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An Examination of Public Capital's Role in Production

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Table of contents

Abstract

This paper examines whether or not the long-term government bond rate could reasonably be employed as the rate of return on public capital when calculating public sector gross domestic product. It finds that the rate of return on public capital is lower than often reported and is roughly consistent with the rate of return on private capital. Given that there is a range of estimates that are plausible, the paper concludes that the long-run government bond rate could be used as a conservative estimate for the rate of return for public infrastructure.

Previous studies have shown that production function estimates tend to find rates of return that are implausibly large, while cost function estimates appear more reasonable. This paper shows that public capital and multifactor productivity (MFP) growth behave similarly, and argues that production function estimates for the impact of public capital overstate its impact as a result, catching part of what belongs in estimates of MFP. It also shows that the similarity between the growth in public capital and MFP leads to a large confidence interval around public capital elasticity estimates derived from the production function framework. The paper then proceeds by generating a confidence interval from the production function estimated first with and then without MFP growth. It then uses a cost function to pinpoint more precisely estimates for the marginal cost savings from public capital. Importantly, the estimate derived from the cost function is found in the lower part of the confidence interval derived from the production function. The rate of return associated with the overlapping estimates is then shown to cover a range that extends from the average long-run government bond rate to the rate of return on private capital.

Keywords: infrastructure, rate of return, outlier

Executive summary

Public capital provides the foundation for the Canadian economy. The roads, water and sewer systems that make up the majority of public capital allow for lower transportation costs and greater concentrations of people and firms, promote agglomeration economies and provide access to broader, deeper markets.

Despite the contribution that public capital makes to the economy, it has proven difficult to generate a robust estimate of the rate of return that investment in public capital produces. In Canada, public capital provision is primarily funded through taxation. It does not have a private sector equivalent that could be used as a proxy for its rate of return, nor does it have commercial markets for its output. As a result, it is necessary to econometrically estimate the rate of return.

Econometric techniques, however, have not led to a consensus about what a reasonable rate of return may be. Depending on the method used, estimates as high as 50% and as low as 0% have been generated.

The uncertainty around the estimates has proven sufficiently large that no economic rate of return is attached to public capital when public sector gross domestic product (GDP) is estimated. Only the depreciation of public capital enters the calculation for GDP in the public sector.

This paper investigates issues surrounding the uncertainty arising from rate of return estimates. It uses a variety of econometric techniques, and it pays special attention to time series issues in estimation. Through the course of the paper, a number of questions surrounding the uncertainty are addressed.

- Why is it so difficult to estimate the rate of return?

  The growth in public capital and multifactor productivity (MFP) are very similar. As a result, including MFP and public capital in a regression leads to multi-colinearity. This is a data problem that hinders accurate estimation of the rate of return. Estimates of public capital's impact are either captured by MFP leading to no impact, or capture elements of MFP inflating public capital's impact. This is especially true of aggregate production functions.

- Which approach is preferable: a cost or a production function?

  Cost function estimates tend to suggest that the impact of public capital is positive and lower in magnitude than production function estimates. They are generally viewed as more credible. This paper assumes that both approaches contain some useful information. It uses cost and production function estimates to 'triangulate' on what a reasonable impact from public capital could be. The 'triangulation' suggests that an elasticity between 0.10 and 0.15 and a rate of return centered on 17% is appropriate.

- Which estimation approach is preferable?

A variety of estimation techniques are employed to verify the robustness of the estimates. Time series analysis, for elements such as unit roots, is employed in order to avoid spurious results. As well, because the analysis uses panel data sets the estimation procedure controls for unit specific fixed effects. Outside of these considerations, the analysis is robust to changes in estimation strategy.

- What is a reasonable rate of return for public capital?

The paper shows that, while it is difficult to place an exact number on the rate of return from public capital, it is larger than zero. Moreover, it also shows that high rates of return presented in the literature likely result from the elasticity estimate capturing elements of MFP growth. More precision requires data sets that allow for more variation in the underlying series. This paper does so by moving to provincial cost data. The 'triangulated' range supports both of these arguments. Rates of return are produced with a mean 17%, but they continue to cover a relatively wide range, from near 5% to 29%, Nevertheless, they support the contention that the average long-term government bond rate can be used as a conservative estimate for public capital's rate of return.

1 Introduction

The public capital invested in roads, air navigation, canals, and water and sewage systems expands the productive capacity of an economy. Public capital enables greater geographic concentrations of economic resources and facilitates the movement of goods and people. It allows for broader market access and a greater range of choice for employers and employees. It affects input and output markets, helps determine spatial development patterns and provides a large network at low cost to individual users. Public capital is, in short, the foundation upon which the economy is built.

A body of literature based on these ideas suggests that public capital plays an important, often overlooked, role in private production (see, for example, Aschaeur 1989; Munnel 1990a, 1990b; Shah 1992; Berndt and Hanson 1992; Lynde and Richmond 1992; Nadiri and Mamuneas 1994; Conrad and Seitz 1994; Morrison and Schwartz 1996; Fernald 1999; Pereira 2000; and, Ramirez 2004).

These studies employ production and cost functions to estimate the elasticity of output with respect to public capital, or the marginal savings to the private sector of an extra unit of public capital, respectively. They argue that the impact of capital is noteworthy, that the impact is different over time and that changes in public investment in infrastructure have a differential impact across industries.

These arguments are examined using Canadian data in Harchaoui (1997), Harchaoui and Tarkhani (2003) and Brox and Fader (2005). All three papers employ cost functions for estimation. Their models assume that the real level of public capital enters the cost function as an unpaid factor of production. Firms minimize costs over private capital and labour, which constitute the variable cost function for the firm, but take public capital as given in the total cost function. Changes in public capital are assumed to change the height of the variable cost curve. If public capital is cost saving, then there will be a negative elasticity of total cost with respect to public capital. This means that an increase in the level of public capital reduces the total cost of private production.

Harchaoui (1997) uses a trans-log cost function and a panel of Canadian industry data for the 1961-to-1997 period. The author finds that the impact of public capital is significant, accounting for about 12% of overall business sector productivity growth. Harchaoui and Tarkhani (2003) re-examine the relationship using an expanded panel of Canadian industries from 1961 to 2000. They report that on average an increase in public capital reduces production costs in the private sector. Brox and Fader (2005) perform a similar exercise, arguing that public capital is an important input for firms. Taken together, the cost function studies imply that firms use public capital, and that its provision can affect cost structures.

Despite the general agreement that public capital enters a private production or cost function, there is little agreement on what a reasonable rate of return from public investment is. And there has been little discussion of how robust the estimates are to alternate formulations and methods used to deal with particular econometric problems.

The studies present a diverse set of rate of return estimates based on differing aggregations of economic time series, estimation techniques, sample spans and modelling approaches. The range of estimates makes it difficult to ascertain which rate of return is most plausible, or which method is relatively robust.

Investigating the robustness of econometric estimates, and their implied rates of return, is important because estimates of the latter are needed in order to guide those interested in evaluating the need for more infrastructure or those National Accountants who are trying to incorporate public infrastructure into the National Accounts. Both require estimates of the rate of return on public capital. But these estimates are not readily available—outside of econometric exercises.

National Accountants need an estimate of the value of public capital that is incorporated into public output. But markets for public sector products are rarely available and therefore the expenditure of final products and the sum of value added approaches for calculating public sector gross domestic product (GDP) in the National Accounts are not available to National Accountants. Instead, public sector GDP is estimated using factor payments

Using this approach, it is possible to calculate labour remuneration from payroll information; however, without a robust estimate for public capital's rate of return it is difficult to calculate the remuneration to publicly financed capital that should be added into public sector output. Because research to date has not reached a consensus about what rate of return is earned from public capital, the System of National Accounts (SNA) has assumed that the return from public capital is only equal to its depreciation rate. There is no economic return from public capital in current public sector GDP estimates. Robust public capital rate of return estimates are, therefore, necessary if an economic return from government assets is to be included in public sector GDP.

Estimating a robust rate of return is more difficult than it appears. Although the public sector owns buildings and machinery and equipment, the majority of public capital consists of roads, bridges, and water and sewer systems (Baldwin and Dixon 2008). These assets have no market price, and, in most cases, lack close parallels with privately owned capital goods in Canada. Because of this lack of information, it is not possible to directly calculate the gross return from public capital, nor is it possible to use the returns from privately owned assets as proxies in Canada. The lack of substitutes forces economists to rely on econometric techniques to infer the return from public capital, fuelling the debate about what a reasonable rate of return is by offering multiple methods for estimation and widely different answers.

This paper's primary purpose is to examine how robust the estimates of the impact of public capital are to alternative estimation methods and to ask what the range of rates of return to public capital is. It uses a variety of estimators and explicitly examines the time series properties of the data. Using a simple functional form, the relationship between real output and public capital, and the relationship between per unit costs and public capital, is estimated and used to infer public capital's rate of return. The paper then attempts to see whether information from the different methods can be combined in such a way as to 'triangulate' on a preferable estimate of public capital's rate of return.

The study is organized as follows. Section 2 examines how aggregate real GDP and public capital covary over time and shows the difficulty in disentangling the effects of public capital and growth in multifactor productivity. Section 3 describes the panel data sets employed to estimate the elasticity of public capital. Section 4 examines production function estimates of the elasticity of public capital, while section 5 examines cost function estimates for the elasticity of cost savings. Section 6 concludes the paper.

2 Aggregate gross domestic product and public capital

The relationship between real gross domestic product (GDP) and public capital is complex because public capital is an enabling resource. Unlike most types of private capital, if public capital were to be removed from the economy, it would rapidly collapse.[1] Moreover, public capital acts as a network connecting geographically separated economic agents. As a result, public capital's economic contribution is the full set of interactions that the network enables. It may, therefore, be difficult to accurately capture public capital's contribution to private sector value added.

Due to the complexity of the relationship, it is useful to start the examination of the relationship between public capital and real GDP using aggregate data before attempting to use sophisticated estimation techniques. Because public capital forms part of the foundation of the economy, one interpretation of how public capital affects real GDP is that public capital helps to define its trend.

This interpretation stems from how real GDP and the real stock of public capital co-vary over time (Figure 1). For most of the 1961-to-2005 period, real GDP and public infrastructure track each other closely. The only significant deviations occur during the recessions of the early 1980s and early 1990s.

In the absence of inputs, estimates of trend GDP have been interpreted as multifactor productivity (MFP), which is a proxy variable for intangible or difficult to measure production inputs. Since public capital is similar to trend GDP, Figure 1 implies that disentangling MFP and the marginal impact of public capital may be difficult.

