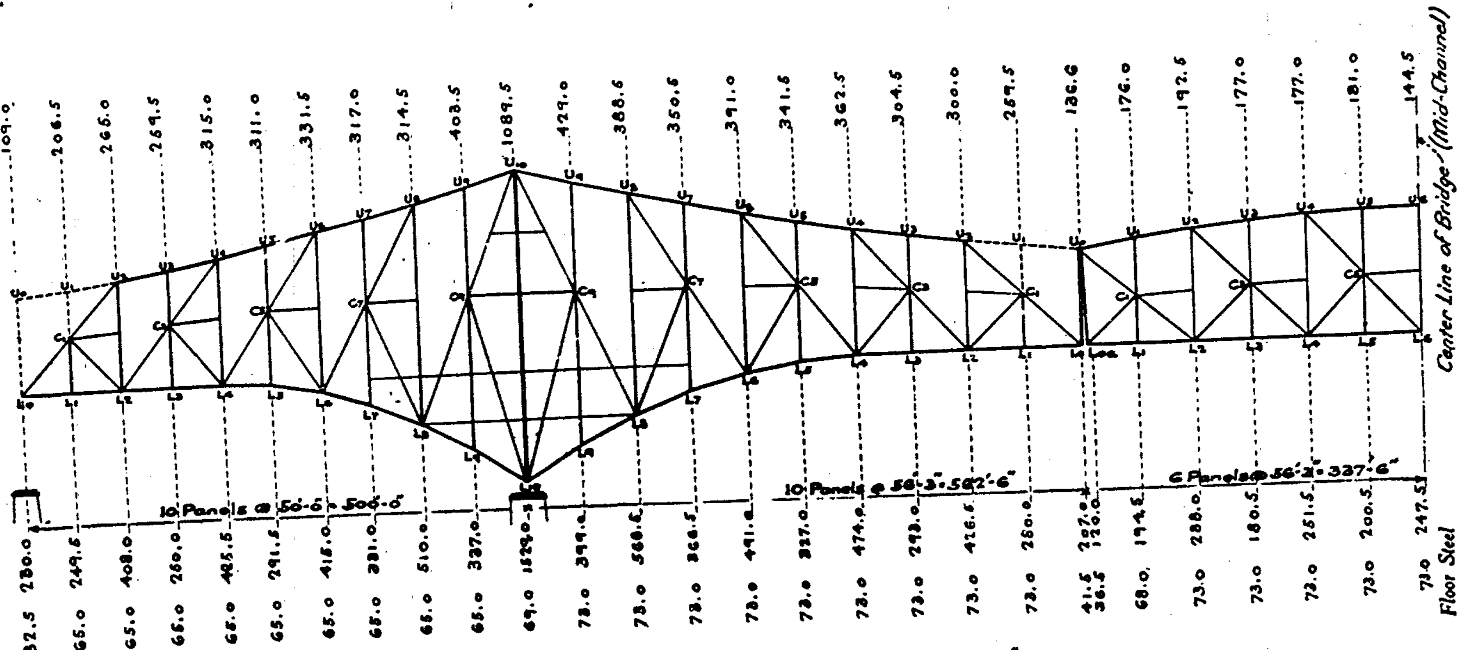
For the cantilever arms, the full wind on the suspended span should be considered.

A snow load of 1,600 lbs. per foot of bridge should be used.

Report on Quebec Bridge.

Appendix B.

Dead Load Concentrations at Panel Points.



Weights are for one Truss and are given in units of 1000#

Report on Quebec Bridge. Appendix B.

Tabulated Statement of Strains, Sectional Areas and Unit Strains.

The strains in the members are given in thousands of pounds, the unit strains in pounds, and the following notations are used;-

+ denotes Tension,
− " Compression,
A " Sectional Area of Member, in sq. inches,
r " Least Radius of Gyration of Member, in inches,
D " Strain resulting from Dead Load,
L " " " " Live ",
S " " " " Snow ",
W " " " " Wind Pressure,
E " Maximum Strain occurring August 29, 1907.
u_e " Unit Strain " " "

*φ denotes coefficient by which the specified minimum unit strain of 12000 lbs. pr. sq. in. for Tension, or $(12000-50\frac{l}{r})$ lbs. pr. sq. in for Compression, is to be multiplied in order to ascertain the permissible unit strain.

u denotes unit strain for Dead, Live and Snow Loads,
I- as required by Specifications;
II- as would actually occur in completed structure.

u_1 denotes unit strain for dead, 1½ live and snow loads, combined with ⅓ wind presure,
I- as required by Specifications,
II- as would actually occur in completed structure.

* For strains of one kind only, $\varphi = 1 + \frac{min.}{max.}$

For combined Strains, $\varphi = 1 + \frac{D-L_1}{D+L+L_1}$

For reversed Strains, $\varphi = \frac{L_1}{D+L+L_1}$

Anchor Arm – Upper Chord.

Member		U_2-U_3	U_3-U_4	U_4-U_5	U_5-U_6	U_6-U_7	U_7-U_8	U_8-U_9	U_9-U_{10}
A		309	309	555	559	696	698	707	711
D		+ 4290	+ 4305	+ 8500	+ 8535	+ 11510	+ 11555	+ 12510	+ 12585
L		+ 1998	+ 2006	+ 3475	+ 3490	+ 4156	+ 4176	+ 4047	+ 4068
		− 749	− 751	− 968	− 973	− 772	− 776	− 380	− 382
S		+ 365	+ 365	+ 700	+ 705	+ 915	+ 915	+ 960	+ 965
W		+ 70	+ 10	+ 130	0	+ 150	+ 320	+ 1200	+ 1400
D+L+S		+ 6653	+ 6676	+ 12675	+ 12730	+ 16581	+ 16646	+ 17517	+ 17618
$D+1\frac{1}{2}L+S+\frac{1}{3}W$		+ 7675	+ 7682	+ 14455	+ 14475	+ 18709	+ 18841	+ 19940	+ 20119
φ		$1+\frac{3541}{7037}=1.50$	$1+\frac{3554}{7062}=1.50$	$1+\frac{7532}{12943}=1.58$	$1+\frac{7562}{12998}=1.58$	$1+\frac{10738}{16438}=1.65$	$1+\frac{10779}{16507}=1.65$	$1+\frac{12130}{16937}=1.72$	$1+\frac{12203}{17035}=1.72$
u	I	18000	18000	19000	19000	19800	19800	20800	20600
	II	21500	21600	22800	22800	23800	23800	24800	24800
u_1	I	24000	24000	24000	24000	24000	24000	24000	24000
	II	24800	24900	26100	25900	26900	27000	28200	28300
E		+ 4185	+ 4205	+ 8245	+ 8280	+ 11095	+ 11150	+ 12020	+ 12090
u_e		13500	13600	14900	14800	15900	16000	17000	17000

ANCHOR ARM – LOWER CHORD.

Member-	Lo-L1	L1-L2	L2-L3	L3-L4	L4-L5	L5-L6	L6-L7	L7-L8	L8-L9	L9-L10
A	302	302	542	542	702	709	729	768	781	843
$\frac{l}{r}$	$\frac{600}{18.7}=32.1$	$\frac{600}{18.7}=32.1$	$\frac{600}{16.5}=36.3$	$\frac{600}{16.5}=36.3$	$\frac{600}{16.2}=37.0$	$\frac{609}{16.2}=37.6$	$\frac{626}{16.2}=38.6$	$\frac{652}{16.2}=40.2$	$\frac{684}{16.2}=42.2$	$\frac{722}{16.2}=44.5$
D	− 3985	− 3985	− 8110	− 8110	− 11425	− 11585	− 12755	− 13275	− 13690	− 14455
L {	− 1965	− 1965	− 3413	− 3413	− 4212	− 4270	− 4181	− 4349	− 4021	− 4249
	940	+ 840	+ 1037	+ 1037	+ 840	+ 852	+ 438	+ 455	+ 43	+ 45
S	− 335	− 335	− 670	− 670	− 910	− 920	− 985	− 1025	− 1025	− 1080
W	− 660	− 1420	− 2430	− 3260	− 4170	− 5160	− 5570	− 6870	− 7370	− 9060
D+L+S	− 6285	− 6285	− 12193	− 12193	− 16547	− 16775	− 17921	− 18649	− 18736	− 19784
D+1½L+S+½W	− 7487	− 7740	− 14709	− 14986	− 20043	− 20630	− 21868	− 23113	− 23203	− 24928
φ	$1+\frac{3145}{6790}=1.46$	$1+\frac{3145}{6790}=1.46$	$1+\frac{7073}{12560}=1.56$	$1+\frac{7073}{12560}=1.56$	$1+\frac{10585}{16477}=1.64$	$1+\frac{10733}{16707}=1.64$	$1+\frac{12317}{17374}=1.71$	$1+\frac{12820}{18079}=1.71$	$1+\frac{13647}{17754}=1.77$	$1+\frac{14410}{18749}=1.77$
u { I	17500	17500	18700	18700	19700	19700	20500	20500	21200	21200
u { II	20800	20800	22500	22500	23600	23700	24600	24300	24000	23500
u_1 { I	24000	24000	24000	24000	24000	24000	24000	24000	24000	24000
u_1 { II	24800	25600	27100	27600	28600	29100	30000	30100	29700	29600
E	− 3915	− 3915	7885	− 7885	− 11050	− 11200	− 12260	− 12755	− 13125	− 13870
u_e	13000	13000	14500	14500	15700	15800	16800	16600	16800	16500

Anchor Arm – Vertical Posts.

Member	L2-U2	L4-U4	L6-U6	L8-U8 Upper	L8-U8 Middle	L8-U8 Lower	L10-U10 Upper	L10-U10 Middle	L10-U10 Below Floor.	L10-U10 Lower
A	371	355	277	175	163	163	472	514	514	472
$\frac{l}{r}$	$\frac{720}{14.5}=50$	$\frac{837}{14.6}=57$	$\frac{1058}{15.3}=69$	$\frac{1169}{16.15}=72$	$\frac{920}{16.3}=56$	$\frac{603}{16.3}=37$	$\frac{759}{17.5}=43$	$\frac{920}{18}=51$	$\frac{596}{18}=33$	$\frac{742}{17.5}=42$
D	− 4180	− 4480	− 3510	− 1225	− 1225	− 1380	− 8065	− 8065	− 8250	− 8250
L {	− 1848	− 1499	− 895	− 633	− 633	− 827	− 1444	− 1444	− 1619	− 1619
	+ 692	+ 310	+ 67	+ 323	+ 323	+ 323				
S	− 335	− 325	− 220	− 25	− 25	− 65	− 375	− 375	− 420	− 420
W	− 270	− 230	− 200	− 810	− 810	− 810	− 4830	− 4830	− 4830	− 4830
D+L+S	− 6363	− 6304	− 4625	− 1883	− 1883	− 2272	− 9884	− 9884	− 10289	− 10289
D+1½L+S+⅓W	− 7377	− 7130	− 5139	− 2469	− 2469	− 2955	− 12216	− 12216	− 12708	− 12708
φ	$1+\frac{3488}{6720}=1.52$	$1+\frac{4170}{6289}=1.66$	$1+\frac{3443}{4472}=1.77$	$1+\frac{902}{2181}=1.41$	$1+\frac{902}{2181}=1.41$	$1+\frac{1057}{2530}=1.42$	$1+\frac{8065}{9509}=1.85$	$1+\frac{8065}{9509}=1.85$	$1+\frac{8250}{9869}=1.836$	$1+\frac{8250}{9869}=1.836$
u { I	14400	15200	15100	11900	13000	14400	18200	17500	18900	18200
u { II	17200	17800	16700	10800	11600	13900	20900	19200	20000	21800
u_1 { I	19000	18300	17100	16800	18400	20300	19700	18[illegible]00	20700	19800
u_1 { II	19900	20100	18500	14100	15100	18100	25900	23800	24700	26900
E	− 4155	− 4475	− 3550	− 1365	− 1365	− 1470	− 7380	− 7380	− 7510	− 7510
u_e	11200	12600	12800	7800	8400	9000	15600	14400	14600	15900

Anchor Arm - Main Diagonals.

Member	Lo-C1	C1-U2	L2-C3	C3-U4	L4-C5	C5-U6	L6-C7	C7-U8	L8-C9	C9-U10
A	451	454	396	400	330	300	150	149	163	225
$\frac{l}{r}$									$\frac{984}{14.8} = 66$	$\frac{816}{13.8} = 59.$
D	+ 5975	+ 6325	+ 6335	+ 6650	+ 5595	+ 5115	+ 2285	+ 1815	+ 35	− 360
L {	+ 2944	+ 2944	+ 2357	+ 2332	+ 1550	+ 1371	+ 1731	+ 832	+ 914	+ 994
	− 1258	− 1105	− 616	− [illegible]3	− 187	− 107	− 62	− 326	− 961	− 1179
S	+ 515	+ 540	+ 505	+ 530	+ 410	+ 365	+ 125	+ 80	− 70	− 105
W	+ 420	+ 410	+ 340	+ 330	+ 40	+ 280	+ 770	+ 1005	− 990	− 2350
D+L+S	+ 9434	+ 9809	+ 9197	+ 9512	+ 7555	+ 6851	+ 3141	+ 2727	− 996	− 1644
$D+1\frac{1}{2}L+S+\frac{1}{2}W$	+ 11046	+ 11418	+ 10488	+ 10788	+ 8343	+ 7629	+ 3763	+ 3478	− 2139	− 3016
φ	$1+\frac{4717}{10177}=1.464$	$1+\frac{5220}{10374}=1.503$	$1+\frac{5719}{9308}=1.614$	$1+\frac{6167}{9465}=1.651$	$1+\frac{5408}{7332}=1.738$	$1+\frac{5008}{6593}=1.760$	$1+\frac{2223}{3078}=1.727$	$1+\frac{1489}{2973}=1.500$	$\frac{961}{1910}=0.503$	$\frac{994}{2533}=0.392$
u { I	17600	18000	19400	19800	20800	21100	20700	18000	4380	3550
u { II	20900	21600	23200	23800	22900	22800	20930	18300	6110	7300
u_1 { I	24000	24000	24000	24000	24000	24000	24000	24000	17400	18100
u_1 { II	24500	25100	26500	27000	25300	25400	25100	23300	13100	13400
E	+ 5885	+ 6200	+ 6145	+ 6435	+ 5380	+ 4870	+ 2150	+ 1675	− 25	− 415
u_e	13000	13700	15500	16100	16300	16200	14300	11200	150	1800

REPORT ON QUEBEC BRIDGE.