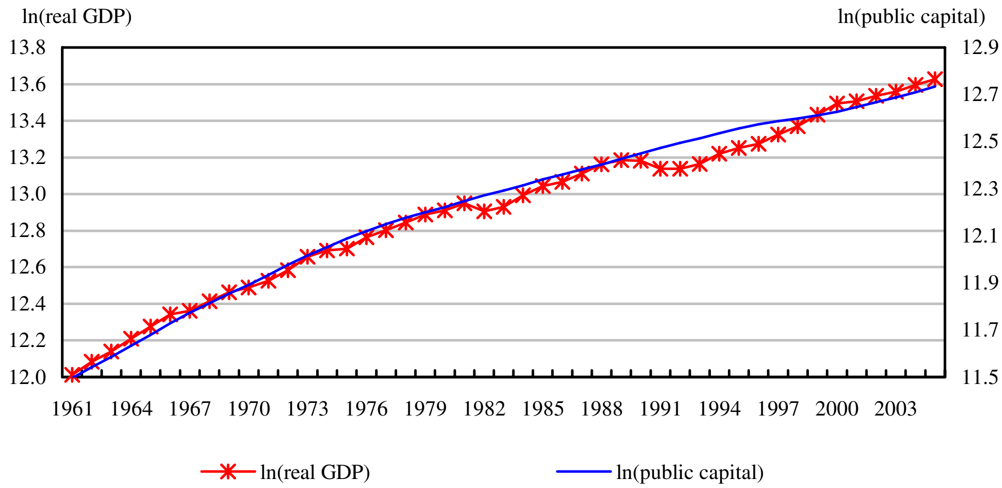
A simple experiment helps to illustrate this. Suppose, for the moment, that the following models are specified:

Model 1: $\ln(GDP) = \alpha + \delta t + \gamma t^2 + e$;
Model 2: $\ln(GDP) = \alpha + \beta \ln(\text{public capital}) + e$.

1. The impact will be similar if any type of infrastructure is removed, including private sector infrastructure such as telecommunications or power networks.

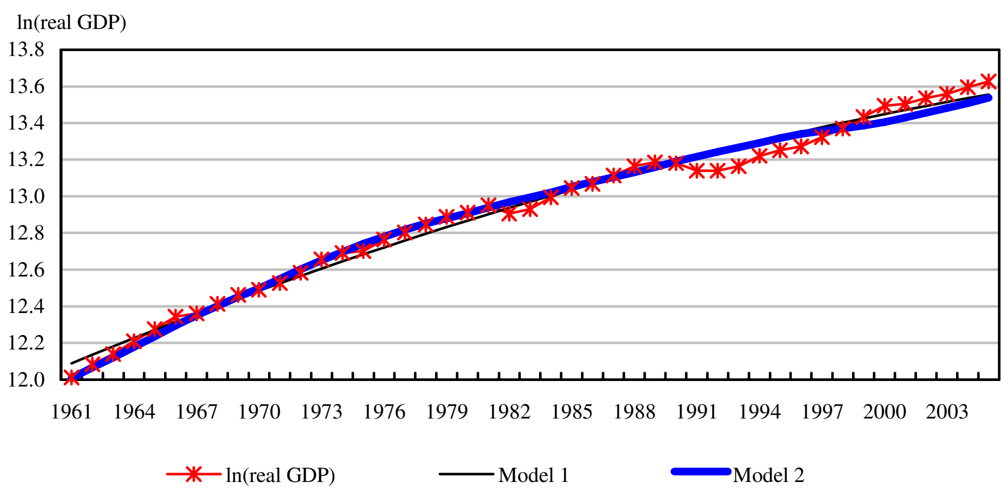
Model 1 supposes that the trend in log-level of GDP can be approximated by a quadratic trend while Model 2 assumes trend GDP can be modelled using public capital. If public capital and the commonly used time trend for MFP are capturing a similar feature of real GDP growth, then it is expected that the fitted values of the two models should be approximately the same. Moreover, the fitted values of the two models should mimic trend GDP growth.

Figure 1
Public capital versus real gross domestic product (GDP)



Source: Statistics Canada.

Figure 2
Trend estimates versus real gross domestic product (GDP)



Source: Statistics Canada.

When the fitted values are plotted against the log-level of real GDP, the hypothesized relationship emerges (Figure 2). Model 1 and Model 2 provide fitted values that closely resemble each other. Both models also track trend GDP well. The trend implied by public capital appears to better represent trend GDP during the first half of the period, while both models have difficulty during recessions and after the late 1990s when GDP overshoots the fitted values. Nevertheless, preliminary analysis supports the assertion that it will be difficult to separate trend changes in GDP from the contribution of public capital.

Of course, MFP is calculated as the difference between GDP growth and a weighted average of labour and capital input growth. Nevertheless, the same point applies. The growth in public capital is so closely related to the growth in overall GDP that it will be difficult to statistically separate its impact from other inputs that move in the same smooth fashion as does public capital.

3 Data sets

Because trend gross domestic product (GDP) and the real stock of public capital appear similar, it will be difficult to ascertain the elasticity of public capital when multifactor productivity (MFP) and public capital are placed in the same model. In an effort to improve parameter estimates, panel datasets that potentially provide more variation are employed. A provincial panel is used to estimate a production function so as to provide more degrees of freedom and more variability between the growth in GDP and the growth in public capital than can be derived from estimates for the economy as a whole. Subsequently, an industry panel is employed to estimate the cost savings associated with an additional unit of public capital.

Provincial panel data

The provincial panel spans 1981 to 2005. This covers the period over which trend GDP growth is approximately linear. The panel comprises business sector real GDP, capital stock, hours worked and public capital stock by province. The real GDP estimates are formed by removing the estimates of government expenditure and the value added from owner-occupied dwellings from provincial real GDP estimates.

The hours worked estimates are taken from the Canadian Productivity Accounts (CPA) estimates of provincial hours worked. The estimates are consistent with the hours worked estimate for Canada currently produced for the CPA. Estimates of public and private capital stocks are taken from the Investment and Capital Stock Division of Statistics Canada.

Industry panel data

The industry panel data variables are taken from the Input–Output tables developed by the Industry Division of Statistics Canada and from the Capital, Labour, Energy, Materials and Services (KLEMS) dataset produced by the Micro-economic Analysis Division of Statistics Canada as part of the Productivity Accounts. Estimates of business sector nominal and real GDP, the GDP deflator and labour cost are taken from the KLEMS dataset (for more information, see

Baldwin and Gu 2007; Gellatly, Tanguay and Yan 2003; Harchaoui and Tarkhani 2003; and, Gu et al. 2003).

The user cost of capital is estimated as $p_{i,k,t} = q_{i,k,t-1}\left[\left(r_t + \delta_{k,t} - \pi_{i,k,t}\right)\times\tau_{i,k,t} + \phi_{i,t}\right]$ where $q_{i,k,t-1}$ is the past price of the capital good, r_t is the nominal rate of return paid for the asset, $\delta_{k,t}$ is the asset specific depreciation rate, $\pi_{i,k,t}$ is the asset price change from $t-1$ to t, $\tau_{i,k,t}$ is the effective tax on capital income and $\phi_{k,t}$ captures property taxes.

While estimates of the user cost of capital are available in the KLEMS dataset, they are derived using an endogenous rate of return estimate. This may not be an appropriate measure for estimating a cost function because the internal rate of return adjusts to exhaust the economic surplus making the user cost of capital volatile. The internal rates of return respond quickly and, in some cases, strongly when industries experience demand and supply shocks. In these instances the change in the cost of capital is due to short-run extraordinary gains or losses that can be quickly unwound. The cost of capital estimates can become excessively volatile and lead to problems with econometric routines.

The short-term volatility induced in the endogenous rate of return can create an errors–in-variables problem if the true cost of capital does not adjust as quickly. One solution for errors-in-variables problems is the use of instrumental variables that are uncorrelated with the short-run shock but highly correlated with the true rate of return. In this paper, the five-year moving average of the long-term government bond rate is used (See Baldwin and Gu 2007 for a discussion of these issues).

Estimates of nominal transportation costs for the North American Industry Classification System (NAICS) industry classification are taken from the L-level input–output tables. As will be explained later, these estimates are used to capture the cost of using public infrastructure, and ultimately define an upper bound for the impact of public capital.

The KLEMS dataset provides estimates of economic aggregates for a range of NAICS industries at the L-level aggregation. Not all of the industries are employed here. Education, Health and Public Administration are removed from the dataset because they are not private-sector industries.

4 The production function approach

The production function approach was the first widely employed method for examining the impact of public capital in private production. It extends back at least as far as Meade (1952) who develops production function specifications that include public capital. More recently, Arrow and Kurz (1970) and Grossman and Lucas (1974) argue that the provision of public capital should be included in private-sector production functions. Despite these arguments, the impact of public capital did not receive widespread attention until empirically analysed by Aschauer (1989).

Aschauer (1989) uses production function estimates to ignite a debate about the role of public capital in private production, and its role in the productivity slowdown in the United States during the 1970s. Additional estimates are provided for the United States by Holtz-Eakin (1988) and Munnell (1990a, 1990b) that support the notion that output is responsive to the level of public capital.

For Canada, Wylie (1996) adopts the approach taken by Aschauer (1989) to estimate the elasticity of public capital in Canada. Using a trans-log production function, and Canadian aggregate data from 1946 to 1991, he finds that government capital has a positive elasticity. He concludes by arguing that his results support the finding for the United States that public capital plays an important role in business sector output and productivity growth.

These studies, particularly Aschaeur's, have been criticized for failing to account for non-stationarity and omitted variable bias (Tatom 1991a, 1993), simultaneity bias (Berndt and Hanson 1992), and because of the magnitude of the coefficient estimates, which critics claim are improbably large (Aaron 1990). The present study addresses many of these issues, presenting estimates that explicitly account for time series features of the data.

Production function specification

Consistent with previous studies, this paper assumes that although public capital investment is financed through taxation, the marginal quantities consumed by firms are 'free of charge.' Firms view public capital as an unpaid factor of production when maximizing profit. Their output is assumed to be a function of private capital (K), labour input (L), public capital (G), and multifactor productivity $MFP(t)$:

$$Y = MFP(t)F(K, L, G) .$$

This paper uses the Cobb-Douglas function to generate parameter estimates of $MFP(t)F(K, L, G)$. The Cobb-Douglas function is employed because it is commonly used in the literature, making estimates comparable with previous studies. Additionally, the resulting elasticity estimates should be similar to labour and capitals' share of income, which provides a transparent, widely understood set of prior expectations against which the impact of including public capital can be examined.