APPENDIX B.

Anchor Arm – Suspenders.

Member	L_1-C_1	L_3-C_3	L_5-C_5	L_7-C_7 Upper	L_7-C_7 Lower	L_9-C_9 Upper	L_9-C_9 Middle	L_9-C_9 Lower
A	41 net	41 net	149 gr.	141 gr.	97 gr.	125 gr.	108 gr.	96 gr.
$\frac{l}{r}$			$\frac{933}{10.7}=87$	$\frac{974}{10.8}=90$	$\frac{333}{11.4}=29$	$\frac{927}{11.0}=84$	$\frac{598}{11.4}=52$	$\frac{328}{11.7}=28$
D	+ 315	+ 315	− 1055	− 1115	− 1270	− 1080	− 1245	− 1245
L	+ 412	+ 412	− 521	− 494	− 494	− 436	− 436	− 436
			+ 497	+ 458	+ 70	+ 420	+ 8	+ 8
S	+ 40	+ 40	− 70	− 75	− 115	− 70	− 110	− 110
W	0	0	− 570	− 720	− 720	− 870	− 870	− 870
D+L+S	+ 767	+ 767	− 1646	− 1684	− 1879	− 1586	− 1791	− 1791
$D+1\frac{1}{2}L+S+\frac{1}{3}W$	+ 973	+ 973	− 2096	− 2171	− 2366	− 2094	− 2299	− 2299
φ	$1+\frac{315}{727}=1.433$	$1+\frac{315}{727}=1.433$	$1+\frac{558}{2073}=1.269$	$1+\frac{657}{2067}=1.318$	$1+\frac{1200}{1834}=1.654$	$1+\frac{660}{1936}=1.341$	$1+\frac{1237}{1689}=1.732$	$1+\frac{1237}{1689}=1.73$[illegible]
u I	17200	17200	9700	9900	17500	10500	16300	13400
u II	18700	18700	11000	11900	19400	12700	16600	18700
u_1 I	24000	24000	15300	15000	21100	15600	18800	21200
u_1 II	23700	23700	14100	15400	24400	16700	21300	24000
E	+ 260	+ 260	− 1060	− 1110	− 1215	− 1085	− 1200	− 1200
u_e	6300	6300	7100	7900	12500	8700	11100	12500

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Anchor Arm – Diagonal Sub struts.

Member.		C1-L2	C3-L4	C5-L6	C7-L8	C9-L10
A		51.4 gr.	51.4 gr.	59.6 net	59.6 net	52.6 net
$\frac{l}{r}$		$\frac{889}{11.3} = 79$	$\frac{969}{11.3} = 86$			
D		− 345	− 310	+ 535	+ 620	+ 555
L		− 276	− 255	+ 363	+ 355	+ 317
		−	+ 25	− 319	− 275	− 250
S		− 25	− 25	+ 50	+ 55	+ 55
W		0	0	+ 340	+ 430	+ 500
D+L+S		− 646	− 590	+ 948	+ 1030	+ 927
D+1½L+S+⅓W		− 784	− 717	+ 1242	+ 1350	+ 1252
φ		$1+\frac{345}{621}=1.556$	$1+\frac{285}{590}=1.483$	$1+\frac{216}{1217}=1.177$	$1+\frac{345}{1250}=1.276$	$1+\frac{305}{1122}=1.272$
u	I	12500	11400	14100	15300	15300
	II	12600	11500	15900	17300	17600
u_1	I	16100	15400	24000	24000	24000
	II	15200	13900	20800	22600	23800
E		− 315	− 280	+ 560	+ 620	+ 565
u_e		6100	5400	9400	10400	10700

Report on Quebec Bridge.

Appendix B

Cantilever Arm - Upper Chord.

Member.	U_2-U_3	U_3-U_4	U_4-U_5	U_5-U_6	U_6-U_7	U_7-U_3	U_8-U_9	U_9-U_{10}
A	435	437	572	574	649	650	664	669
D	+ 3540	+ 3550	+ 7420	+ 7440	+ 10530	+ 10560	+ 11900	+ 11950
L	+ 1505	+ 1508	+ 2773	+ 2784	+ 3564	+ 3576	+ 3745	+ 3760
S	+ 325	+ 325	+ 640	+ 645	+ 860	+ 865	+ 930	+ 930
W	+ 350	+ 352	+ 608	+ 610	+ 728	+ 731	+ 990	+ 1140
D+L+S	+ 5370	+ 5383	+ 10833	+ 10869	+ 14954	+ 15001	+ 16575	+ 16640
$D+1\frac{1}{2}L+S+\frac{1}{3}W$	+ 6239	+ 6254	+ 12422	+ 12464	+ 16979	+ 17033	+ 18777	+ 18900
φ	$1+\frac{3540}{5045}=1.702$	$1+\frac{3550}{5058}=1.702$	$1+\frac{7420}{10193}=1.728$	$1+\frac{7440}{10224}=1.728$	$1+\frac{10530}{14094}=1.747$	$1+\frac{10560}{14136}=1.747$	$1+\frac{11900}{15645}=1.760$	$1+\frac{11950}{15710}=1.760$
u { I	20400	20400	20700	20700	21000	21000	21100	21100
u { II	12300	12300	18900	18900	23000	23100	25000	24900
u_1 { I	24000	24000	24000	24000	24000	24000	24000	24000
u_1 { II	14300	14300	21700	21700	26200	26200	28300	28300
E	+ 4405	+ 4410	+ 7890	+ 7910	+ 10575	+ 10610	+ 11635	+ 11680
u_e	10100	10100	13800	13800	16300	16300	17500	17500

Cantilever Arm - Lower Chord

Member	L_0-L_1	L_1-L_2	L_2-L_3	L_3-L_4	L_4-L_5	L_5-L_6	L_6-L_7	L_7-L_8	L_8-L_9	L_9-L_{10}
A	456	456	602	602	708	728	728	767	767	841
$\frac{l}{r}$	$\frac{675}{17.7}$ = ·38	$\frac{675}{17.7}$ = 38	$\frac{675}{16.5}$ = 41	$\frac{675}{16.5}$ = 41	$\frac{676}{16.2}$ = 42	$\frac{685}{16.2}$ = 42	$\frac{702}{16.2}$ = 43	$\frac{726}{16.1}$ = 45	$\frac{756}{16.1}$ = 47	$\frac{792}{16.1}$ = 49
D	− 3205	− 3205	− 7060	− 7060	− 10815	− 10655	− 12220	− 12630	− 13410	− 14050
L	− 1443	− 1443	− 2733	− 2733	− 3652	− 3699	− 3910	− 4042	− 3951	− 4140
S	− 295	− 295	− 610	− 610	− 865	− 875	− 960	− 990	− 1005	− 1055
W	− 700	− 1427	− 2453	− 3275	− 4154	− 5136	− 5518	− 6790	− 7244	− 8883
D+L+S	− 4943	− 4943	− 10403	− 10403	− 15032	− 15229	− 17090	− 17662	− 18366	− 19245
D+1½L+S+⅓W	− 5879	− 6140	− 12587	− 12861	− 18243	− 18790	− 20884	− 21946	− 22756	− 24276
φ	$1+\frac{3205}{4648}$=1.690	$1+\frac{3205}{4648}$=1.690	$1+\frac{7060}{9743}$=1.721	$1+\frac{7060}{9743}$=1.721	$1+\frac{10815}{14167}$=1.742	$1+\frac{10655}{14364}$=1.742	$1+\frac{12220}{16130}$=1.757	$1+\frac{12630}{16672}$=1.757	$1+\frac{13410}{17361}$=1.772	$1+\frac{14050}{18190}$=1.772
u I	20300	20300	20600	20600	20900	20900	21100	21100	21300	21300
u II	10800	10800	17300	17300	21200	20900	23500	23000	24000	22900
u_1 I	24000	24000	24000	24000	24000	24000	24000	24000	24000	24000
u_1 II	12900	13500	20900	21400	25800	25800	28700	28600	29700	28900
E	− 3860	− 3860	− 7545	− 7545	− 10605	− 10740	− 11965	− 12380	− 12925	− 13545
u_e	8500	8800	12500	12500	15000	14800	16400	16100	16900	16100

Cantilever Arm – Vertical Posts.

Member.	Lo–Uo	L2–U2	L4–U4	L6–U6	L8–U8 Upper	L8–U8 Middle	L8–U8 Lower.
A	249	241	283	259	196	184	184
$\frac{l}{r}$	$\frac{1086}{16.9} = 64$	$\frac{718}{15.6} = 46$	$\frac{835}{15.3} = 55$	$\frac{1060}{15.4} = 69$	$\frac{1174}{16.6} = 71$	$\frac{918}{16.8} = 55$	$\frac{601}{16.8} = 36$
D	– 2905	– 3210	– 3880	– 3480	– 1840	– 1840	– 2025
L	– 2235	– 1250	– 1285	– 962	– 570	– 570	– 854
					+ 133	+ 133	+ 133
S	– 265	– 265	– 285	– 215	– 45	– 45	– 90
W	– *570 – 340	– 280	– 240	– 190	– 990	– 990	– 990
D+L+S	– 5405	– 4725	– 5450	– 4657	– 2455	– 2455	– 2969
D+1½L+S+⅓W	– 6825	– 5443	– 6172	– 5201	– 3070	– 3070	– 3726
φ	$1+\frac{2905}{5140}=1.565$	$1+\frac{3210}{4460}=1.720$	$1+\frac{3880}{5165}=1.751$	$1+\frac{3480}{4442}=1.783$	$1+\frac{1707}{2548}=1.671$	$1+\frac{1707}{2543}=1.671$	$1+\frac{1892}{3012}=1.628$
u I	13800	16700	16200	15300	14100	15500	16600
u II	21700	19600	19200	18000	12500	13300	16100
u_1 I	17600	19400	18500	17100	16900	18500	20400
u_1 II	27400	22600	21800	20100	15700	16700	20200
E	– 1600	– 2220	– 3565	– 3140	– 1590	– 1590	– 1715
u_e	6400	9200	12600	12100	8100	8600	9300

* Bending.

Cantilever Arm - Main Diagonals.

Member-	Lo-C1	C1-U2	L2-C3	C3-U4	L4-C5	C5-U6	L6-C7	C7-U8	L8-C9	C9-U10
A	300	315	330	345	300	300	180	180	191 gr. 174 net	276 gr. 205 net
$\frac{l}{r}$									$\frac{984}{14.5}$ — 68	$\frac{816}{13.6}$ — 60
D	+ 4470	+ 4910	+ 5355	+ 5760	+ 5260	+ 4970	+ 2760	+ 2450	+ 950	+ 690
L	+ 2007	+ 2085	+ 2043	+ 2224	+ 1678	+ 1524	+ 805	+ 791	+ 673	+ 684
							− 26	− 202	+ 655	− 821
S	+ 420	+ 450	+ 445	+ 470	+ 380	+ 350	+ 150	+ 115	+ 15	+ 45
W	+ 487	+ 487	+ 391	+ 387	+ 19	+ 280	+ 1040	+ 1350	± 1840	± 2154
D+L+S	+ 6897	+ 7445	+ 7843	+ 8484	+ 7318	+ 6844	+ 3715	+ 3356	+ 1623	+ 1374
D+1½L+S+⅓W	+ 8062	+ 8649	+ 8994	+ 9695	+ 8163	+ 7699	+ 4464	+ 4201	+ 2572	+ 2434
φ	$1+\frac{4470}{6477}=1.691$	$1+\frac{4910}{6995}=1.702$	$1+\frac{5355}{7398}=1.724$	$1+\frac{5760}{7984}=1.721$	$1+\frac{5260}{6938}=1.758$	$1+\frac{4970}{6494}=1.765$	$1+\frac{2734}{3591}=1.761$	$1+\frac{2248}{3443}=1.653$	$1+\frac{295}{2278}=1.130$	$\frac{821}{2196}=0.374$
u { I	20300	20400	20700	20700	21100	21200	21100	19800	13600	4490
u { II	23000	23600	23800	24500	24400	22800	20600	18600	9300	6700
u_1 { I	24000	24000	24000	24000	24000	24000	24000	24000	24000	24000
u_1 { II	26900	27400	27200	28100	27200	25700	24800	23300	14800	11900
E	+ 2560	+ 3260	+ 4795	+ 5185	+ 4630	+ 4280	+ 2210	+ 1860	+ 515	+ 235
u_e	8500	10300	14500	14900	15400	14300	12300	10300	3000	1100

Cantilever Arm - Suspenders.

Member-	L_1-C_1	L_3-C_3	L_5-C_5	L_7-C_7 Upper	L_7-C_7 Lower	L_9-C_9 Upper	L_9-C_9 Lower
A	60 net	60 net	117 gr.	117 gr.	89 gr.	117 gr.	93 gr.
$\frac{l}{r}$			$\frac{937}{11.2}=84$	$\frac{974}{11.2}=87$	$\frac{335}{11.7}=29$	$\frac{918}{11.2}=82$	$\frac{601}{11.6}=52$
D	+ 355	+ 355	− 735	− 850	− 1035	− 845	− 1040
L	+ 458	+ 458	− 402	− 414	− 414	− 388	− 388
			+ 412	+ 415		+ 416	
S	+ 45	+ 45	− 55	− 60	− 100	− 60	− 100
W	0	0	− 507	− 650	− 650	− 771	− 771
D+L+S	+ 858	+ 858	− 1192	− 1324	− 1549	− 1293	− 1528
$D+1\frac{1}{2}L+S+\frac{1}{3}W$	+ 1087	+ 1087	− 1562	− 1748	− 1973	− 1744	− 1979
φ	$1+\frac{355}{813}=1.437$	$1+\frac{355}{813}=1.437$	$1+\frac{323}{1549}=1.208$	$1+\frac{435}{1679}=1.259$	$1+\frac{1035}{1449}=1.714$	$1+\frac{429}{1649}=1.260$	$1+\frac{1040}{1428}=1.728$
u I	17200	17200	9400	10000	18100	10000	16200
u II	14300	14300	10200	11300	17400	11000	16400
u_1 I	24000	24000	15600	15300	21100	15800	18800
u_1 II	18100	18100	13300	14900	22200	14900	21300
E	+ 760	+ 300	− 825	− 900	− 1030	− 860	− 995
u_e	12700	5000	7100	7700	11600	7400	10700

Report on Quebec Bridge.

Appendix B.

Cantilever Arm – Diagonal Sub struts.

Member		C1–L2	C3–L4	C5–L6	C7–L8	C9–L10
A		64 gr.	64 gr.	64 net	64 net	64 net.
$\frac{l}{r}$		$\frac{951}{11.4} = 83$	$\frac{1029}{11.4} = 90$			
D		− 440	− 405	+ 325	+ 420	+ 380
L		− 328	− 306	+ 283	+ 300	+ 279
			+ 16	− 265	− 264	− 261
S		− 30	− 25	+ 40	+ 50	+ 45
W		0	− 4	+ 332	+ 412	+ 465
D+L+S		− 798	− 736	+ 648	+ 770	+ 704
D+1½L+S+⅓W		− 962	− 890	+ 900	+ 1057	+ 993
φ		$1+\frac{440}{768} = 1.573$	$1+\frac{389}{727} = 1.535$	$1+\frac{60}{873} = 1.069$	$1+\frac{156}{984} = 1.158$	$1+\frac{119}{920} = 1.129$
u	I	12300	11500	12800	13900	13500
	II	12500	11500	10100	12000	11000
u_1	I	15700	15000	24000	24000	24000
	II	15000	13900	14100	16500	15600
E		− 705	− 360	+ 390	+ 455	+ 395
u_e		11000	5600	6100	7100	6200

Suspended Span–Upper Chord

Member	$U_0 U_1$	$U_1 U_2$	$U_2 U_3$	$U_3 U_4$	U_4 [illegible]	$U_5 U_6$
A	158 gr. 136 net	158 gr. 136 net	224 gr. 191 net	224 gr. 191 net	242 gr. 207 net	242 gr. 207 net
$\frac{l}{r}$	$\frac{686}{13.9} = 49$	$\frac{683}{13.9} = 49$	$\frac{680}{13.5} = 50$	$\frac{677}{13.5} = 50$	$\frac{676}{13.5} = 50$	$\frac{675}{13.5} = 50$
D	− 2400	− 2385	− 3415	− 3400	− 3700	− 3695
L	− 1098	− 1091	− 1582	− 1577	− 1717	− 1714
S	− 225	− 225	− 325	− 325	− 350	− 350
W	− 181	− 330	− 445	− 528	− 577	− 594
D + L + S	− 3723	− 3701	− 5322	− 5302	− 5767	− 5759
$D + 1\frac{1}{2}L + S + \frac{1}{3}W$	− 4332	− 4356	− 6261	− 6266	− 6817	− 6814
φ	$1+\frac{2400}{3498}=1.686$	$1+\frac{2385}{3476}=1.686$	$1+\frac{3415}{4997}=1.683$	$1+\frac{3400}{4977}=1.683$	$1+\frac{3700}{5417}=1.683$	$1+\frac{3695}{5409}=1.683$
u	20200	20200	20200	20200	20200	20200
	23600	23400	23800	23700	23800	23800
u_1	24000	24000	24000	24000	24000	24000
	27400	27600	28000	28000	28200	28200
E	+ 380	+ 380				
u_e	2800	2800				

Suspended Span – Lower Chord.

Member		Loa–L1	L1–L2	L2–L3	L3–L4	L4–L5	L5–L6	
A		251 gr. 220 net	251 gr. 220 net	208 gr. 186 net	208 gr. 186 net	260 net	260 net	
$\frac{l}{r}$		$\frac{588}{12.15} = 48$	$\frac{675}{12.15} = 56$	$\frac{675}{12.6} = 54$	$\frac{675}{12.6} = 54$			
D		+ 210	+ 210	+ 2500	+ 2500	+ 3550	+ 3550	
L	{	+ 261	+ 261	+ 1167	+ 1167	+ 1646	+ 1646	
		− 18	− 18					
S		+ 20	+ 20	+ 235	+ 235	+ 335	+ 335	
W		+ 227	+ 625	+ 947	+ 1189	+ 1350	+ 1431	
D+L+S		+ 491	+ 491	+ 3902	+ 3902	+ 5531	+ 5531	
$D+1\frac{1}{2}L+S+\frac{1}{2}W$		+ 697	+ 829	+ 4801	+ 4881	+ 6804	+ 6831	
φ		$1+\frac{192}{489} = 1.393$	$1+\frac{192}{489} = 1.393$	$1+\frac{2500}{3667} = 1.682$	$1+\frac{2500}{3667} = 1.682$	$1+\frac{3550}{5196} = 1.683$	$1+\frac{3550}{5196} = 1.683$	
u	I	16700	16700	20200	20200	20200	20200	
	II	2230	2230	21000	21000	21300	21300	
u_1	I	24000	24000	24000	24000	24000	24000	
	II	3170	3770	25800	26200	26200	26300	
E		− 1790	− 1790					
u_e		7100	7100					

SUSPENDED SPAN:-

	Vertical Posts.		Main Diagonals.					
Member-	L_2-U_2	L_4-U_4	U_0-C_1	C_1-L_2	U_2-C_3	C_3-L_4	U_4-C_5	C_5-L_6
A	134 gr. 119 net	78 gr. 70 net	204	191	118	114	75 net	94 gr. 82 net
$\frac{l}{r}$	$\frac{748}{10.7} = 70$	$\frac{792}{11.6} = 68$						$\frac{1015}{11.2} = 91$
D.	− 1215	− 300	+ 3085	+ 2840	+ 1495	+ 1285	+ 445	+ 215
L	− 646	− 373	+ 1433	+ 1365	+ 834	+ 813	+ 509	+ 494
	+ 95	+ 235	−	− 31	− 106	− 196	− 282	− 387
S	− 100	− 15	+ 285	+ 265	+ 145	+ 120	+ 45	+ 20
W	0	0	0	0	0	0	0	0
D+L+S	− 1961	− 688	+ 4803	+ 4470	+ 2474	+ 2218	+ 999	+ 729
D+1½L+S+⅓W	− 2284	− 874	+ 5519	+ 5152	+ 2891	+ 2624	+ 1253	+ 976
φ	$1+\frac{1120}{1956}=1.572$	$1+\frac{65}{908}=1.072$	$1+\frac{3085}{4518}=1.683$	$1+\frac{2809}{4236}=1.663$	$1+\frac{1389}{2435}=1.570$	$1+\frac{1089}{2294}=1.475$	$1+\frac{163}{1236}=1.132$	$\frac{387}{1096}=0.353$
u I	13400	9200	20200	20000	18800	17700	13600	4240
u II	14600	8800	23500	23400	21000	19400	13300	8900
u_1 I	17000	17200	24000	24000	24000	24000	24000	24000
u_1 II	17000	11200	27000	27000	24500	23000	16700	11900
E	− 540		+ 2155	+ 1870	+ 535			
u_e	4000		10600	9800	4500			

SUSPENDED SPAN

Suspenders

Member-	Loa-Uo	[illegible]	L3-C3	L5-C5
A	50		42 net	42 net
$\frac{l}{r}$				
D	+ 340	265	+ 255	+ 275
L	+ 471	+ 405	+ 458	+ 458
	− 17			
S	+ 40	+ 40	+ 45	+ 45
W	0	0	0	0
D+L+S	+ 851	+ 710	+ 758	+ 778
D+$1\frac{1}{2}$L+S+$\frac{1}{3}$W	+ 1086	+ 912	+ 987	+ 1007
φ	$1+\frac{323}{828}=1.390$	$1+\frac{265}{670}=1.396$	$1+\frac{255}{718}=1.358$	$1+\frac{275}{733}=1.375$
u I	16700	16800	16300	16500
u II	17000	16900	18000	18500
u_1 I	24000	24000	24000	24000
u_1 II	21700	21700	23500	24000
E	+ 340	+ 205	+ 110	
u_e	6800	4900	2600	

Diagonal Substruts

Loa-C1	L2-C3	L4-C5
49 gr.	53 gr.	53 gr.
$\frac{328}{10.15}=32$	$\frac{969}{11.55}=84$	$\frac{1016}{11.55}=88$
− 260	− 210	− 230
− 292	− 280	− 265
+ 24	+ 26	+ 21
− 25	− 25	− 25
0	0	0
− 577	− 515	− 520
− 723	− 655	− 652
$1+\frac{236}{376}=1.410$	$1+\frac{184}{516}=1.357$	$1+\frac{209}{516}=1.405$
13500	12700	13100
11800	9700	9800
15800	15600	15200
14800	12400	12300
− 305	− 535	
6200	10100	

REPORT ON QUEBEC BRIDGE. APPENDIX B.

Table of Permissible Unit Strains for Compression Members.

p - permissible strain in lbs. pr. sq. in.

l - unsupported length of member in inches.

r = least radius of gyration in inches.

$p = 21000 - 90 \frac{l}{r}$

$\frac{l}{r}$	p	$\frac{l}{r}$	p	$\frac{l}{r}$	p	$\frac{l}{r}$	p
30	18300	51	16410	72	14520	93	12630
31	18210	52	16320	73	14430	94	12540
32	18120	53	16230	74	14340	95	12450
33	18030	54	16140	75	14250	96	12360
34	17940	55	16050	76	14160	97	12270
35	17350	56	15960	77	14070	98	12180
36	17760	57	15870	78	13980	99	12090
37	17670	58	15780	79	13890	100	12000
38	17580	59	15690	80	13800	101	11910
39	17490	60	15600	81	13710	102	11820
40	17400	61	15510	82	13620	103	11730
41	17310	62	15420	83	13530	104	11640
42	17220	63	15330	84	13440	105	11550
43	17130	64	15240	85	13350	106	11460
44	17040	65	15150	86	13260	107	11370
45	16950	66	15060	87	13170	108	11280
46	16860	67	14970	88	13080	109	11190
47	16770	68	14880	89	12990	110	11100
43	16680	69	14790	90	12900	111	11010
49	16590	70	14700	91	12810	112	10920
50	16500	71	14610	92	12720	113	10830

$p = 24000 - 100 \frac{l}{r}$

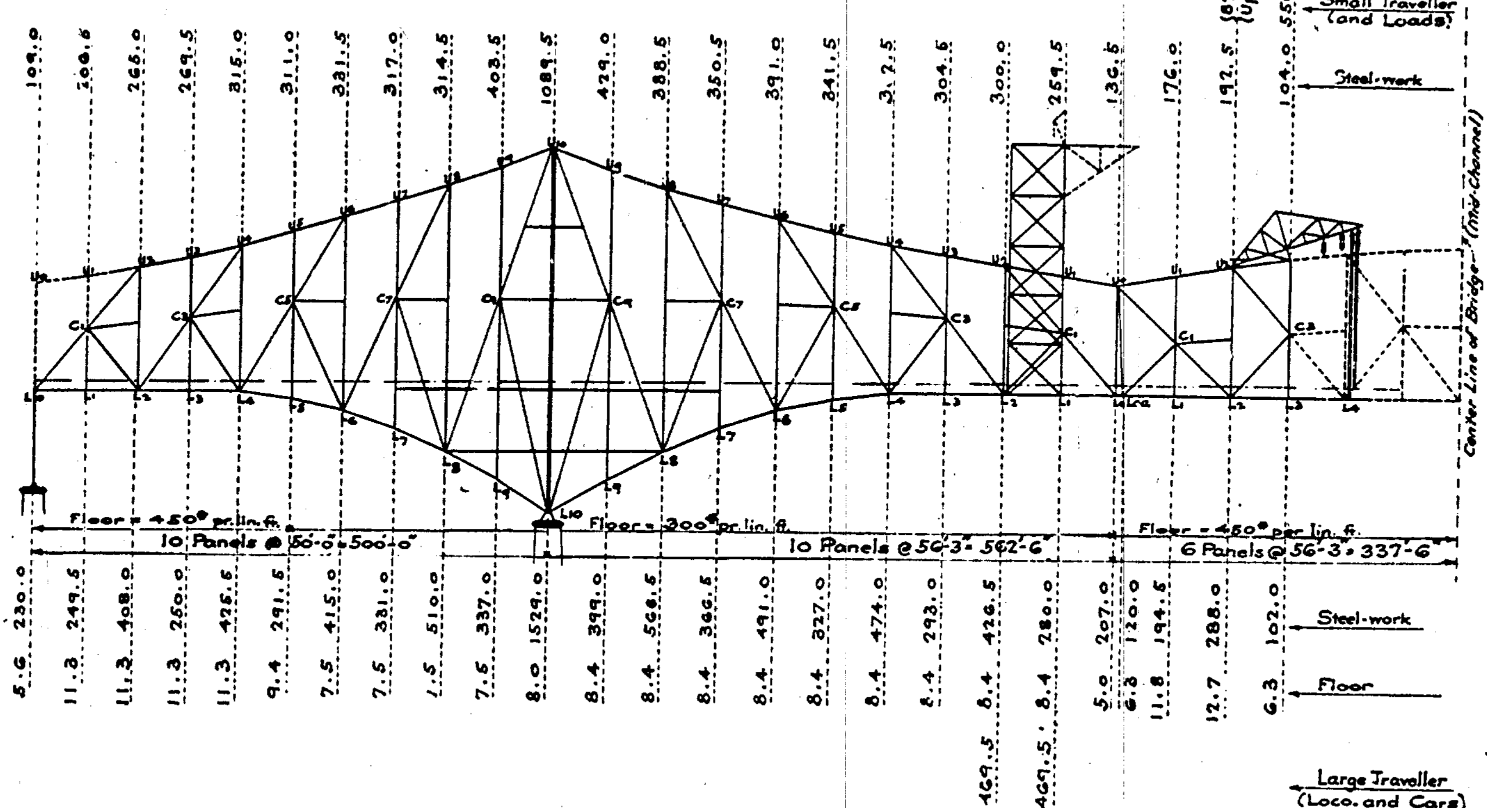
$\frac{l}{r}$	p	$\frac{l}{r}$	p	$\frac{l}{r}$	p	$\frac{l}{r}$	p
30	21000	51	18900	72	16800	93	14700
31	20900	52	18800	73	16700	94	14600
32	20800	53	18700	74	16600	95	14500
33	20700	54	18600	75	16500	96	14400
34	20600	55	18500	76	16400	97	14300
35	20500	56	18400	77	16300	98	14200
36	20400	57	18300	78	16200	99	14100
37	20300	58	18200	79	16100	100	14000
38	20200	59	18100	80	16000	101	13900
39	20100	60	18000	81	15900	102	13800
40	20000	61	17900	82	15800	103	13700
41	19900	62	17800	83	15700	104	13600
42	19800	63	17700	84	15600	105	13500
43	19700	64	17600	85	15500	106	13400
44	19600	65	17500	86	15400	107	13300
45	19500	66	17400	87	15300	108	13200
46	19400	67	17300	88	15200	109	13100
47	19300	68	17200	89	15100	110	13000
48	19200	69	17100	90	15000	111	12900
49	19100	70	17000	91	14900	112	12800
50	19000	71	16900	92	14800	113	12700