Because government capital is an unpaid factor, it is unclear what the production function's returns to scale should be. Viable alternatives present in the literature include constant returns to scale across all inputs and constant returns to scale across private inputs (See Aschauer 1989, Holtz-Eakin 1994). The first supposes that the public capital is taken as an input in the same fashion as private capital and labour. Implicitly, this functional form assumes that public capital's share of private sector income is captured by private sector inputs. However, there is little guidance about how public capital's share of income is distributed between the private sector inputs, and various authors have approached the problem in different ways.

The constant returns to scale across private sector inputs assumption posits that because public capital can increase the concentration of economic agents, and because it acts as a network, including public capital can lead to increasing returns to scale across all inputs while firms face constant returns to scale in private inputs. Because firms take variables such as land cost and transportation time into account when making investment decisions, public capital has a significant impact on where firms locate. However, once that choice is made, the influence of public capital does not affect its input decisions. Firms choose the amount of labour and private capital in order to minimize costs and maximize investor returns. While public capital is used in the production process (through shipping, for example), its marginal quantities are employed free of change. As a result, at the margin, public capital does not affect investment decisions about the optimal amounts of capital and labour that should be employed. The firm, therefore, may face constant returns to scale across private inputs with public capital leading to overall increasing returns to scale.

Because there is no consensus about which returns to scale assumption is most valid, both specifications were analysed for this paper. The results reported below examine only the function with constant returns to scale across private inputs because it provides estimates that appear to fit the data the most reasonably.[2]

The Cobb-Douglas function with constant returns to scale across private inputs imposes the constraint that $\beta_l + \beta_k = 1$ so that the production function can be written as:

$$y_{p,t} - l_{p,t} = MFP(t)_{p,t} + \beta_k (k_{p,t} - l_{p,t}) + \beta_g g_{p,t} + e_{p,t} \quad (1)$$

where a lower case letter denotes the log-level, β_k, β_l, and β_g are the elasticities private capital, labour input, and public capital respectively, $MFP(t)$ is a term that captures changes in intangible, difficult to measure, inputs such as management structure or research and development, p indexes provinces and t indexes time. Finally, e_t is an identically and independently established error term.

2. The alternative set of estimates is available upon request.

4.1 Econometric methodology

Pre-testing

The provincial panel estimates for the 1981-to-2005 period contain trends over time. When variables have trends, it is important to ascertain the nature of the trends in order to avoid spurious results. Spurious results can arise in two situations. The first situation occurs when the levels of variables that do not share a stochastic trend are used in a regression. When this occurs, the changing levels over time can make it appear that a strong statistical relationship exists when there is no actual relationship. The second situation occurs when a variable with a deterministic trend is used in a regression on a variable with a stochastic trend. This often occurs in level regressions that include time trends. As with the first situation, a strong statistical relationship can be found between the variables because the levels change over time, not because of an economic relationship. Only when all variables have a deterministic trend, or all variables follow a common stochastic trend (are co-integrated), will a regression using levels provide meaningful inference. As a result, it is important to investigate the time series nature of the data prior to estimation.

Over the period in question, business sector gross domestic product (GDP), hours worked, private capital and public capital series have a particular relationship: labour input adjusts quickly to short-term economic fluctuations; private capital adjusts moderately; and, public capital tends to mimic trend GDP growth. It is important to note, however, that this is a generalization and that the pattern is not fixed. There are differences across provinces that can be exploited to increase the accuracy of the parameter estimates.

An IPS (Im, Pesaran and Shin 1997) unit root test is applied to gauge how the data should be treated. The IPS test aggregates individual augmented Dickey-Fuller unit root tests into a panel unit root test statistic. The test attempts to increase the power of the individual unit root tests by exploiting the cross sectional observations present in the panel. The null hypothesis is that all series contain a unit root, while the alternative hypothesis is that at least one series does not contain a unit root.

Table 1
IPS (Im, Pesaran and Shin) unit root test provincial panel data, 1981 to 2005

	H_O: All series contain unit roots H_A: At least one series does not contain a unit root	
	Test statistic	P-value
ln(GDP)	-2.31	0.00
ln(K)	-1.52	0.39
ln(L)	-1.13	0.85
ln(G)	-0.57	1.00

Note: GDP stands for gross domestic product; K stands for private capital; L stands for labour input; and G stands for public capital.
Source: Statistics Canada.

The IPS test statistic rejects the null hypothesis that all log-level GDP series contain a unit root while it fails to reject the null hypothesis of a unit root in labour input, private capital and public capital (Table 1).

The IPS tests imply that a mixture of processes are present in the underlying series. To avoid generating spurious results the series are transformed by taking first differences to remove their trends over time. The resulting equation, which is estimated below, is:

$$dy_{p,t} - dl_{p,t} = \alpha_p + \beta_k (dk_{p,t} - dl_{p,t}) + \beta_g dg_{p,t} + u_{p,t} \,. \tag{2}$$

Estimation strategy

The provincial panel data provides a rich dataset; however, it will be necessary to control for fixed effects and contemporaneous shocks. Two system estimators are used to control for these effects. In order to make the impact of using the more sophisticated estimators clear, their results are compared with simple ordinary least squares (OLS) estimates.

The first set of estimates based on Equation (2) come from applying OLS to pooled log-differenced data. At this stage, the function is estimated without the public capital variable to generate a base case. The base case is then compared with OLS estimates from the pooled data that include public capital, and with a specification where multifactor productivity (MFP) is constrained to zero. Through imposing the constraint that MFP is zero, it is possible to examine the degree to which public capital and MFP are capturing similar features of real GDP growth in the sample.

Using OLS on pooled data potentially misses important fixed effects across provinces and covariances between provincial economic shocks. Two systems estimators that account for potential variance-covariance problems and fixed effects are, therefore, calculated to provide more efficient, consistent estimates.

The first system estimator ($FGLS_1$) treats each province as a separate equation. It allows for province specific MFP and variances as well as a contemporaneous covariance between provinces. The equations are stacked as:

$$Vec(dy_{t,h} - dl_{t,h}) = vec\left[MFP_h + \beta_k (dk_{t,h} - dl_{t,h}) + \beta_g dg_{t,h} + e_t \right]$$

where the subscript h indexes provinces. Provincial dummy variables are included to account for province-specific MFP growth. The elasticities of private capital, labour and public capital are constrained to be equal across provinces.

The system is then estimated as a generalized least squares problem. The variance-covariance matrix for the system, which allows for heterogeneous errors across provinces and contemporaneous correlation between provinces, is written as:

$$E(V) = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1m} \\ \sigma_{21} & \sigma_{22} & & \vdots \\ \vdots & & \ddots & \\ \sigma_{m1} & \cdots & & \sigma_{mm} \end{bmatrix} \otimes I$$

where $\sigma_{i,j}$ is the covariance between provinces i and j if $i \neq j$, or the variance of province i if $i = j$.

The second system estimator (FGLS$_2$) modifies FGLS$_1$ to account for residual serial correlation. It adds an additional level of complexity by expanding the variance-covariance matrix as:

$$E(V) = \begin{bmatrix} \sigma_{11}\Omega & \sigma_{12}\Omega & \cdots & \sigma_{1m}\Omega \\ \sigma_{21}\Omega & \sigma_{22}\Omega & & \vdots \\ \vdots & & \ddots & \\ \sigma_{m1}\Omega & \cdots & & \sigma_{mm}\Omega \end{bmatrix} \otimes I$$

where Ω is a matrix containing a first-order serial correlation correction (See Greene 2000 ch. 15).

Evaluation of estimates

The base case equation using pooled log-difference data to estimate (2) provides an estimate for the elasticity of capital of 0.31, similar to capital's share of income (Column (1) of Table 2). The elasticity of labour is 0.69 and is similar to labour's share of income. The estimate of MFP growth is 1.04% per year and is of a reasonable magnitude for the 1981-to-2005 period. All estimates are statistically significant and the data provide estimates that fit well with prior expectations.

When public capital is included in the production function, the elasticities of hours worked and private capital remain unchanged at 0.31 and 0.69, respectively (Column (2) of Table 2). The estimate of MFP growth increases slightly to 1.15% per year. The estimate for the elasticity of public capital, however, is negative and statistically insignificant. When MFP growth is constrained to zero, there is little change in the elasticity of hours worked and private capital. The estimate for the elasticity of public capital, however, increases to 0.41 and becomes statistically significant (Column (3) of Table 2).[3] The impact of public capital then is difficult to disentangle from overall productivity growth. While estimates of overall productivity growth probably incorporate some influence of public capital when that variable is not included, it is unlikely that when public capital is included, MFP growth should be zero—unless all of it comes from public capital.

3. Note that t-statistics are not exact when multifactor productivity is constrained to be zero. The residual sums are not zero so the inference is only approximate.

Table 2
Panel production function estimates

	Base	OLS	OLS (no MFP)	$FGLS_1$	$FGLS_1$ (no MFP)	$FGLS_2$	$FGLS_2$ (no MFP)
	dy-dl	dy-dl	dy-dl	dy-dl	dy-dl	dy-dl	dy-dl
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
dk-dl	0.31	0.31	0.35	0.31	0.37	0.30	0.37
	(4.73) **	(4.72) **	(5.33) **	(5.72) **	(6.94) **	(5.67) **	(6.94) **
dg	…	-0.06	0.41	0.04	0.31	0.02	0.31
	…	(0.3)	(3.45) **	(0.23)	(3.17) **	(0.16)	(3.12) **
MFP	1.04	1.15	...	1.01	...	1.05	...
	(4.43) **	(2.73) **	...	(1.68)	...	(1.79)	...
N.L.	...	...	...	0.62	...	0.61	...
	...	...	...	(0.62)	...	(0.62)	...
P.E.I.	...	...	...	-0.32	...	-0.33	...
	...	...	...	(0.35)	...	(0.37)	...
N.S.	...	...	...	0.17	...	0.15	...
	...	...	...	(0.22)	...	(0.2)	...
N.B.	...	...	...	0.03	...	0.01	...
	...	...	...	(0.05)	...	(0.02)	...
Que.	...	...	...	-0.42	...	-0.43	...
	...	...	...	(1.15)	...	(1.22)	...
Man	...	...	...	0.13	...	0.11	...
	...	...	...	(0.17)	...	(0.15)	...
Sask.	...	...	...	0.45	...	0.44	...
	...	...	...	(0.49)	...	(0.5)	...
Alta.	...	...	...	-0.20	...	-0.20	...
	...	...	...	(0.29)	...	(0.29)	...
B.C.	...	...	...	-0.76	...	-0.77	...
	...	...	...	(1.12)	...	(1.17)	...
Observations*	240	240	240	240	240	240	240