Report on Quebec Bridge.

Appendix B.

Conditions on Aug. 29, 1907,

Showing Concentrations on Panel Points produced by Dead Load and Travellers.



Loads are for one Truss and are given in units of 1000#

REPORT ON DESIGN OF QUEBEC BRIDGE BY C. C. SCHNEIDER.

APPENDIX C.

THEORY OF COLUMNS.

A REVIEW OF EXISTING LITERATURE AND EXPERIMENTS.

An ideal column with a straight axis and of uniform material, loaded in the direction of its axis, would fail in direct compression by crushing. In practice, a column will fail by buckling caused by lateral deflection.

Failure by direct compression or tension is caused by strains which exceed the resistance of the material. Since these strains are in direct proportion to the loads causing them, it became customary to measure the safety of a structure by the ratio of the working strain to the ultimate strength, instead of by the ratio of the permissible load to the load causing failure.

Failure by buckling, however, is not necessarily the result of overstraining the material, as the strains are not in direct proportion to the corresponding loads (see examples page 192), but depend upon certain conditions which influence the strength of a column considered as a member of a structure.

Perhaps the clearest conception of buckling can be obtained by considering it as the result of unstable equilibrium between the external and internal forces. Assuming a steel spring (Fig. 1) rigidly fixed at the bottom, and loaded at the top with a weight W, then the spring will slightly deflect laterally, but will remain in equilibrium. If W is gradually increased, a condition will be reached where equilibrium is no more possible, and the weight will drop suddenly. The spring has lost its supporting power at this moment of unstability, but the weight may go to the bottom without producing any excessive strains in the spring.

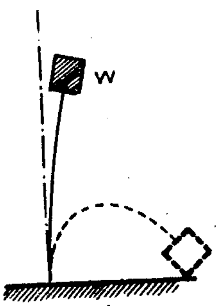


Fig. 1

The lateral deflection of a column is caused by an initial eccentricity as the load will not be exactly in the center, nor the axis be mathematically straight and the material uniform throughout the column, owing to irregularities in rolling, or caused by straightening, riveting, drifting, &c. (In an I-beam 8 feet long, Bauschinger found a variation of 5 per cent. in the elastic modulus and in the ultimate strength.)

This initial eccentricity and the deflection produced by it will cause bending and shearing strains in the column in addition to direct compression.

The *average* compressive strain obtained by dividing the buckling load—that is, the load under which the column fails—by the area of its section is called the *buckling strain*.

I. Long Columns.

In order to find a formula for the buckling strain, long columns which fail with a buckling strain within the elastic limit will first be considered. To apply to these the theory of elasticity is not strictly correct, as the maximum fibre strain may have exceeded the elastic limit; however, this, as will be shown later, affects the buckling load only very slightly. The true elastic limit for wrought iron and steel is almost identical with the limit of proportionality between strain and deformation.

Let it be assumed that an elastic column with hinged ends free to move in the direction of its original axis, and subjected to an axial load P, has been deflected laterally (see fig 2). Neglecting the shortening of the column and the influence of the shearing strains, and assuming $s=x$, the elastic line is represented by the differential equation

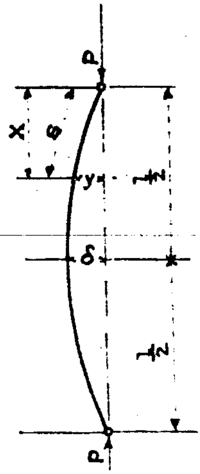


Fig. 2

$$\frac{d^2y}{dx^2}=\frac{M}{EI} \quad \ldots \ldots \quad (1)$$

where bending moment $M=Py$, $I=$ Moment of Inertia of the section, and $E=$ Modulus of Elasticity of the material of the column.

Twice integrating,

$$\text{then } y=\delta \sin x\sqrt{\frac{P}{EI}} \quad \ldots \ldots \quad (2)$$

where $\delta=$ deflection at the centre.

The elastic line, therefore, is a sinus curve for $x=l$, and $y=o$, then from equation (2)

$$P_o=\pi^2\frac{EI}{l^2} \quad \ldots \ldots \quad (3)$$

as the load which holds the internal strains in equilibrium.

This formula is known as Euler's formula, having been first introduced by Euler in 1759. Since this formula does not contain δ, P_o is the load which, after a lateral deflection is once started, may increase this deflection, and with it the fibre strain, rapidly and finally produce buckling. This buckling load, therefore, is independent of the strength of the material as long as E remains the same.

According to Euler's formula, a column made of steel containing 3 per cent nickel, with an ultimate strength about 50 per cent higher than ordinary carbon steel, could safely carry a load only about 4 per cent greater than an identical column made of ordinary carbon steel; that is, in proportion of the moduli of elasticity.

On account of the assumptions made in deriving formula (3), P_o does not correctly represent the buckling load. More correct formulae have been derived by Grashof, (Festigkeits Lehre, published 1866) who gives

$$P=\frac{\pi^2 EI}{l^2}\left(1+\frac{\pi^2\delta^2}{8l^2}\right) \quad \ldots \ldots \quad (4)$$

and by Wm. Cain, (Trans. A. S. C. E., Vol. XXXIX.) who derives

$$\delta^2=16\left[\frac{l}{\pi}\sqrt{\frac{EI}{P}}-\frac{EI}{P}\right] \quad \ldots \ldots \quad (5)$$

An investigation of formulae (4) and (5) shows that if P exceeds $P_o=\frac{\pi^2 EI}{l^2}$, a certain deflection δ corresponds to the load P; but that a very small increase over P_o is sufficient to make the deflection excessive and cause failure; so that P_o can practically be regarded as the buckling load. In these formulæ for $\delta=o$, $P=P_o$, in Euler's formula; in other words, P_o represents the load under which bending just begins, so that for smaller loads than P_o the compressive strains are uniformly distributed over the section.

In formulae (3), (4) and (5), the initial eccentricity 'e' (fig. 3) has been assumed negligible as compared with the deflection δ. Investigation of the formula given on page 191 for eccentric loading shows that any load P, even below P_o, can produce deflection; but if the eccentricity 'e' is small, the buckling load will be only slightly smaller than P_o, although the maximum fibre strain produced thereby may be higher than the buckling strain. This is another reason for regarding P_o as the actual buckling load. A greater initial eccentricity will reduce the buckling load by giving fibre strains above the limits of safety.

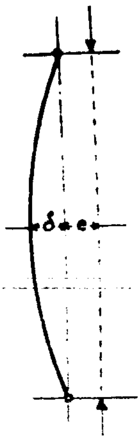


Fig. 3

On this basis, many attempts have been made to derive formulae giving the load which would cause failure by excessive fibre strains. (See J. M. Moncrieff, Trans. A. S. C. E., Vol., XLV.)

The yield point must be regarded as the highest safe fibre strain, because as soon as it is exceeded the deflection increases rapidly until finally failure occurs.

On the other hand, an initial bend in the column can counteract the initial eccentricity of the load, keeping the column in stable equilibrium even for a greater load than P_o. These cumulative influences explain the different actions of columns in testing as regards deflections and breaking loads.

As it is impossible to determine for every case the initial eccentricity, a buckling formula has to be derived for the case of an ideal, or nearly ideal column; provided this formula agrees with the results of experiments made under conditions as nearly as possible like those of the ideal column.

In determining the safe working load, the lowest test result should be used with a margin of safety.

Similar conditions occur in bending. The permissible strain for bending is derived from the ultimate strength with the provision that under the worst condition the fibre strain shall remain below the yield point.

Column tests, especially those with point bearings made by Tetmajer and Bauschinger, prove that for long columns, which fail with a buckling strain within the elastic limit, Euler's formula gives correct results. (See L. v. Tetmajer, 'Die Gesetze der Knickungs festigkeit,' 3rd edition, Leipzig and Wien, 1903, also, 'Mitteilungen der Materiel Prüfungsanstalt,' München, 1887, by Bauschinger.)

Euler's formula (3) does not give the greatest strains actually existing in a column. This has caused the introduction of various formulæ which apparently express the relation between the load and the corresponding greatest strain. Since, however, as has been seen, strains in buckling are very uncertain, all the formulæ based on strains contain one or more coefficients, the values of which have to be derived empirically from the buckling load of column tests. Dividing the buckling strain thus found by a factor of safety, the formulæ represent more or less correctly safe loads, but they do not give the actual safe unit strains.

One of these is the extensively used 'Rankine Formula.'

$$k_o = \frac{k_u}{1 + c\,\frac{l^2}{r^2}} \qquad (6)$$

where k_o = buckling strain, k_u an assumed constant approximately equal to the yield point and c a constant to be derived from tests. It has been proven, however, by experiments and analytically, that c is not constant but varies not only with the material, but also with the value of $\frac{l}{r}$ and with the average unit strain. Tetmajer found by tests a variation of $c = 0{\cdot}000448$ to $0{\cdot}000136$ for wrought iron, and $c = 0{\cdot}000370$ to $0{\cdot}000130$ for steel.

The fact that it is possible to give k_u and c such values that k_o corresponds fairly well with observed buckling strains within the practical limits of $\frac{l}{r}$ makes the formula applicable to practical use. It thus becomes an empirical formula.

Dividing P_o by the area, Euler's formula takes the following form:—

$$k_o = \pi^2 E \frac{r^2}{l^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where k_o represents the buckling strain. Giving k_o the value of the elastic limit, and solving for $\frac{l}{r}$, the limit for Euler's formula is found to be

$$\frac{l}{r} = \pi \sqrt{\frac{E}{k_o}}$$

Tetmajer found the following values:—

For wrought iron with an elastic limit $= 22,600$, $\frac{l}{r} = 112$

soft steel " " $= 27,100$, $\frac{l}{r} = 105$

medium steel " " $= 28,400$, $\frac{l}{r} = 105$

and with $E = 28,450,000$, $30,580,000$ and $32,000,000$ respectively, Euler's formula becomes

$$k_o = 280,800,000 \left(\frac{r}{l}\right)^2 \text{ for wrought iron,}$$

$$k_o = 301,800,000 \left(\frac{r}{l}\right)^2 \text{ " soft steel,}$$

$$k_o = 315,900,000 \left(\frac{r}{l}\right)^2 \text{ " medium steel.}$$

II. Short Columns.

Thus far this subject has been considered theoretically only, in order to give a clear account of the nature of buckling. Columns which fail with buckling strains above the elastic limit will now be considered. These include the majority of cases occurring in actual practice.

Since, as shown above, Euler's formula is limited, and is applicable to steel columns only whose $\frac{l}{r}$ exceeds 105, it appears desirable to consider the subject wholly from a practical standpoint and endeavour to find an empirical buckling formula based on experiments.

The first question to be considered is: What is the buckling strain for a very short column (theoretically $\frac{l}{r} = o$)?

Fig. 4 represents the typical deformation diagram for wrought iron or steel, the abscissas ϵ representing the elongations corresponding to the stresses k as ordinates. P denotes the limit of proportionality, or elastic limit, and Y the yield point. Up to the elastic limit, the modulus of elasticity E for wrought iron and steel is constant, but is variable for higher strains. If the values of E for strains above the elastic limit were known and applied to Euler's formula, it would still express correctly the buckling load.