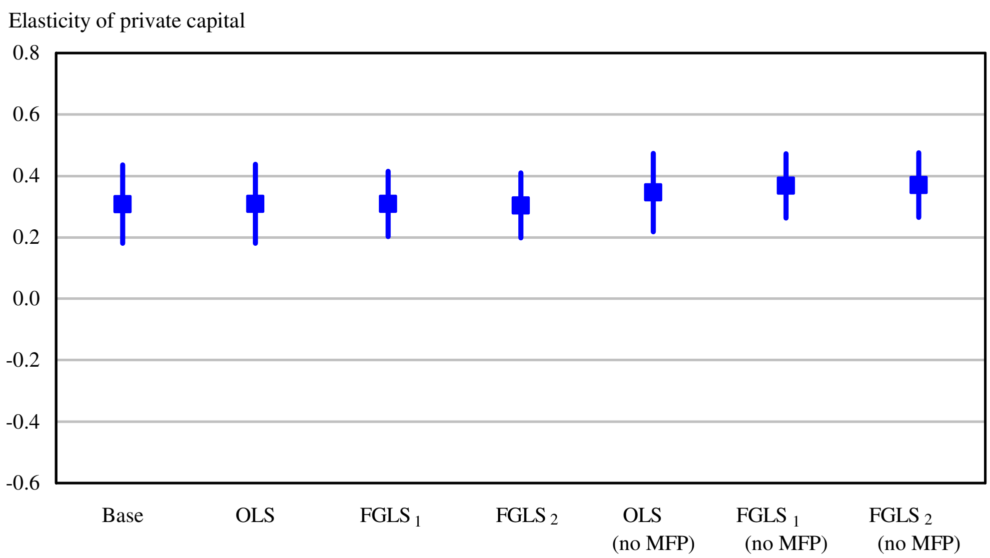
… not applicable
* significant at 5%
** significant at 1%
Note: OLS stands for ordinary least squares; MFP stands for multifactor productivity; $FGLS_1$ stands for the first system estimator; and $FGLS_2$ stands for the second system estimator. Absolute value of t statistics in parentheses.
Source: Statistics Canada.

When the relationship is estimated using $FGLS_{1,}$ the estimate for the elasticity of capital is 0.31 and statistically significant. The estimate for MFP growth is 1.01% and not statistically different from zero when public capital is included (Column (4) of Table 2). Estimates of provincial MFP do not differ statistically from Ontario, which is used as the reference level. If MFP growth is constrained to zero, the estimate for the elasticity of private capital increases to 0.37 and the estimate for the elasticity of public capital increases to 0.31 (Column (5) of Table 2). Both estimates are statistically significant at the 5% level. Moving to $FGLS_2$ provides similar results. Once more, it is difficult to disentangle the effects of public capital from the estimates of MFP.

Panel estimates of the elasticity of private capital are close to private capital's share of income, regardless of the inclusion of public capital and the exclusion of MFP growth. Overall they appear to offer a robust set of private input elasticity estimates. Using this specification, estimates for the elasticity of public capital are intertwined with MFP growth. This supports the finding in Harchaoui and Tarkhani (2003) that MFP growth and public capital growth are related. While Harchaoui and Tarkhani (2003) argue that approximately 12% of MFP growth is accounted for by public capital using cost data, the production function data employed here do not permit the disentanglement of these two factors.

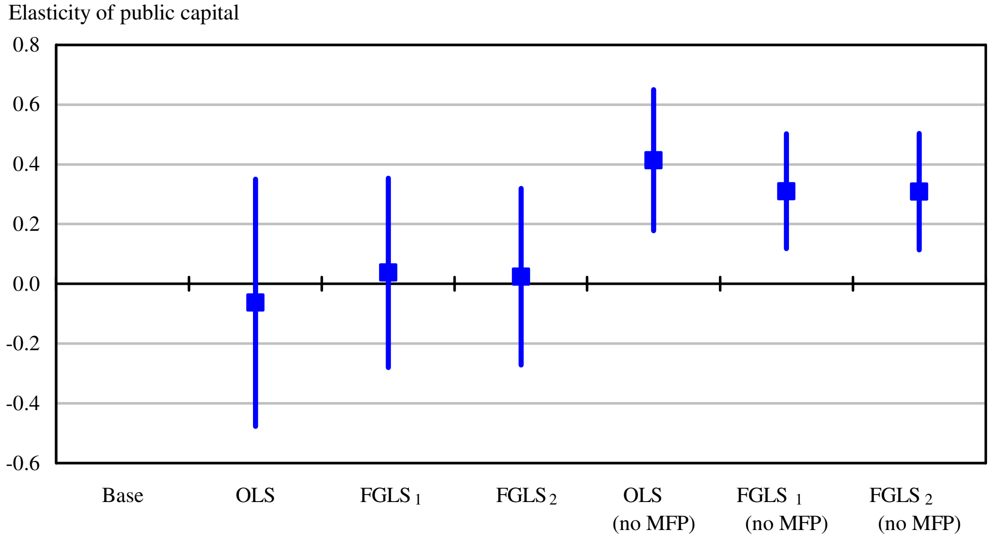
The clearest way to summarize the results is through a series of figures that plot point estimates with their 95% confidence intervals. Because estimates for the elasticity of capital are invariant to the inclusion of government capital and the suppression of MFP growth they provide a stable, and robust, set of estimates against which the impact of public capital can be analysed (Figure 3). The private input elasticity estimates are little changed across specifications and estimation techniques with relatively tight confidence intervals. The point estimate for the elasticity of public capital changes noticeably, moving from near zero to the 0.3 to 0.4 range depending on whether or not MFP growth is included in the equation (Figure 4). The confidence interval for public capital shrinks when MFP is constrained to zero. The MFP point estimate is little changed when government capital is included; however, its confidence interval expands and the estimates become statistically insignificant (Figure 5).

Figure 3
Production function estimates of the elasticity of private capital



Note: OLS stands for ordinary least squares; MFP stands for multifactor productivity; $FGLS_1$ stands for the first system estimator; and $FGLS_2$ stands for the second system estimator.
Source: Statistics Canada.

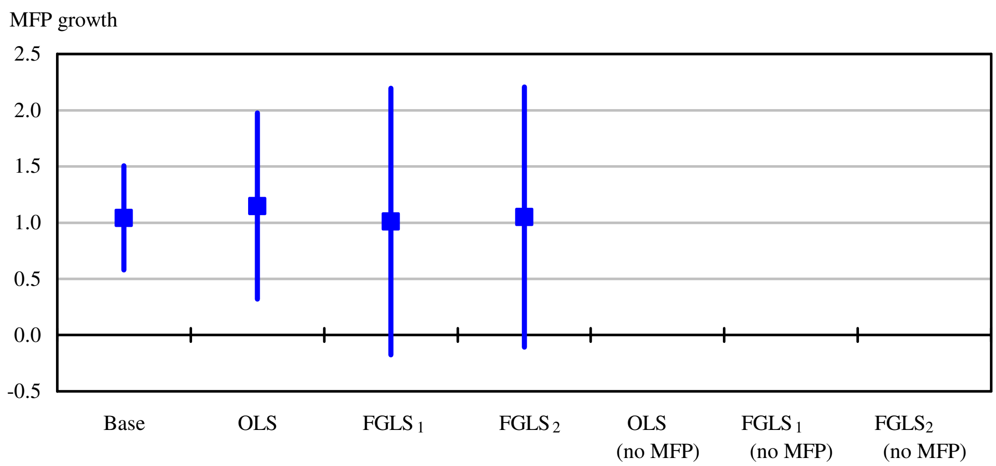
Figure 4
Production function estimates of the elasticity of public capital



Note: OLS stands for ordinary least squares; MFP stands for multifactor productivity; $FGLS_1$ stands for the first system estimator; and $FGLS_2$ stands for the second system estimator.
Source: Statistics Canada.

Figure 5

Production function estimates of multifactor productivity (MFP) growth



Note: OLS stands for ordinary least squares; MFP stands for multifactor productivity; $FGLS_1$ stands for the first system estimator; and $FGLS_2$ stands for the second system estimator.
Source: Statistics Canada.

What does the production function approach tell us?

Production function results show that MFP estimates and the elasticity of public capital are capturing similar features of real GDP. Consequently, it is likely that, where the impact of public capital is excluded from the production function, MFP estimates are capturing the influence of public capital. However, it is unclear what fraction of MFP growth can be attributed to public capital. This is not an econometric or theoretical problem. It is a data problem that reflects the fact that changes in the provision of public capital do not vary greatly over time. In order to gain greater precision on the elasticity of public capital, it will be necessary to find different data or an alternate method for calculating public capital services.

This feature of public capital is not widely recognized in the literature sparked by Aschauer (1989). When panel estimates from the literature are compared with the results from this paper, it appears that the same data problem may be present in other studies (Table 3). Including MFP and public capital in the same equation generates results that suggest they may be capturing the same feature of GDP. Notably, including a trend tends to reduce the impact of public capital.

Table 3
Comparison of panel data results

	Munnel 1990b	Holtz-Eakin 1994	Garcia-Miller et al. 1996	Garcia-Miller et al. 1996	Macdonald 2007	Macdonald 2007
	Pooled OLS	Fixed effects	Fixed effects	Fixed effects	Pooled OLS	System estimator ($FGLS_2$)
	Ln(GDP) (1)	Ln(GDP) (2)	Ln(GDP) (3)	dLn(GDP) (4)	dLn(GDP) (5)	dLn(GDP) (6)
Ln(L)	0.59 *	0.69 *	0.70 *	…	…	…
Ln(K)	0.31 *	0.30 *	0.52 *	…	…	…
Ln(G)	0.15 *	-0.05 *	…	…	…	…
Ln(Highways)	…	…	0.13 *	…	…	…
Ln(Water and Sewer)	…	…	0.06 *	…	…	…
Ln(Other)	…	…	-0.07 *	…	…	…
dLn(L)	…	…	…	0.99 *	0.69 *	0.63 *
dLn(K)	…	…	…	0.35 *	0.31 *	0.37 *
dLn(G)	…	…	…		-0.06	0.31 *
dLn(Highways)	…	…	…	-0.06	…	…
dLn(Water and Sewer)	…	…	…	-0.03	…	…
dLn(Other)	…	…	…	-0.02	…	…
MFP	5.75 *	Present but not reported	Present but not reported	Present but not reported	1.15 *	…

… not applicable
*significant at the 5% level
Note: OLS stands for ordinary least squares; GDP stands for gross domestic product; $FGLS_2$ stands for the second system estimator; L stands for labour input; K stands for private capital; and G stands for public capital.
Source: Statistics Canada.