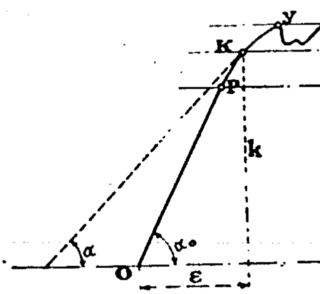


Fig. 4

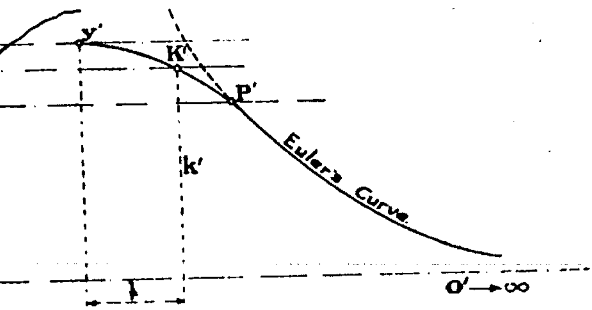


Fig. 5

Drawing a tangent to the curve at a point K, the corresponding modulus of elasticity may be represented by

$$E = \frac{dk}{d\epsilon} = tg.\,a;$$

introducing this in Euler's formula and solving for $\frac{l}{r}$

then $\frac{l}{r} = \pi\sqrt{\frac{tg.\,a}{k}}$

This equation enables us to construct the curve of Fig. 5, where the abscissas represent the values $\frac{l}{r}$ and the ordinates the strains k. If point K travels on the straight line from O to P, E is constant $= E_0 = tg\ a_0$ and point K' follows Euler's curve from O' to P', the corresponding values of $\frac{l}{r}$ for point P' being those given on page 186.

If point K continues from P to Y, $tg\ a$ gradually decreases from $tg\ a_0$ to zero, while point K' travels over curve $P'\ Y'$ and $\frac{l}{r}$ gradually becomes zero.

This means that a very short column becomes unstable when the buckling strain reaches the yield point, since this is the point of first horizontal tangency. As is well known, the yield point, commercially called elastic limit, manifests itself in testing by the sudden drop of the test load.

Cast iron does not follow the law of proportionality, nor has it a yield point (see deformation diagram, fig. 6).

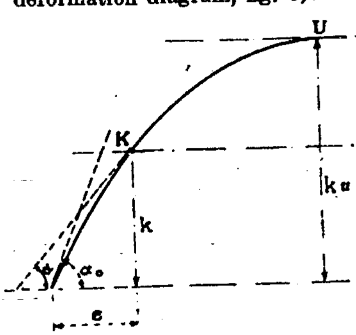


Fig. 6

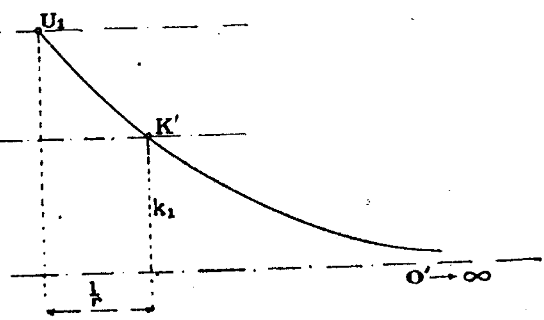


Fig. 7

$tg\ \alpha$ decreases from the point of zero strain, and becomes zero where the tangent to the deformation curve becomes horizontal; that is, at the point U of ultimate strength k_u. Point K' does not follow Euler's curve (fig. 7) but reaches U_1 for $\frac{l}{r} = 0$ in a more regular parabolic curve. The buckling strain for very short columns $\left(\frac{l}{r} = 0\right)$ is therefore equal to the ultimate strength. This explains the fact that short cast iron columns show a much higher resistance to buckling than wrought iron or ordinary steel columns.

If tests were made with short columns of very hard steel, in which the yield point and ultimate strength are close together, these tests would evidently also show a proportionately greater buckling strength than those of ordinary steel.

While some engineers are of the opinion that the ultimate strength in tension should be regarded as the buckling strain for $\frac{l}{r} = 0$, others have recognized the yield point as this ultimate buckling strain. (See J. B. Johnson's 'Modern Framed Structures,' p. 159.)

What is generally called yield point (about 60 to 70 per cent of the ultimate strength for steel) is an apparent strain obtained from tension tests based on the original area of the bar. Since the area of the bar has decreased, the true yield point must be higher, and this is equal to the true yield point in compression. The apparent yield point in compression based on the original area of the compression member is still higher since in compression the area has increased and this yield point must be regarded as ultimate buckling strain, because the latter is also based on the original area.

Since the increase of the area is not known, the ultimate buckling strain must be found from tests. Undoubtedly a column of say $\frac{l}{r} = 5$ in the testing machine acts practically the same as one of $\frac{l}{r} = 0$; that is, the strain is uniformly distributed up to the breaking point, since any accidental eccentricity would cause only very small bending strains. The buckling strains thus found can, therefore, be considered as the ultimate buckling strain for $\frac{l}{r} = 0$.

Tetmajer found for this strain which he calls 'a kind of compressive strength, different from, but comparable to the crushing strength of cubes,' the following values:

	Lbs. per sq. in.
For wrought iron	$k_u = 43{,}100$
" soft steel	$k_u = 44{,}100$
" medium steel	$k_u = 45{,}700$

A rational column formula should contain these values as the limiting buckling strain for $\frac{l}{r} = 0$ and give buckling strains decreasing from this limit with increasing $\frac{l}{r}$.

The curve representing this formula should, moreover, intersect Euler's curve at the point for which k_0 is equal to the true elastic limit. As this latter strain as well as the yield point is more or less variable, even in the same material, it is evident that points P' and Y' (fig. 5) can be chosen within certain limits. Owing to the greatly varying test results, it is also evident that a great number of different curves can be drawn between points P' and Y' as representing approximately the average of the plotted test results.

For all practical purposes, the simplest curve is naturally the best and that is the straight line.

The writer considers that all these more or less complicated analytical formulæ (like Rankie's, &c.) are not justified. Analytical formulæ based on the theory of proportionality between the strain and deformation (with a constant E) cease to be correct for the buckling strains which are now being considered; and have been made applicable to the latter by merely choosing empirical coefficients.

The following publications contain the results of column tests and diagrams, with the results plotted and the different curves representing the formulæ. An examination will show that the straight line fits at least as well as any curve:—

1.—L. F. G. Bouscaren. Trans. A.S.C.E., Vol. IX.

2.—J. Christie, 'Experiments on the Strength of Wrought Iron Struts.' Trans. A.S.C.E., Vol. XIII.

3.—T. H. Johnson, 'On the Strength of Columns.' Trans. A.S.C.E., Vol. XV.

4.—C. A. Marshall. Trans. A.S.C.E., Vol. XVII.

5.—C. L. Strobel, 'Experiments upon Z-Iron Columns.' Trans. A.S.C.E., Vol. XVIII.

6.—Tests of Metals made at Watertown Arsenal. Vols. 1881, 1882, 1883, 1884 and 1885.

7.—A. Marston, 'On the Theory of the Ideal Column.' Trans. A.S.C.E., Vol. XXXIX.

8.—J. M. Moncrieff, 'The Practical Column.' Trans. A.S.C.E., Vol. XLV.

9.—Johnson, Bryan and Turneaure, 'The Modern Framed Structures.' 8th Edition, page 168.

10.—G. Lanza, 'Applied Mechanics,' page 416.

11.—L. v. Tetmajer, 'Die Gesetze der Knickungs festigkeit.' 3rd Edition, 1903.

12.—Prof. Bauschinger, 'Mitteilungen der Material prüfungsanthalt München.' 15th Vol.

The straight line formula

$$k_o = k_u - c\frac{l}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

was first proposed in 1886 by T. H. Johnson (see Trans. A.S.C.E., Vol. XV.) and is now generally used. He derived it from tests of wrought and cast iron and steel columns made by Hodgkinson, Christie and others under greatly varying conditions, and proposed for columns with round ends, the following buckling strength:—

Wrought iron,	$42{,}000 - 203\,\frac{l}{r}$,	upper limit	$\frac{l}{r} = 138$
Carbon 0·12%, soft steel,	$52{,}500 - 284\,\frac{l}{r}$,	"	$\frac{l}{r} = 123$
" 0·36%, hard steel,	$80{,}000 - 534\,\frac{l}{r}$,	"	$\frac{l}{r} = 100$

They represent straight lines drawn from k_u tangent to Euler's curve. By referring to the above-mentioned tests, it is evident that k_u is too high for steel, while the point of meeting Euler's curve is too low. A less inclined line, taking the former point lower and the latter higher would give more correct results.

Based on his own numerous tests of wrought iron and steel columns with point bearings, L. v. Tetmajer introduced a stright line formula, at the same time proving the correctness of Euler's formula for buckling strains lower than the elastic limit.

(See 'Mitteilungen der Material Prüfungsanstalt, Zürich,' Vol. VIII, also L. v. Tetmajer, 'Die Gesetze der Knickungs festigkeit,' 3rd part, 1903.) He proposed for

Wrought iron, $k_o = 43{,}100 - 183 \frac{l}{r}$, $\frac{l}{r} \overline{\overline{<}} 112$

Ultimate strength < 57,000, soft steel, $k_o = 44{,}100 - 162 \frac{l}{r}$, $\frac{l}{r} \overline{\overline{<}} 105$.

" > 57,000, medium steel, $k_o = 45{,}700 - 165 \frac{l}{r}$, $\frac{l}{r} \overline{\overline{<}} 105$

Since steel columns with $\frac{l}{r} > 105$ are used for unimportant parts only, and the difference between Euler's and the straight line formula is only small for $\frac{l}{r}$ from 105 to 120 (which is generally the practical limit), it is justifiable to use the straight line formula throughout.

The permissible unit strain for tension is usually deduced from the ultimate strength; while that for compression must be deduced from the considerably lower buckling strain. For compression therefore, a smaller factor of safety is permissible than for tension, since the strains in either case must remaiin with a margin of safety below the true yield point.

If, in accordance with usual practice, a unit strain of 16,000 pounds per square inch in tension for structural steel (55,000 to 65,000 ultimate strength) is used,the same strain is permissible for a column with $\frac{l}{r} = O$ in compression. For longer columns, this strain has to be reduced by the formula in order to have the same factor of safety for all ratios of $\frac{l}{r}$. The formula for the permissible unit strain

$$s_o = 16000 - 70 \frac{l}{r} \quad \text{. . . . . (9)}$$

which was adopted by the Committee on Steel Structures of the American Railway Engineering and Maintenance of Way Association will allow a safety of about 3.

Thus far only the case of a column with ends free to rotate has been considered. This case, however, does not occur in practice; the ends will always offer more or less resistance to turning. All cases, however, can be treated similarly by assuming the so-called buckling length; that is, the distance between points of contraflexure.

The assumption of the buckling length is mainly a matter of practical judgment, since in practice no column will correspond either to theory or to experiments.

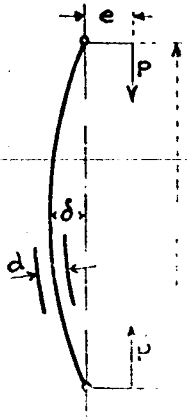
For compression members with hinged ends, the friction of the hinges should be entirely neglected and even for compression members with riveted, and, therefore, partly fixed ends, the free buckling length should not be assumed less than the distance between connections on account of the secondary strains due to the elastic deformation of the truss. These secondary strains, as will be seen from Appendix D, are the result of bending moments which may partly or entirely counteract the fixity of the ends.

III. The Eccentrically Loaded Column.

Since in practice a column is always more or less eccentrically loaded, this case must be considered in order to determine to what degree an eccentricity can affect the buckling load of the ideal column. This will also show the increase of the fibre strains when the load increases. Of course, only comparatively small eccentricities are considered; such as may occur in compression members of trusses.

In a column loaded eccentrically and parallel to the original axis, the deflection δ can be accurately determined; hence also the bending moments and the fibre strains, provided the latter do not exceed the limit of proportionality. In order to get comparative figures, however, the following formula will be used for strains up to the yield point:—

With the notations of fig. 8, the extreme fibre strain k can be expressed by the well-known Navier formula



$$k = k_0 \left[1 + \frac{(e+\delta)\, d}{2\, r^2} \right] \qquad (10)$$

and the deflection by the formula

$$\delta = \frac{e}{\pi^2 \dfrac{E\, r^2}{k_0\, l^2} - 1} \qquad (11)$$

where 'e' is either an initial eccentricity or an initial bend and k_0 the load per square inch.

This formula shows that however small the eccentricity 'e' may be, the deflection δ will increase to excessive proportions and the column will fail absolutely when the denominator approaches zero.

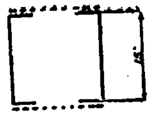
But $\pi^2 \dfrac{E\, r^2}{k_0\, l^2} - 1 = 0$

is nothing else than Euler's formula, and it is seen at once that for very small 'e,' δ, and with it the fibre strain, becomes unsafe only in case the load approaches k_0 of Euler's formula.

Assuming that the column would fail when the maximum fibre strain reaches the yield point (according to Tetmajer's tests of eccentrically loaded columns this assumption is justified); that is, making k of equation (10) equal to the yield point and introducing the value of δ from formula (11) into equation (10), then an expression for the breaking load is found by solving for k_0.

A few examples, however, will better illustrate the relation between load and strains than the investigation of such a formula.

For all examples, a column composed of 15-in. ['s, 33 lbs. will be assumed with $r=5·62$ in. and $d=15$ in. and $\pi^2E=300,000,000$.