It, therefore, seems reasonable that if one were to constrain MFP growth to zero and re-estimate the relationship in the other studies that the elasticity of public capital would increase. Unfortunately, this does not help to accurately estimate the elasticity of public capital in Canada or the rate of return associated with it. An alternative method for capturing the impact of public capital is necessary.

5 Cost function approach

The cost function approach uses input prices as explanatory variables. These are more likely to be exogenous than the input variables used in a production function. Thus, the cost function is viewed by many economists as a better way to estimate the impact of public capital. In the cost function approach, public capital is assumed to be an unpaid factor of production that affects the level of the variable cost curve. Typically a flexible functional form such as a trans-log or generalized Leonteif specification is employed. Estimates using cost functions continue to generate a positive impact from public capital (see, for example, Morrison and Schwartz 1996, Nadiri and Mamuneas 1994, Conrad and Seitz 1994, Lynde and Richmond 1992, Shah 1992, and Berndt and Hanson 1992). The magnitude of the returns are smaller than those derived from the primal approach leading authors to conclude that the cost function estimates are more realistic.

For Canada, Harchaoui (1997), Harchaoui and Tarkhani (2003) and Brox and Fader (2005) provide estimates of the cost savings associated with public capital using industry data sets. Their models assume that the real level of public capital enters the cost function as an unpaid factor of production. Firms are represented as minimizing costs over private capital and labour, which constitute the variable cost function for the firm, but take public capital as given in the total cost function. Changes in public capital are assumed to change the height of the variable cost curve. Their approach also includes a demand function.

Using this approach, Harchaoui (1997) finds that the impact of public capital is significant, accounting for about 12% of overall business sector productivity growth. Harchaoui and Tarkhani (2003) re-examine the relationship using an expanded data set, reporting that on average a one-dollar increase in public capital reduces private production costs by 17 cents. Brox and Fader (2005) perform a similar exercise, arguing that the elasticity of public capital with respect to private costs is -0.48.[4]

We extend these analyses with an alternate approach to modelling the impact of public capital in the cost function.

4. Harchaoui (1997) and Harchaoui and Tarkhani (2003) use internally consistent databases developed at Statistics Canada. Brox and Fader (2005) use their own economic time series for analysis.

Cost function specification

The relationship between costs and public capital that follows is based on the relationship outlined in Fernald (1999). Suppose a representative firm faces a standard cost minimization problem:

$$Min \quad w \bullet z \tag{3}$$

$$S.T. \quad f\left(z\right) \geq q \tag{4}$$

where w is a vector of input prices, z is a vector of input quantities, $f\left(z\right)$ is the transformation function describing the level of output that is produced with z inputs and q is the minimum desired level of production The vectors w and z contain information on capital and labour. Under standard first order conditions, a cost function and conditional factor demand functions can be described.

Public capital is introduced by assuming that firms use public capital in the production process. It is, therefore, a factor of production in the transformation function:

$$f(z) = MFP \times F(K, L, T(V, R)) \tag{5}$$

Where MFP, K and L are standard inputs and $T(V, R)$ is a combinatorial function that generates transportation services from public capital (R) and vehicles (V). Note that the majority of public capital consists of roads (Baldwin and Dixon 2008). Unlike functional forms where the stock of public capital is assumed to enter directly, this paper follows Fernald (1999) and assumes that $T(V, R)$ combines roads and vehicles, which allows the impact of public capital to be approximated as proportional to the input share of transportation services.

As a result, it is possible to write down equilibrium conditions and a cost function where per unit cost is a function of capital, labour, multifactor productivity (MFP) and the reflection of public capital in transport cost shares. In this setting, private inputs exhaust the economic surplus so that the sum of private inputs is one. Public capital is an unpaid factor that is assumed to affect the height of the total cost curve.

By assuming that public capital usage is proportional to the transportation cost share, it is possible to generate from Equation (5) a measure of the elasticity of public capital that varies over time and across industries. Moreover, the measure exhibits enough variability that it will capture a distinct feature of cost changes due to public capital that differs from MFP.

After taking logarithms, the following Cobb-Douglas cost equation is employed for estimation:

$$c_{i,t} - y_{i,t} = MFP(t) + \beta_{i,K} p_{i,t,K} + \beta_{i,L} p_{i,t,L} + \beta_{i,Tc} p_{i,t,Tc} + e_{i,t} \tag{6}$$

where c is total cost, y is output, p is the price of capital (k), labour (l) or public capital use (Tc), i indexes industries and t indexes time. Homogeneity of degree 1 in prices and constant returns to scale across private inputs are imposed on the function.

5.1 Econometric methodology

The Capital, Labour, Energy, Materials and Services (KLEMS) database contains volatile, disaggregated data that pose a number of challenges for researchers wishing to use it (see Macdonald 2007). Because the data are noisy and contain unusual, or aberrant, observations, less common estimators can be useful for generating parameter estimates.

The most common estimator employed by economists is ordinary least squares (OLS), which is sensitive to aberrant observations. Aberrant observations can influence OLS slope and intercept estimates because they are formed from sample averages, variances and covariances. These measures are inflated by aberrant observations and can lead to poor inference in their presence.

In the presence of aberrant observations, estimation techniques that minimize the impact of unusual observations can be useful for describing statistical relationships. In this paper, Rousseeuw and Yohai's (1984) S-estimator is employed to juxtapose OLS estimates, and to provide insight into how public capital affects private costs. The S-estimator searches through sub-samples of the data to find the one that produces estimates where the dispersion of the residuals is the smallest (for more information, see Rousseeuw and Yohai 1984, and, Chen 2002).

By minimizing the estimate of the residual variance across sub-samples, the algorithm chooses parameter estimates that represent a majority of the observations. Aberrant observations, which increase the residual variance, are not employed and can be identified from diagnostic measures for further evaluation. Moreover, once identified, the aberrant observations can be re-weighted to remove their influence and OLS can be applied to the remaining sample.

Pre-testing

In the presence of level shifts and trend changes, unit root testing is challenging. These types of observations can make a stationary series appear to follow a unit root process and lead to poor inference (Madalla and Kim 2003). The KLEMS dataset is affected by both of these types of events (Macdonald 2007). As a result, testing the unit root hypothesis is problematic and caution needs to be exercised.

Panel unit root tests are available that combine the cross sectional information from the panel in an attempt to increase the power of the tests. These tests should lead to better inference about the presence of a unit root. However, in the presence of aberrant observations and trend changes, it is not clear that this is accomplished.

Table 4
Hadri LM panel unit root tests

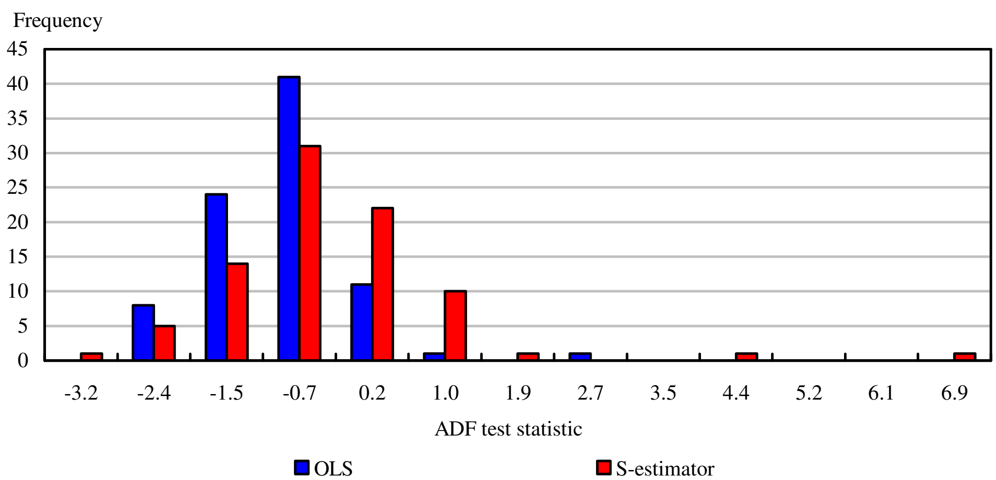
	H_O: All time series are stationary			
	c-y	p_l	p_k	Tc
Homoskedastic	138.91 *	176.84 *	57.1 *	95.14 *
Heteroskedastic	128.87 *	174.5 *	63.28 *	72.25 *
Serial correlation	12.72 *	14.65 *	6.87 *	13.39 *

* Denotes rejection at the 10% significance level
Source: Statistics Canada.

Inspection of the price series suggests that Hadri's LM test may provide sufficient inference. However, the dataset does contain aberrant observations that can affect results. Hadri's LM test is a panel unit root test that follows the same vein as the univariate KPSS test (Kwaitkowski, Phillips, Schmidt and Shin 1992). The null hypothesis assumes the series follow a linear trend over time.

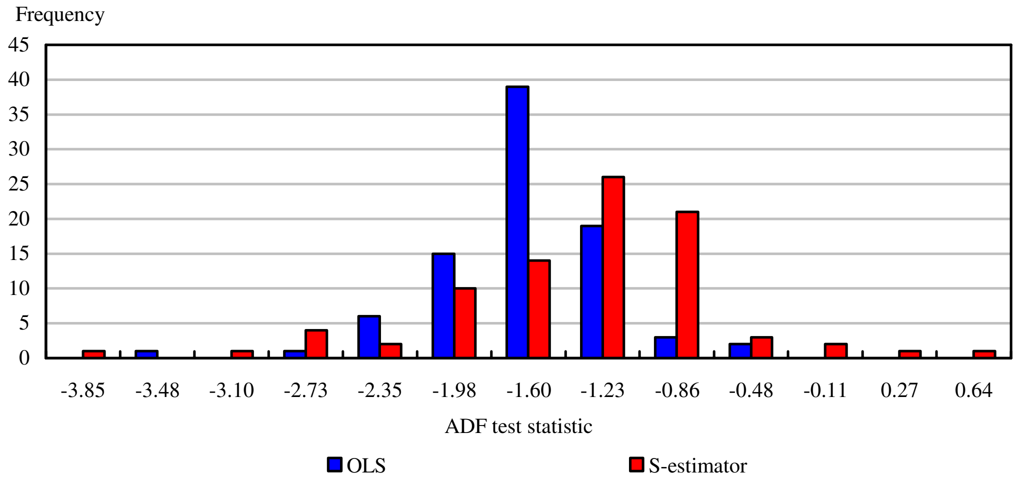
The results imply that all series follow a unit root process (Table 3). However, the underlying data are subject to unusual observations that can affect results. The data are, therefore, subjected to a second unit root examination.