$\frac{l}{r}$	Buckling strain for axial load.	Safe buckling strain, $16,000-70\frac{l}{r}$	Assumed eccentricity. e	Assumed load in lb. p. sq. in. k_o.	Deflection. δ	$e+\delta$	Bending strain k_c.	Maximum fibre strain k_u.
			In.		In.			
				7,600	0·06	0·16	300	7,900
				10,000	0·09	0·19	500	10,500
	$\pi^2E\left(\frac{r}{l}\right)^2$	....	0·1	15,000	0·25	0·35	1,200	16,200
				18,000	0·62	0·72	3,000	21,000
				20,000	2·5	2·6	9,500	29,500
				7,600	0·6	1·6	2,900	10,500
				10,000	0·9	1·9	4,500	14,500
			1·0	15,000	2·5	3·5	12,500	27,500
120	20,800	7,600		17,700	5·5	6·5	27,300	45,000
				7,600	2·9	7·9	14,200	21,800
				10,000	4·6	9·6	22,700	32,700
			5·0	12,000	6·8	11·8	33,500	45,500
				10,400	0·03	0·13	300	10,700
	$45,000-160\frac{l}{r}$	....	0·1	20,000	0·07	0·17	800	20,800
				30,000	0·18	0·28	2,000	32,000
				10,400	0·29	1·29	3,200	13,200
			1·0	20,000	0·74	1·74	8,200	28,200
80	32,200	10,400		28,000	1·47	2·47	16,500	44,500
				10,400	1·45	6·45	15,800	25,800
			5·0	16,500	2·7	7·7	30,000	46,500
				13,200	0·08	1·08	3,300	16,500
	$45,000-160\frac{l}{r}$	....	1·0	20,000	0·12	1·12	5,300	25,300
				30,000	0·19	1·19	8,500	38,500
				13,200	0·38	5·38	16,800	30,000
40	38,600	13,200	5·0	20,000	0·60	5·60	26,500	46,500

The loads underlined are approximately the buckling strains caused by excessive fibre strains.

Applying the foregoing to a straight column apparently centrally loaded, it is seen at once that its safety cannot be judged by merely comparing the working load (including impact, if any) with the buckling load, but that also the possibility of an eccentricity must be considered, since under unfavourable conditions the maximum fibre strains may become excessive under the working load. It is, however, not necessary to keep these strains within the same limits as allowed for tension or direct compression, but is sufficient if they remain within the yield point, since they are only accidental.

In this respect, columns differ from beams or tension members, as for these load and strain are in direct proportion so that only the one condition has to be fulfilled to keep the working strain, under the most unfavourable condition, within the yield point.

What should be considered as the most unfavourable condition as to eccentricity is a matter of judgment. But from the foregoing examples, it is evident that for columns of lengths such as used in practice there is sufficient safety against excessive accidental fibre strains when using for static loads the permissible unit strain given by the formula $16{,}000 - 70 \frac{l}{r}$, since the eccentricities which would cause excessive fibre strains under the working load are evidently greater than those likely to occur in good practice.

It must be remembered that the column with frictionless hinged ends is considered here. In practice more or less fixity of the ends counteracts the influence of a possible eccentricity; that is, the free buckling length will be reduced, unless the eccentricity is excessive, or secondary strains are likely to occur.

In good practice, the latter cases should be carefully considered and, if found of importance, special provision should be made in designing the column.

The writer has endeavoured to treat this subject merely from a practical standpoint, applying theory only so far as necessary to explain some fundamental principles, as the many elaborate theories advanced on this subject have been productive of more or less confusion.

Considering that static computations are only approximations in any case, the writer is of the opinion that our knowledge of the behaviour of compression members under strain is sufficient to enable us to design columns with as much approach to accuracy as any other member of a structure subject to bending. Additional tests on large columns, corresponding to those used in modern practice, made under the supervision of experienced experimenters, would tend to further reduce the factor of ignorance on this subject.

THE DESIGN OF LATTICING OF COLUMNS.

If a column is made up of several shapes or parts, they have to be connected in such a manner that they will act as a unit. In an ideal column each part would take its share of the load and no connection would be required. In practice, however, as stated before, bending will occur before the buckling load is reached, causing shearing strains which have to be transferred through the connections, as latticing, tie plates or cover plates. These connection parts have, therefore, to perform the same function as the web of a girder or the web system of a truss. It has also been previously explained that, due to the variety of causes producing an initial eccentricity, it is not possible to figure exactly the bending strains caused by a given load, not even at the time of breaking. And, since the shearing strains depend on the bending strains, the same uncertainty applies to these. The design of latticing, therefore, will remain largely a matter of practical judgment like the design of other details, until by means of numerous comparative tests, an empirical basis can be established.

There is, however, a rational method of dimensioning latticing analytically, which agrees well with actual examples found in existing bridges of usual dimensions.

When a column is bending the maximum fibre strain will exceed the average buckling strain, the difference being the bending strain. As a very short column theoretically $\frac{l}{r} = O$) will fail when the *average* buckling strain has reached the yield point, while a longer column whose *maximum* fibre strain has reached the yield point, will deflect rapidly and fail under a small increase of the load, it is reasonable to assume that a column will fail by buckling when the maximum fibre strain reaches the yield point; in other words, when the bending strain is equal to the difference between the yield point and the buckling strain.

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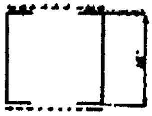

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Extremely long columns which may buckle without their fibre strain reaching the yield point (see example given for a steel spring) are not used in structural work and are, therefore, beyond the scope of this investigation.

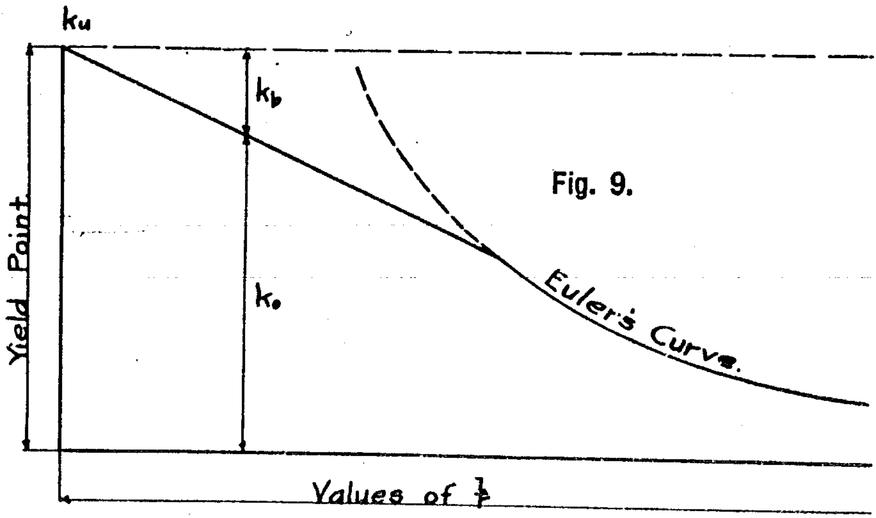


Fig. 9.

In the straight line formula $k_o = k_u - c \frac{l}{r}$, the bending strain is therefore, $k_b = c \frac{l}{r}$, represented in the diagram of buckling strains (fig. 9) by the ordinates between the buckling curve k_o and the horizontal line through the yield point k_u.

It is evident that every part of the column must be able to resist the bending corresponding to the strain k_b, as otherwise its full strength would not be developed.

Some lacing bars are in compression and others in tension. Those in compression must be treated in the same way as the column; using the same unit strain k_u, but reduced according to their $\frac{l}{r}$. Those in tension become ineffective when they stretch, as their elongation would permit a sudden increase in the deflection of the column and have, therefore, to be proportioned for the yield point in tension. A column thus proportioned has a uniform resistance against failure in all its parts, and if, instead of the respective yield points, the same permissible unit strains are used in proportioning column and latticing, a uniform safety is obtained for the column as a whole.

In order to find the shear due to the bending, the shape of the axis of the deflected column has to be assumed. As mentioned before, the elastic line of an axially loaded column is a sinus curve. If, however, the column has an initial eccentricity, the elastic line will approach a circular curve the more the greater the eccentricity compared to the resulting deflection. We will, therefore, assume the elastic line as a parabola which lies between the two curves. (Fig. 10.)

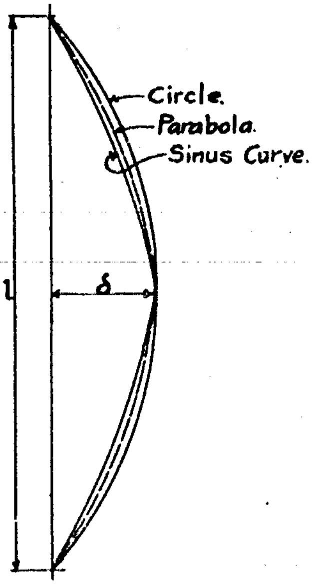


Fig. 10.

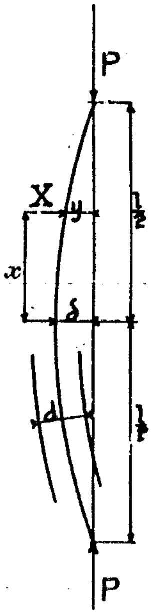


Fig. 11.

The equation of the elastic line with the notations taken from fig. 11 will then be

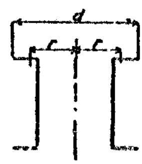
$$y = x^2 \frac{4\,\delta}{l^2} \text{ and } \frac{d\,y}{d\,x} = \frac{8\,\delta}{l^2}\,x$$

The maximum bending strain we have assumed as

$$k_b = c\,\frac{l}{r}\text{. which must be equal to} =$$

$$\frac{M \max}{R} = \frac{P\,\delta}{R}$$

where R = Moment of Resistence = $\frac{2\,ar^2}{d}$, a = area, r = radius of gyration, d = width of column.



Therefore we have

$$M \max = R\ c\,\frac{l}{r} = 2\ c\,\frac{ar}{d}\,l = P\,\delta$$

Since the bending moment at any point X is $M = Py$, the shear at the same point is

$$S = \frac{dM}{dx} = P\,\frac{dy}{dx} = \frac{8\ P\delta}{l^2}\,x$$

Substituting for $P\,\delta$ the value given above, we get

$$S = 16\ xc\,\frac{ar}{dl} \quad \ldots\ldots\ldots\ldots \quad (1)$$

and for $x = \frac{l}{2}$

$$S \max = 8\ c\,\frac{ar}{d} \quad \ldots\ldots\ldots\ldots \quad (2)$$

NOTE.—'r' is the radius of gyration laterally and d the width of the member, also laterally; that is, in the plane of the lacing. The 'a' is not the actual area used, but is the area required for the lateral radius of gyration and the corresponding l. In ordinary cases, however, the actual area can be used as 'a.'

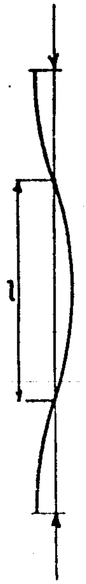


Fig. 12.

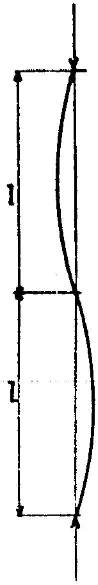


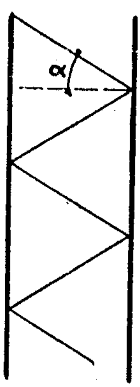
Fig. 13.

From equation (1), it follows that the shear decreases toward the middle of the column. In practice, however, the ends are always more or less fixed so that the elastic line will take the shape shown in fig 12 or fig. 13, and l will be the distance between points of contraflexure.

Since S max (according to equation (2)) is entirely independent of the length of the column, and since it occurs at the point of contraflexure, it follows, that it may occur at almost any point. The latticing should, therefore, be proportioned for the maximum shear throughout the entire length of the column.

The following gives the proportioning of various systems of column latticing:—

Fig. 14.



(1) If the column consists of two segments (fig. 14) connected by one system of single lacing bars, the shear S (under which we will now always understand S max) has to be taken up by one bar. The required area A of the bar is

$$A = \frac{S}{k} \sec a \text{ and since } S = 8\,c\,\frac{ar}{d}$$

$$A = 8\,\frac{c}{k}\,\frac{ar}{d}\sec a \quad \ldots\ldots\ldots\ldots \quad (3)$$

k being the yield point in tension for a tension bar, and $k = k_u - c\,\frac{l}{r}$ for a compression bar; $\frac{c}{k}$ being a constant for the same bar, the size of the bar is a function of the properties of the column section only, and does not depend on the column length or on any strains. We can, therefore, in any given case, without knowing the column load and the permissible unit strain, judge if the latticing be sufficient for the section of the column. Thus we follow the accepted practice of designing the connections to develop the full strength of the member. Since the strains have only relative values, the permissible unit strains of 16,000 pounds for tension and $(16{,}000 - 70\,\frac{l}{r})$ for compression will be used hereafter instead of the final values of k given above.

Fig. 15.

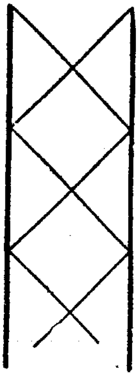
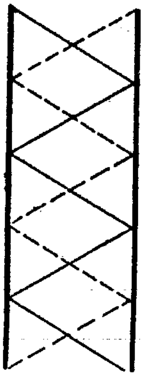


Fig. 16.



If the system is double (fig. 15), or single and on two sides of the column (fig. 16), the area required in the bar is, of course, only half of that given by formula (3).

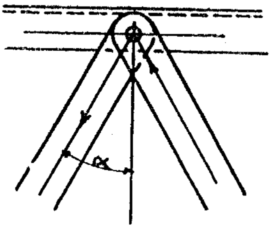
To find the number of rivets N required to connect the lattice bar, we have to remember that the allowed unit for shear is assumed ¾ of that in tension. If A_r = area of rivet, and A the net area required in the tension bar, we have

$$A = \frac{3}{4} N A_r, \text{ from which}$$

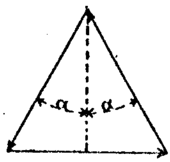
$$N = \frac{4}{3}\,\frac{A}{A_r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If the bars are connected by one rivet as in fig. 16, this rivet transmits the resultant of the two lattice strains. The strain on one bar is

Fig. 17.



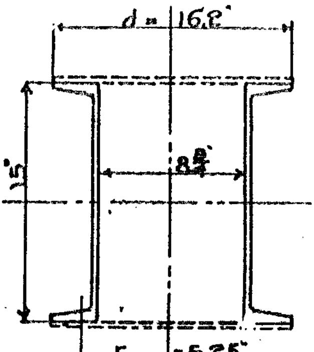
If the bars are connected by one rivet as in fig. 17, the rivet transmits the resultant of the two lattice strains.



The strain on one bar for single latticing on both sides is

$$S \frac{\sec a}{2}$$

Fig. 18.



Example:—

Column section 2 – 15-in. ['s 50 lbs. (fig. 18)

We will assume that the area 'a' required for buckling in either direction be the same: a = 29·4 sq. in. and that the column shall have single lacing of $a = 30°$ on both sides.

$$\text{For } \frac{c}{k} = \frac{70}{16{,}000} = 0{\cdot}0044$$

since sec a = 1·16, we find by formula (3) the net area of one bar.

$$A = \tfrac{1}{2} \times 8 \times 0{\cdot}0044\, \frac{29{\cdot}4 \times 5{\cdot}25}{16{\cdot}2} \times 1{\cdot}16 = 0{\cdot}195 \text{ sq. in.}$$

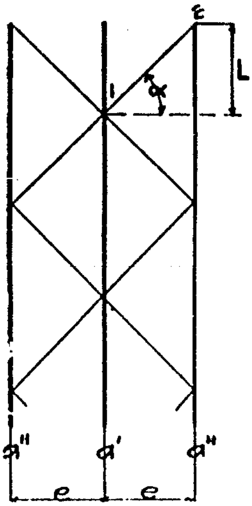
1 bar 2¼ × ¼ = 0·56 square inch will be ample.

Number of $\frac{7}{8}$-in. rivets required $= N = \frac{4}{3}\ \frac{0 \cdot 195}{0 \cdot 6} = 0 \cdot 43$

per bar use one for 2 bars as in Fig. 17.

2.—*Columns with 3 Webs:* (Fig. 19.)

Fig. 19.



Required area of column $a = a' + 2a''$, where a' and a'' are the actual areas of the ribs reduced in proportion of the total required area 'a' to the actual area.

The longitudinal shear S' between two ribs for one panel length L has to be taken up by the diagonal 1 – 2 of that panel.

The longitudinal shear per lineal inch is found from the transverse shear S by formula

$$t = \frac{SM}{I} = \frac{SM}{ar^2}$$

where M = Static Moment of the outer rib about the column axis $= a''e$, e being the distance of *centre of gravity* of the rib from the column axis.

't' of course decreases with S towards the point of maximum deflection, and S' could be found by integration for the length L. The error will, however, be small if we assume 't' constant for one panel length. We then get

$$S' = tL = \frac{SM}{ar^2}L = 8c\,\frac{ML}{dr}\left(\text{since } S = 8c\,\frac{ar}{d}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

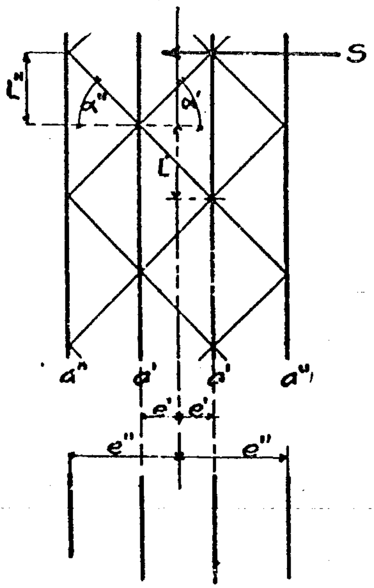
and the area of the bar required.

$$A = \frac{S'}{k}\operatorname{cosec} a = 8\,\frac{c}{k}\,\frac{ML}{dr}\operatorname{cosec} a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

or, $\frac{1}{2}$ of this if there are two sides of latticing.

3.—*Columns with 4 Webs:*

Fig. 20.



Required area of column $a = 2\ (a' + a'')$ (Fig. 20 shows a complete system of latticing for this case).

The longitudinal shear between the outer and inner rib for one panel length L'' is equal to

$$S'' = \frac{SM''}{ar^2}L'' = 8\,c\,\frac{M''L''}{dr} \quad . \quad . \quad (7)$$

where M'' = Static Moment $a''e''$.

Therefore, area of outer bar required

$$A = \frac{S''}{k}\operatorname{cosec} a'' = 8\,\frac{c}{k}\,\frac{M''L''}{dr}\operatorname{cosec} a'' \quad (8)$$

or, $\frac{1}{2}$ of this when there is latticing on two sides.

Correspondingly, we find for the latticing between the inner ribs

$$S' = 8\,c\,\frac{M'L'}{dr} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

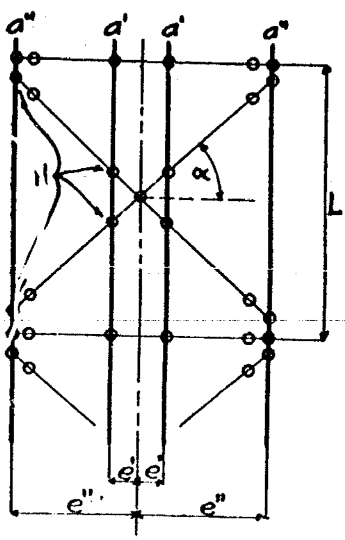
and the area of the inner bar required

$$A = \frac{S'}{k}\operatorname{cosec} a' = 8\,\frac{c}{k}\,\frac{M'L'}{dr}\operatorname{cosec} a' \quad . \quad (10)$$

where M' = Static Moment $a''e'' + a'e'$.

LATTICING OF THE LOWER CHORD (L-9) OF THE QUEBEC BRIDGE.

Fig. 21.



The top latticing as sketched in Fig. 21 (the circles indicating rivets) will first be considered.

The latticing angles between the outer and inner web practically form a complete system, only insignificant bending on the ribs being caused by the centre lines of the diagonals not meeting at the centre line of the ribs. We can apply formula (8) on page 198, to find the area which would be required in these angles, assuming hinges at points H.

Let it be assumed that the actual area has been found by the formula $16{,}000-70\frac{l}{r'}$, where r' is taken parallel to the webs. This area must be multiplied by $\dfrac{16{,}000-70\frac{l}{r'}}{16{,}000-70\frac{l}{r}}$ in order to find the area 'a' required for buckling laterally. The actual area is 781 square inches, $r=19\cdot7$ ins., $l=684$ ins., $r'=16\cdot1$

Therefore

$$a=781\frac{13{,}000}{13{,}600}=746 \text{ sq. in.}$$

and $a'=a''=\dfrac{a}{4}=186\cdot5$ sq. in.

$e'=5\cdot8$ in., $e''=27\cdot2$ in., $d=67\cdot5$ in., $L=73$ in.
$M''=a''e''=5{,}070.$

$$S''=8\times70\times\frac{5070\times73}{67\cdot5\times19\cdot7}=156{,}000 \text{ lbs.}$$

Area of one diagonal required

$$A \text{ net}=\frac{1}{4}\ \frac{156{,}000}{16{,}000}\times1\cdot4=3\cdot40 \text{ sq. in.}$$

$$A \text{ gross}=A \text{ net}\ \frac{16{,}000}{13{,}700}=3\cdot97 \text{ in. gross.}$$

Actually used:

1 angle 4 x 3 x $\frac{3}{8}$ = 2·5 sq. in. gross = 1·1 sq. in. net, as one leg of one angle is cut off at the intersection at centre. Number of $\frac{7}{8}$-in. rivets required in one angle

$$N=\frac{4}{3}\ \frac{3\cdot40}{0\cdot6}=8$$

Actual number of rivets used = 2.

Between the two inner ribs, there is no complete lattice system; the intersecting diagonals have to transmit the longitudinal shear S' of one panel length L, which can be found by formula (9).

$$M' = \frac{a}{4}(e' + e'') = 6060$$

$$S' = 8 \times 70 \frac{6060 \times 73}{67 \cdot 5 \times 19 \cdot 7} = 186{,}000 \text{ lbs.}$$

Area of one diagonal required.

$$A \text{ net} = \frac{1}{4} \; \frac{186{,}000}{16{,}000} \times 1 \cdot 4 = 4 \cdot 07 \text{ sq. in.}$$

Actual effective area as above $= 1 \cdot 1$ sq. in.

Besides there are secondary strains in the lattice angles owing to their continuity, the riveted end connections and the presence of the lateral struts.

The rivets at the inner rib have to transmit the shear

$$S' - S'' = 30{,}000$$

Their number should be

$$N = \frac{1}{2} \; \frac{4}{3} \; \frac{30{,}000}{16{,}000} \; \frac{1}{0 \cdot 6} = 2 \text{ rivets, } \tfrac{7}{8}\text{-in. diameter.}$$

Actually used, 2 rivets $\frac{7}{8}$-in. diameter.

In consideration of these results, the lattice diagonals and their connections are decidedly too weak. It is evident that even under conservative loads certain parts must have been overstrained.

The bottom lacing is somewhat better. The tie plate at the intersection of the angles takes the longitudinal shear and is connected by 4 rivets to *each* rib.

APPENDIX D.

SECONDARY STRAINS IN TRUSSES OF QUEBEC BRIDGE.

In figuring the primary or direct strains in a truss, the truss members are assumed connected to each other by frictionless hinges. This condition is never realized; the members being either riveted and, therefore, unable to turn at their ends, or hinged, which, on account of friction, will permit only a partial turning.

When the truss deflects under the load, the angles between members tend to change. This change, however, cannot take place without bending the members at their ends, which produces bending strains in addition to the direct strains.

These bending strains are called secondary strains. On account of the labour involved in computing these strains, as they can only be determined after the trusses have been designed for the primary strains, they are considered only in rare cases; but provision for them is generally made in the adopted margin of safety.

It would be of no value to compute the secondary strains in every case, since they amount to about the same percentage of the primary strains for trusses of the same type and ordinary spans. They should, however, be carefully considered in unusual designs and in members of unusual proportions.

The secondary strains will depend largely upon the methods of manufacture and erection. In designing, the most unfavourable conditions should be considered; using, however, for the combined strains higher permissible unit strains, which may be the higher the greater the ratio of the secondary strain to the direct strain.

In order to get the maximum secondary strains for all members, different cases of loading should be considered; but generally one case, for instance, that of a total load, will suffice to show their possible magnitude. The secondary strains are the greater the deeper the member, since a bending of the ends has less effect on a slender bar than on a wider member.

As it would lead too far to give here a general theory of secondary strains, only the method followed in computing these strains in the lower chord of the Quebec bridge will be shown.

GENERAL THEORY.

The lower chord of the truss is continuous over the entire length of anchor and cantilever arms; while all the other members are pin-connected. For the present, the friction of the pins will be neglected; all the members connected to the lower chord will, therefore, be considered as turning freely at their ends and receiving no secondary strains under any load. If the lower chord sections, like the other members, were free to turn at their ends, the original angles ζ_1, ζ_2, ζ_3 (see Fig. 1) between two adjacent sections would change under a given load to $\zeta_1 + \Delta\zeta_1$, $\zeta_2 + \Delta\zeta_2$, $\zeta_3 + \Delta\zeta_3$. The change

$\Delta\zeta_2$ at panel point L_2 is equal to the sum of the changes Δa_1, Δa_2, Δa_3, and Δa_4 of the angles a_1, a_2, a_3, a_4.

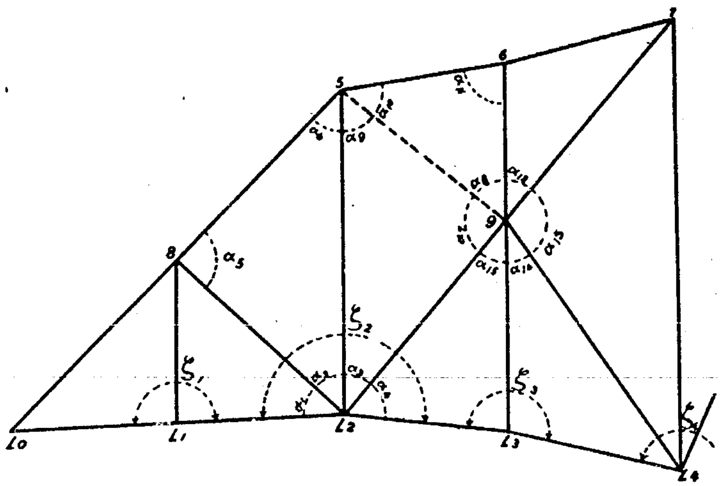


Fig. 1.

The changes Δa in any triangle of the truss, for instance, those in $2-5-8$, are given by the following three equations:—

$$\left.\begin{aligned} E\,\Delta a_3 &= (S_{8-5}-S_{8-2})\text{ ctg. } a_5+(S_{8-5}-S_{5-2})\text{ ctg. } a_4 \\ E\,\Delta a_5 &= (S_{5-2}-S_{8-5})\text{ ctg. } a_4+(S_{5-2}-S_{8-2})\text{ ctg. } a_3 \\ E\,\Delta a_4 &= (S_{8-2}-S_{5-2})\text{ ctg. } a_3+(S_{8-2}-S_{8-5})\text{ ctg. } a_5 \end{aligned}\right\} \quad \ldots\ldots \quad (1)$$

in which S_{8-5}, S_{8-2} and S_{5-2} are the direct unit strains from the given load in the members 8-5, 8-2 and 5-2 forming the triangle, and E = Modulus of Elasticity. The change Δa_2 in the trapezoid $2-9-6-5$ is obtained as follows:—

Let the trapezoid be divided into two triangles by a diagonal $5-9$ and apply to these triangles the above equations as follows:—

$$E\,\Delta a_7 = (S_{2-5}-S_{2-9})\text{ ctg. } a_8 + (S_{2-5}-S_{5-9})\text{ ctg. } a_9$$
$$E\,\Delta a_8 = (S_{5-6}-S_{6-9})\text{ ctg. } a_{11} + (S_{5-6}-S_{5-9})\text{ ctg. } a_{13}$$

from which the imaginary strain in the assumed diagonal is found:

$$S_{5-9} = \frac{(S_{2-5}-S_{2-9})\text{ ctg. } a_8 + (S_{5-6}-S_{6-9})\text{ ctg. } a_{11} + S_{2-5}\text{ ctg. } a_9 + S_{5-6}\text{ ctg. } a_{13} - E\,(\Delta a_7+\Delta a_8)}{\text{ctg. } a_9 + \text{ctg. } a_{13}} \quad (2)$$

wherein

$$E\,(\Delta a_7+\Delta a_8) = -E\,(\Delta a_{12}+\Delta a_{13}+\Delta a_{14}+\Delta a_{15})$$

This enables use to determine Δa_2 from the triangle $2-5-9$. In this way will be determined all the changes $\Delta\zeta$ which would take place under a given load, assuming that the lower chord sections were free to turn at their ends. Owing to the continuity of the chord, these changes in the angles ζ cannot take place without bending the chord. In other words, the forces P at the ends of each bottom chord section are no longer acting axially, but produce bending strains. (See fig. 2.)