Figure 6
ADF unit root test statistics for Ln(C/Y)



Note: ADF stands for augmented Dickey-Fuller, and OLS stands for ordinary least squares.
Source: Statistics Canada.

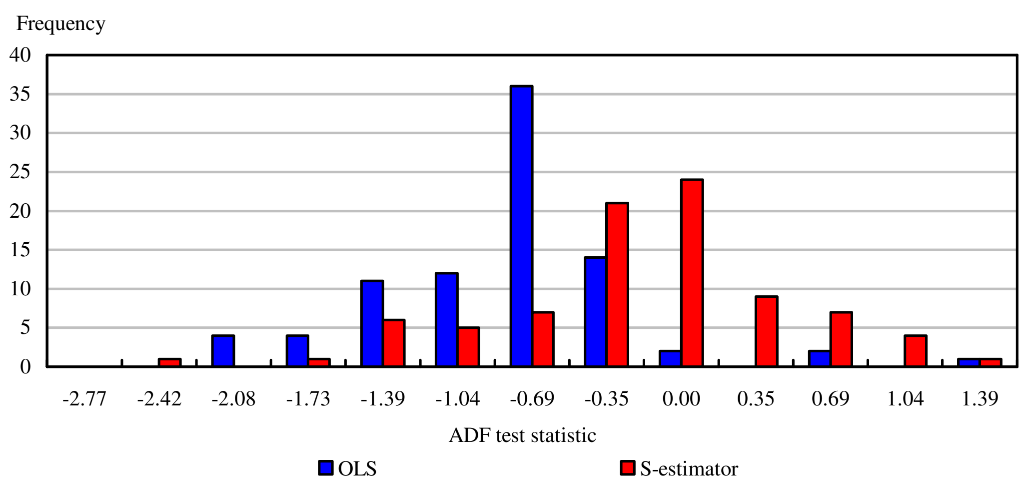
Figure 7
ADF unit root test statistics for Ln(P_L)



Note: ADF stands for augmented Dickey-Fuller, and OLS stands for ordinary least squares.
Source: Statistics Canada.

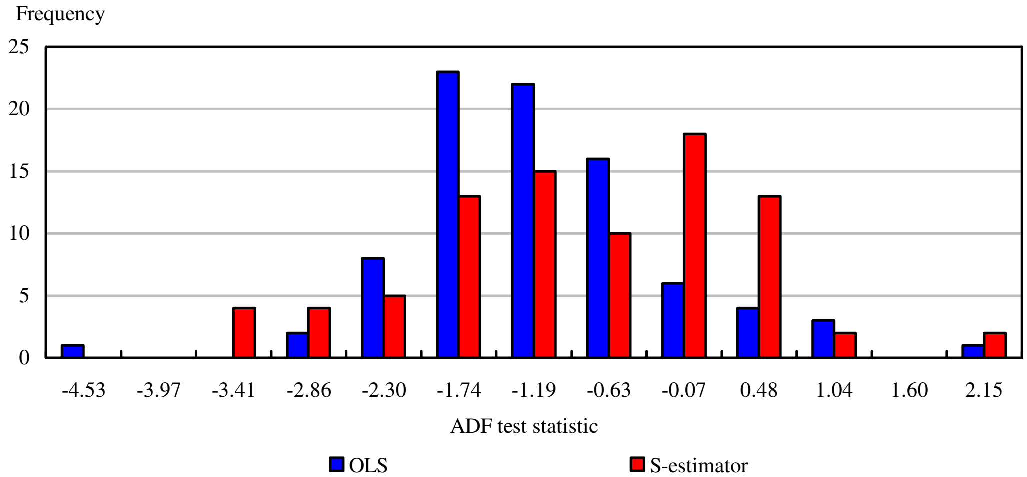
For each of the industries, the augmented Dickey-Fuller (ADF) regression with one lag is run using OLS and the S-estimator. Histograms of the resulting test statistics are presented in Figures 6 through 9. Since the S-estimator is insensitive to aberrant observations, if the aberrant observations are affecting the OLS test statistics, the S-estimator test statistics should give a different outcome.

Figure 8
ADF unit root test statistics for Ln(P_K)



Note: ADF stands for augmented Dickey-Fuller, and OLS stands for ordinary least squares.
Source: Statistics Canada.

Figure 9
ADF unit root test statistics for public capital cost proxy



Note: ADF stands for augmented Dickey-Fuller, and OLS stands for ordinary least squares.
Source: Statistics Canada.

The distribution of responses implies that, for the majority of series, the unit root hypothesis is not rejected for all variables. The result is consistent across estimation strategies; however, in all price series, the OLS distribution is to the left of the S-estimator test distribution, implying that aberrant observations are affecting the OLS test statistics. In this case, the effect is not strong enough to change the test result.

The individual ADF unit root tests and Hadri's LM test imply the series follow unit root processes. The results appear robust to estimation strategy and test type.

Estimation strategy

The unit root tests imply that a unit root process is present in the data. However, it is unclear whether the series are cointegrated or should be first differenced. The presence of aberrant observations in the dataset makes residual distributions non-normal and many cointegration tests unreliable. Rather than attempt to generate panel cointegration tests in this setting, which is less than ideal for them, two approaches are employed and the results compared.

The first approach log-differences Equation (6). It assumes that the contemporaneous log-differences are sufficient for capturing the relationship between the explanatory and response variables. Because the cost share of capital and labour are fairly stable over time, the log-differenced cost function is not estimated as a system. Rather, OLS and S-estimator estimates are formed from the single log-difference equation.

The second approach assumes the data are co-integrated and estimates the level equation including a trend for MFP but asks whether problems appear in the estimates. The data are estimated first as a seemingly unrelated regression (SUR) system using OLS. A second set of estimates is formed using reweighed least squares (RLS). The RLS estimator gives aberrant observations a weight of zero. They are identified by applying Rousseeuw and van Driesen's

(1999) minimum covariance determinant algorithm and the S-estimator to the input variables and Equation (6), respectively.

For both approaches, the units in the panel are treated as distinct entities. They are not pooled, nor are the error processes constrained to have the same variance, which is a commonly applied assumption for panel data models. The units employed here are drawn from a range of industries that have considerable differences between them. Variation in capital intensity makes it difficult to argue that the elasticity of capital, or labour, is similar across industries. Moreover, the small size of some industries makes it difficult to argue that the stochastic processes affecting industries have similar magnitudes.

Therefore, Equation (6) is estimated separately for each industry. This is akin to allowing for unit-specific elasticities of capital and labour, multifactor productivity (MFP), cost savings from public capital and error processes. Once the estimates are generated, their average and individual values are examined.

Parameter estimate evaluation

The average log-difference estimates of MFP accord well with expectations (Tables 5 and 6). The largest MFP estimates are found in primary, construction and manufacturing industries, while services experience the smallest gains.[5] For all industries, the S-estimator and OLS estimates of MFP are similar, and have an average of -1.63% and -1.43%, respectively.[6] For both estimators, manufacturing sector MFP growth is larger than MFP growth in the total economy.

The elasticity of labour estimate is near one for many industries, and notably different from labour's share of income on average. The result does not conform with prior expectations about the elasticity of labour in a level cost function, nor non-parametric estimates of labour's share of income in the aggregate economy. The discrepancy arises from the nature of year-to-year labour and capital cost changes.

The cost of capital is a forward looking variable that amortizes capital expenditures over a number of periods. As a result, it changes less rapidly than wage rates. The log-difference equation estimates reflect the difference by indicating that year-to-year output price changes reflect changes in wage rates to a greater extent than their share of income would suggest. This result tends to occur in cost functions when log-differenced prices are used. In a number of

5. The weighted average of multifactor productivity (MFP) growth across industries is lower than the unweighted average regardless of the estimation technique employed.

	Log-level		Log-difference	
	OLS	RLS	OLS	RLS
Nominal GDP, weighted average of MFP growth	-0.27	-0.42	-1.05	-1.29

Note: GDP stands for gross domestic product; OLS stands for ordinary least squares; and RLS stands for reweighed least squares.

6. On the cost side multifactor productivity captures the extent to which increases in input prices are not passed on to unit costs.

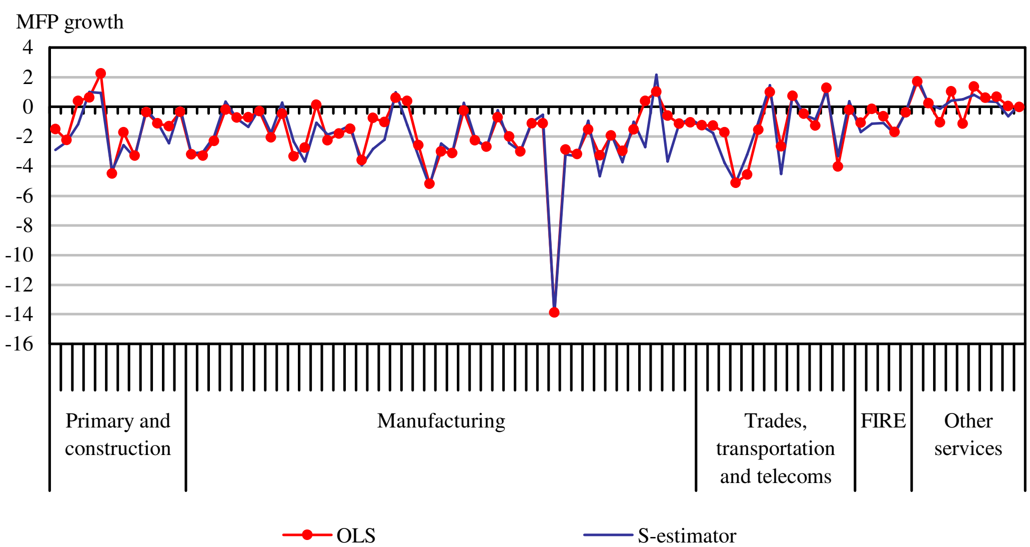
industries, the S-estimate is different from the OLS estimate suggesting that aberrant observations are present.