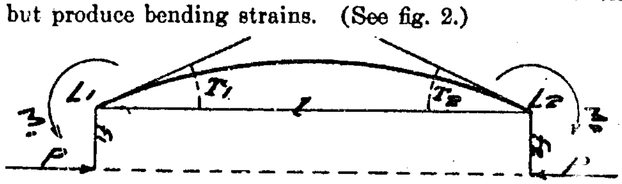


Fig. 2.

These bending strains can be obtained from the bending moments at the ends of the member $M_1 = Pf_1$ and $M_2 = Pf_2$. Between the end moments M_1 and M_2 and the angles T_1 and T_2 which the end tangents form with the original axis, l, the following relations exist:

$$\left.\begin{aligned} T_1 &= \frac{(2M_1 + M_2)l}{6IE} \\ T_2 &= \frac{(2M_2 + M_1)l}{6IE} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (3)$$

These formulae are obtained by integration of the differential equation of the elastic line.

$$\frac{d^2y}{dx^2} = \pm \frac{M}{IE}$$

Two adjacent lower chord members $L_1 - L_2$ and $L_2 - L_3$ (fig. 3) will now be considered. In order to have equilibrium, the two moments M_2^L and M_2^R at the panel point L_2 must be equal $= M_2$. The sum of the angles T_2^L and T_2^R must be equal to the deformation $\Delta \zeta_2$ of the angle ζ_2.

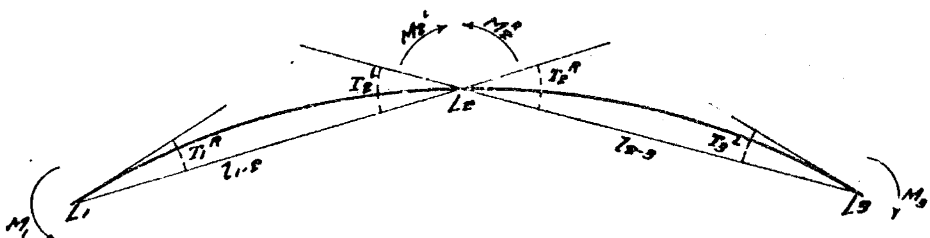


Fig. 3

$$T_2^L + T_2^R = \Delta \zeta_2 \quad \ldots\ldots\ldots\ldots \quad (4)$$

By substituting for T_2^L and T_2^R the values (3) it follows:—

$$\frac{(2M_2 + M_1)l_{1\text{-}2}}{6\, I_{1\text{-}2}\, E} + \frac{(2M_2 + M_3)l_{2\text{-}3}}{6\, I_{2\text{-}3}\, E} = \Delta \zeta_2$$

$$\text{or, } M_1 \frac{l_{1\text{-}2}}{I_{1\text{-}2}} + 2M_2 \left(\frac{l_{1\text{-}2}}{I_{1\text{-}2}} + \frac{l_{2\text{-}3}}{I_{2\text{-}3}} \right) + M_3 \frac{l_{2\text{-}3}}{I_{2\text{-}3}} = 6\, E\, \Delta \zeta_2 \quad \ldots\ldots \quad (5)$$

Each panel point of the lower chord furnishes one equation of this kind; as many equations as there are unknown bending moments are obtained, and these moments can thus be determined. From the moments M the secondary strains in the member are found by the usual formula

$$S = \frac{Me}{I} \quad \ldots\ldots\ldots\ldots \quad (6)$$

wherein e = distance of extreme fibre from the neutral axis.

On account of the continuity of the lower chord, its own weight produces bending moments at the panel points, which cause bending strains in addition to the other secondary strains. If the lower chord section $L_1 - L_2$ were free to turn, it would, under its own weight W_{1-2}, deflect like a uniformly loaded beam on two supports; the bending moment at the centre would be

$$M = \frac{W_{1-2}\ d_{1-2}}{8} \qquad (7)$$

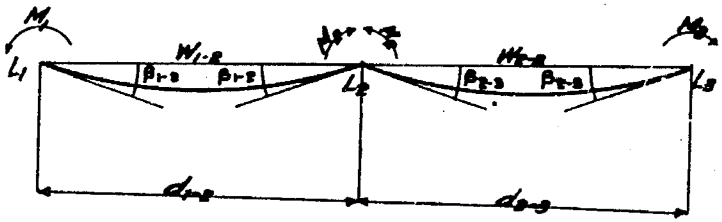


Fig. 4.

and the angles β_{1-2} which the end tangents of the elastic line form with the original axis

$$\beta_{1-2} = \frac{W_{1-2}\ d_{1-2}^2}{24\ E\ I} \qquad (8)$$

The angle between two adjacent lower chord sections $L_1 - L_2$ and $L_2 - L_3$ would increase by the amount

$$\Delta\,\zeta_2 = \beta_{1-2} + \beta_{2-3} \qquad (9)$$

Owing the the continuity of the chord, this increase cannot take place; therefore, bending moments will occur at each panel point. These bending moments have to correspond to equations (5) in which the values (9) have to be substituted for $\Delta\,\zeta$.

For the computation of the secondary strains the following cases of loading have been considered:—

1. Full dead load.
2. A load of 3,000 lbs. per lin. ft. on one truss of the cantilever arm and suspended span.
3. A load of 3,000 lbs. per lin. ft. on one truss of the anchor arm.
4. Own weight of lower chord.

The corresponding strains are given in the attached table, together with the greatest combined strains.

Under the following conditions, the secondary strains in the lower chord from dead load could practically be eliminated in the finished structure:—

1. If during erection the ends of the lower chord members were able to turn freely about the joints.
2. If after the full dead load is on the bridge, the joints would come to uniform bearing.

Both these conditions can only be partly fulfilled. Even if the lower chords were pin-connected, and the splices were not riveted up until completion of erection, friction would partly prevent turning; and it is almost an impossibility for the shop to work so accurately as to fulfil the second condition, especially for a polygonal chord like that of the Quebec bridge.

If, for instance, a butt-joint has an even bearing at the beginning of the erection, the strains would be uniformly distributed over the entire section at that time, but as soon as deformation commences, the strains will be transmitted eccentrically, causing secondary strains which may be as high as if there were no joint at all.

As it is impossible to determine the exact condition under which the joints of the lower chords come to an even bearing, it is equally impossible to ascertain what percentage of the computed secondary strains would come on any one of the chord members.

As the maximum bending moments occur at the panel points, the additional section provided for buckling may help to resist the secondary strains at the panel points where no buckling will take place.

No matter for what condition of loading the length of the members of the trusses may be adjusted to give the joints of the lower chord an even bearing, secondary strains will occur, and it is reasonable to assume that at least those produced by the live load will occur in any case. These range from 3 to 20 per cent of the total direct strains.

The total secondary strains may, therefore, range from the values S_c in the table to the values $S_c + S_d + S_w$, since S_a (from live load of anchor arm) is always of opposite sign to S_d.

The greatest secondary strain occurs in member $L_6 - L_7$ of the anchor arm, where it is between 4,600 and 22,400 lbs. per square inch.

The secondary strains in the lower chord are from 18 to 95 per cent of the corresponding direct strains; this percentage is smallest at the ends of cantilever and anchor arms, and increases towards the pier.

In figuring the secondary strains, the pins have been assumed frictionless. A calculation has shown that the strains caused in the lower chord by friction of the pins are negligible; being less than 1 per cent of the secondary strains where the latter reach the maximum.

The effect of friction of the pins is considerably greater on the eyebars of the upper chord. Approximate computations show that the secondary strains in the eyebars for assumed rigid end connections, would be from 30 to 40 per cent of the direct strains. Since for a coefficient of friction of 0·15, the strains caused by this assumed friction amount to about the same as for rigid end connections, it follows that the ends are prevented from turning under any load and the secondary strains can, therefore, amount to the above given percentages of the direct strain.

It is probable, however, that during erection as well as afterwards, through vibrations from moving loads, the eyebars gradually turn on the pins, thus eliminating partly the secondary strains from the dead load.

The most favourable condition which could be assumed is, that the secondary strains are produced by the live load only. The live load strains in the upper chord bars are from 25 to 30 per cent of the total strains; hence, if the secondary strains are 40 per cent of the primary strains produced by the live load, they will amount to at least from 10 to 12 per cent of the total direct strains.

Secondary Strains on Lower Chord Members of Quebec Bridge

Notation:-

S_d - Unit Strain on extreme fibre from Dead Load.
S_c - " " " " " a Live Load of 3000 lbs. per lin. ft. per truss on Cantilever Arm and Suspended Span.
S_a - " " " " " " " " " " " " " " Anchor Arm.
S_w - " " " " " own weight of member.
\+ denotes tension. The upper sign indicates the strain in the upper fibre.
\- " compression. " lower " " " " " " lower "
All strains given in units of 1000 lbs.

Anchor Arm

	L_0-L_1		L_1-L_2		L_2-L_3		L_3-L_4		L_4-L_5	
	at L_0	at L_1	at L_1	at L_2	at L_2	at L_3	at L_3	at L_4	at L_4	at L_5
S_d	0	± 2.4	± 2.4	∓ 3.5	∓ 2.5	± 4.7	± 4.7	∓ 9.1	∓ 7.2	± 12.2
S_c	0	± 2.5	± 2.5	∓ 2.6	∓ 1.6	± 2.9	± 2.9	∓ 3.6	∓ 2.6	± 4.5
S_a	0	∓ 1.7	∓ 1.7	± 1.9	± 1.4	∓ 1.8	∓ 1.6	± 1.5	± 1.0	∓ 0.9
S_w	0	± 1.1	± 1.1	± 1.1	± 0.6	± 1.2	± 1.2	± 1.5	± 1.0	± 1.2
$S_d+S_c+S_w$	0	± 5.6	± 5.6	∓ 5.0	∓ 3.6	± 8.6	± 8.6	∓ 11.4	∓ 9.0	± 17.7

	L_5-L_6		L_6-L_7		L_7-L_8		L_8-L_9		L_9-L_{10}	
	at L_5	at L_6	at L_6	at L_7	at L_7	at L_8	at L_8	at L_9	at L_9	at L_{10}
S_d	± 12.1	∓ 13.5	∓ 13.2	± 16.6	± 16.0	∓ 14.4	∓ 14.1	± 19.6	± 12.8	∓ 7.9
S_c	± 4.5	∓ 3.1	∓ 3.0	± 4.6	± 4.4	∓ 4.0	∓ 4.0	± 4.6	± 4.5	∓ 4.1
S_a	∓ 0.9	∓ 0.4	∓ 0.4	∓ 0.2	∓ 0.2	± 0.2	± 0.2	∓ 0.5	∓ 0.6	± 1.1
S_w	± 1.2	± 1.2	± 1.2	± 1.2	± 1.2	± 1.2	± 1.2	± 1.2	± 1.1	± 1.4
$S_d+S_c+S_w$	± 17.6	∓ 15.4	∓ 15.0	± 22.4	± 21.6	∓ 17.3	∓ 15.9	± 19.4	± 16.2	∓ 10.6

Cantilever Arm.

	L_{10}-L_9		L_9-L_8		L_8-L_7		L_7-L_6		L_6-L_5	
	at L_{10}	at L_9	at L_9	at L_8	at L_8	at L_7	at L_7	at L_6	at L_6	at L_5
S_d	∓ 7.9	± 9.4	± 10.2	∓ 10.0	∓ 10.0	± 10.5	± 11.1	∓ 9.6	∓ 9.6	± 9.0
S_c	∓ 4.1	± 3.1	± 3.4	∓ 2.5	∓ 2.5	± 3.0	± 3.2	∓ 2.5	∓ 2.5	± 2.6
S_a	± 1.1	∓ 0.5	∓ 0.5	± 0.1	± 0.1	∓ 0.0	0	0	0	
S_w	± 1.4	± 1.6	± 1.6	± 1.5	± 1.5	± 1.5	± 1.6	± 1.5	± 1.5	± 1.5
$S_d+S_c+S_w$	∓ 10.6	± 14.0	± 15.8	∓ 11.1	∓ 11.1	± 15.1	± 16.9	∓ 19.6	∓ 10.6	± 13.1

	L_5-L_4		L_4-L_3		L_3-L_2		L_2-L_1		L_1-L_0	
	at L_5	at L_4	at L_4	at L_3	at L_3	at L_2	at L_2	at L_1	at L_1	at L_0
S_d	± 9.2	∓ 5.6	∓ 6.6	± 3.9	± 3.9	∓ 2.5	∓ 2.9	± 1.9	± 1.9	0
S_c	± 2.6	∓ 1.5	∓ 1.7	± 1.0	± 1.0	∓ 0.6	∓ 0.7	± 0.6	± 0.6	0
S_a										
S_w	± 1.6	± 1.3	± 1.5	± 1.5	± 1.5	± 1.2	± 1.4	± 1.5	± 1.5	0
$S_d+S_c+S_w$	± 13.4	∓ 6.0	∓ 6.8	± 6.4	± 6.4	∓ 1.9	∓ 2.8	± 4.0	± 4.0	0