Public capital generates estimates of the elasticity of cost savings across industries that accord well with prior expectations. Those industries that are most reliant on moving goods experience the largest reductions in costs, due to changing public capital provision over time. The service industries experience small, or zero, reductions. For the majority of industries, the S-estimator and OLS provide similar results. On average, OLS suggests that the elasticity of unit prices with respect to public capital is around -0.15 for the total economy, and around -0.22 for the manufacturing sector.

The averages assume that all industries receive the same benefit from public capital, which is likely to be incorrect, given different industry propensities to transport goods. When nominal gross domestic product (GDP) weights are used instead, the calculated cost savings falls by around three to four percentage points for all industries, and by around four to six percentage points in the manufacturing sector. Nevertheless, the results continue to show that public capital has an important impact on private sector costs.

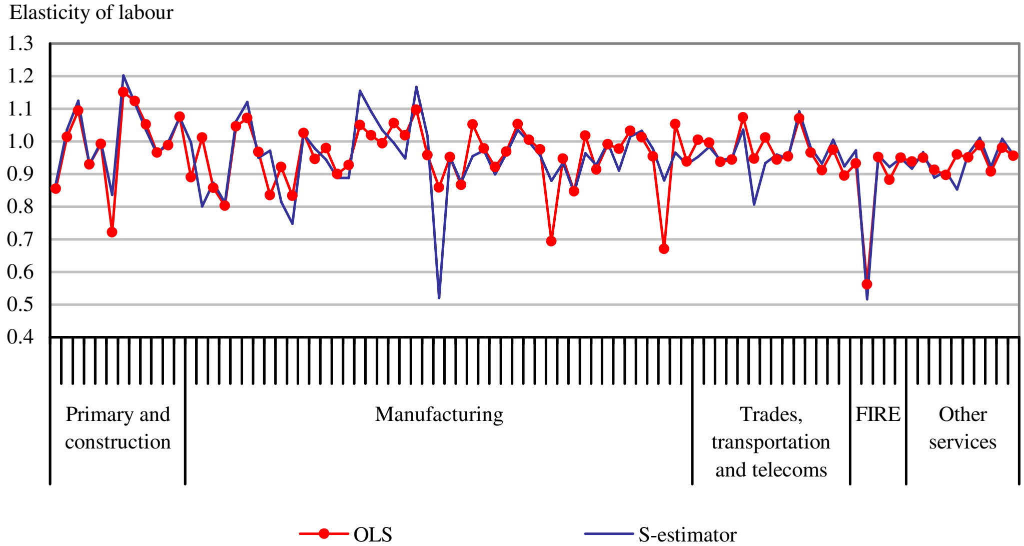
The log-level equation provides slightly lower results for MFP growth compared with the log-difference equation. The OLS and RLS estimates suggest that, on average, MFP growth is -0.92 or -0.81, respectively. In each case, manufacturing is the source of most MFP growth, with a large MFP estimate for computer and peripheral equipment. Service industries experience minimal MFP growth.

Figure 10
Log-difference equation MFP (multifactor productivity)



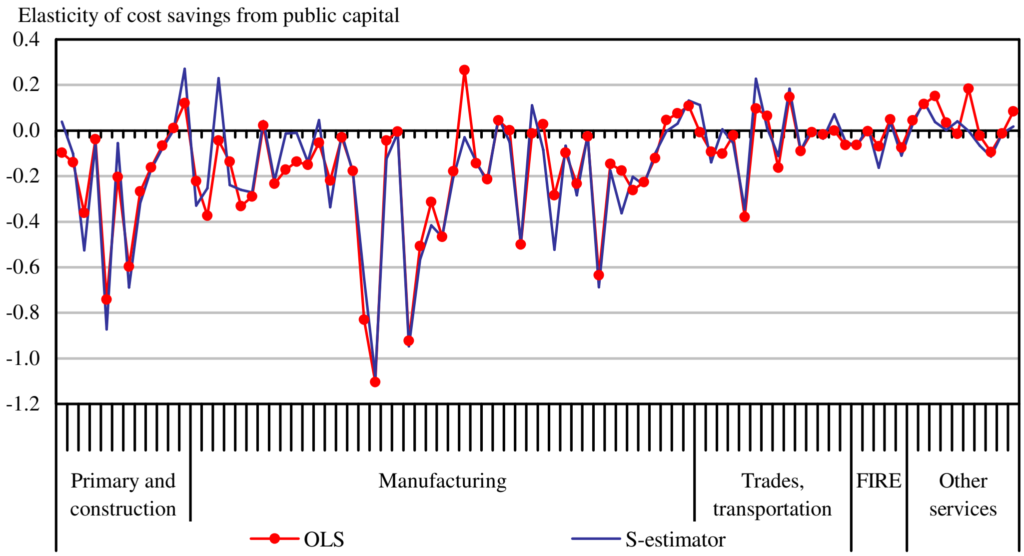
Note: OLS stands for ordinary least squares, and FIRE stands for finance, insurance and real estate.
Source: Statistics Canada.

Figure 11
Log-difference equation elasticity of labour



Note: OLS stands for ordinary least squares, and FIRE stands for finance, insurance and real estate.
Source: Statistics Canada.

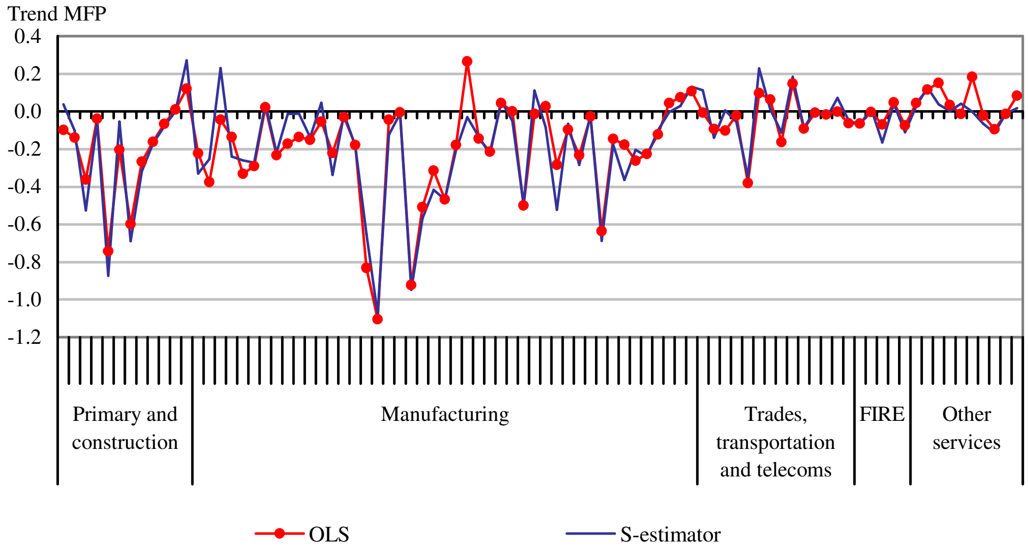
Figure 12
Log-difference equation marginal cost savings of public capital



Note: OLS stands for ordinary least squares, and FIRE stands for finance, insurance and real estate.
Source: Statistics Canada.

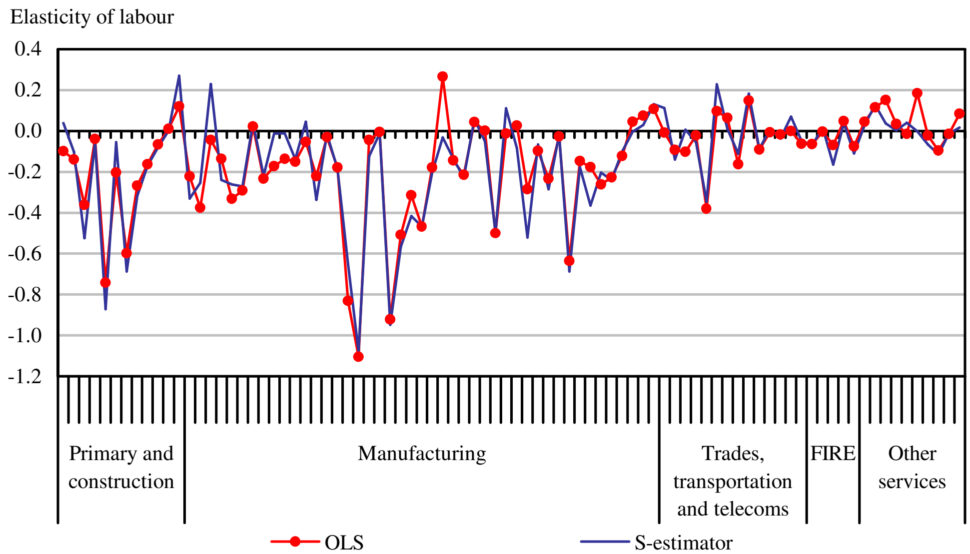
The estimates for the elasticity of capital and labour from the level equation closely accord with their respective income shares on average. OLS yields estimates of 0.37 and 0.63, respectively, while the S-estimates imply that the elasticity of labour is 0.56 and the elasticity of capital is 0.44.

Figure 13
Log-level equation MFP (multifactor productivity) growth



Note: OLS stands for ordinary least squares, and FIRE stands for finance, insurance and real estate.
Source: Statistics Canada.

Figure 14
Log-level equation elasticity of labour

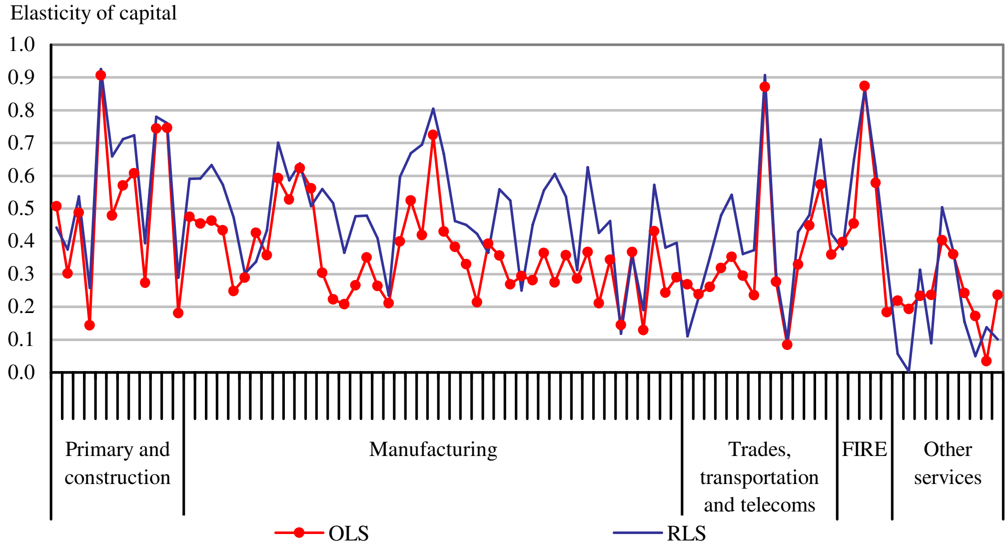


Note: OLS stands for ordinary least squares, and FIRE stands for finance, insurance and real estate.
Source: Statistics Canada.

To this point the results do not differ greatly between the OLS and alternative method that takes into account aberrant observations—the RLS or S-estimator estimates. However, the estimate of the elasticity of the public capital is sensitive to the estimation method. OLS provides estimates that are similar on average and across industries to the first-difference specification, while the

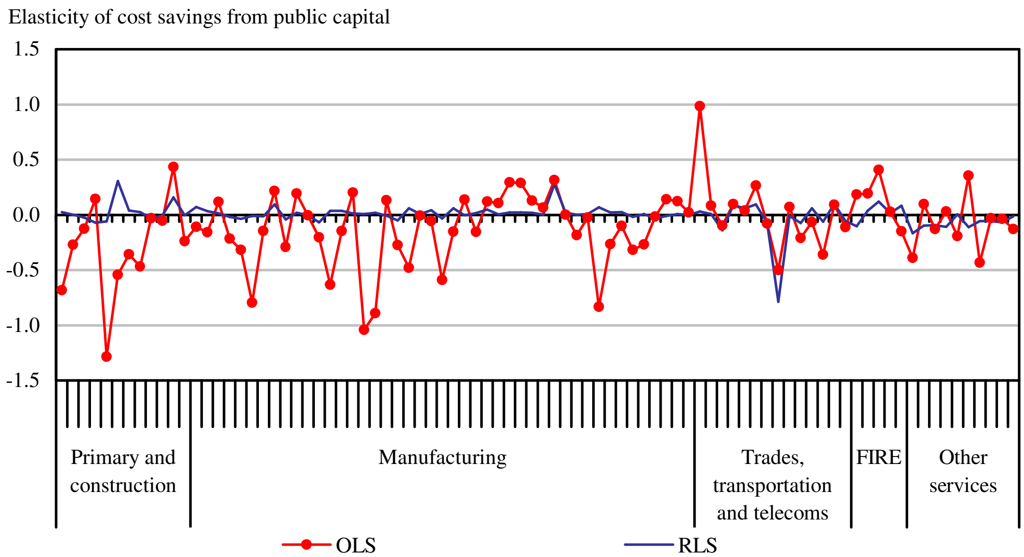
RLS-estimates in the log level approach suggest that the provision of public capital does not lower per unit costs over time.

Figure 15
Log-level equation elasticity of capital



Note: OLS stands for ordinary least squares; RLS stands for reweighed least squares; and FIRE stands for finance, insurance and real estate.
Source: Statistics Canada.

Figure 16
Log-level equation marginal cost saving of public capital



Note: OLS stands for ordinary least squares; RLS stands for reweighed least squares; and FIRE stands for finance, insurance and real estate.
Source: Statistics Canada.

Table 5
Cost function parameter estimate averages (business sector)

	Log-level		Log-difference	
	OLS	RLS	OLS	S-estimator
MFP	-0.92	-0.81	-1.43	-1.63
Elasticity of labour	0.63	0.57	0.96	0.95
Elasticity of capital	0.37	0.43	0.04	0.05
Infrastructure	-0.11	0.03	-0.14	-0.15
Infrastructure (weighted)	-0.08	0.02	-0.12	-0.11

Note: OLS stands for ordinary least squares; RLS stands for reweighed least squares; and MFP stands for multifactor productivity.
Source: Statistics Canada.

Table 6
Cost function parameter estimate averages (manufacturing only)

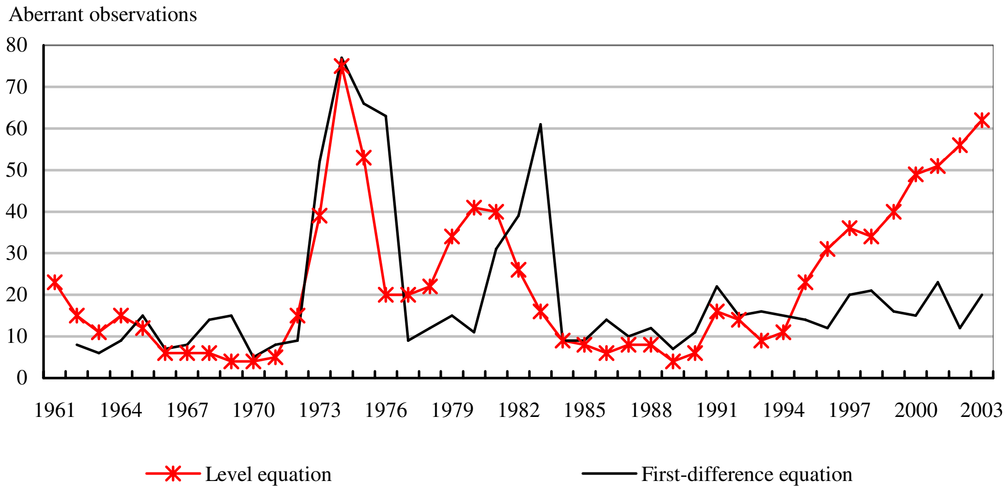
	Log-level		Log-difference	
	OLS	RLS	OLS	S-estimator
MFP	-1.61	-1.24	-1.97	-2.17
Elasticity of labour	0.64	0.56	0.95	0.95
Elasticity of capital	0.36	0.44	0.05	0.05
Infrastructure	-0.14	0.07	-0.21	-0.22
Infrastructure (weighted)	-0.18	0.06	-0.27	-0.27

Note: OLS stands for ordinary least squares; RLS stands for reweighed least squares; and MFP stands for multifactor productivity.
Source: Statistics Canada.

When no corrections are made for aberrant observations, both methods (log-difference and log-level) produce comparable estimates for the elasticity of cost savings from public capital, whether or not a simple or a weighted average is calculated.

The results are similar to the results found in the literature for Canada, in that they provide a negative elasticity of public capital on the cost of the business sector—though they are slightly higher than produced previously. In particular, Harchaoui and Tarkhani (2003) find an elasticity of cost savings of 0.06 for the business sector using a trans-log cost function. Lastly, with the exception of the RLS estimates of the log-level SUR (seemingly unrelated regression) system, the marginal cost savings are robust to aberrant observations.

Figure 17
Aberrant observations by year



Source: Statistics Canada.

Although the RLS estimates for the log level formulation imply there is no impact of public capital when aberrant observations are removed from the sample, this does not mean that an economic relationship is not present. Rather it implies that if the log-level equation is believed, then the marginal cost savings from public capital are generated at particular points in time (Figure 17). For the log-difference and log-level equations, aberrant observations are clustered around the first oil shock and the two recessions in the early 1980s. The log-level equation also has a rising number of aberrant observations toward the end of the period.

The timing of aberrant observations implies that industries that are most able to make use of infrastructure inputs during periods of economic hardship are generating the cost savings seen in the level equation. In other words, during periods of stress, industries that are able to make better use of public capital generate greater cost savings. The explanation may be reasonable during the early part of the period; however, the increase in the number of aberrant observations after 1995 in the log-level equation is difficult to explain economically. Unlike the first oil shock and around the recessions of the early 1980s, there is not an accompanying increase in aberrant observations from the log-difference equation, nor a macroeconomic shock. The discrepancy casts doubt on the veracity of the results for the level equation. In particular, it is possible that the increasing numbers of aberrant observations at the end of the period are due to a misspecification of the trend. Because of this, additional evidence is provided in the next section that the log-level equations suffer other problems.

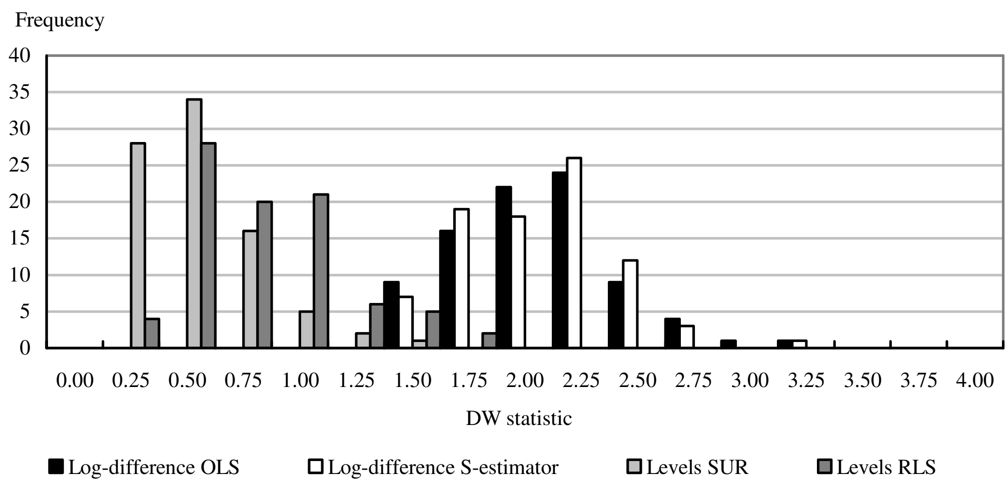
Robustness of the estimates

Incorrectly specifying the trend in a time series regression can lead to spurious results. In particular, if a series has a stochastic trend, which is probable for the price series, and a linear tend is fitted, a relationship between the two variables will only exist because they both increase over time. This is the problem that arises when the SUR and RLS estimates are examined.

The hallmark of a spurious regression is an R-squared statistic that is larger than the Durbin-Watson statistic. This relationship signals a spurious relationship because it can be shown that, in a spurious regression, the R-squared statistic converges to a random variable while the Durbin-Watson statistic converges to zero (for a relatively non-technical discussion, see Granger 2001). The Durbin-Watson and R-squared statistics from the log-difference and log-level models are plotted in Figures 18 and 19, respectively. The level equations have high R-squared values and low Durbin-Watson statistics, making them appear spurious, and supporting Tatom's (1991a, 1993) argument that level regressions are not providing adequate inference.

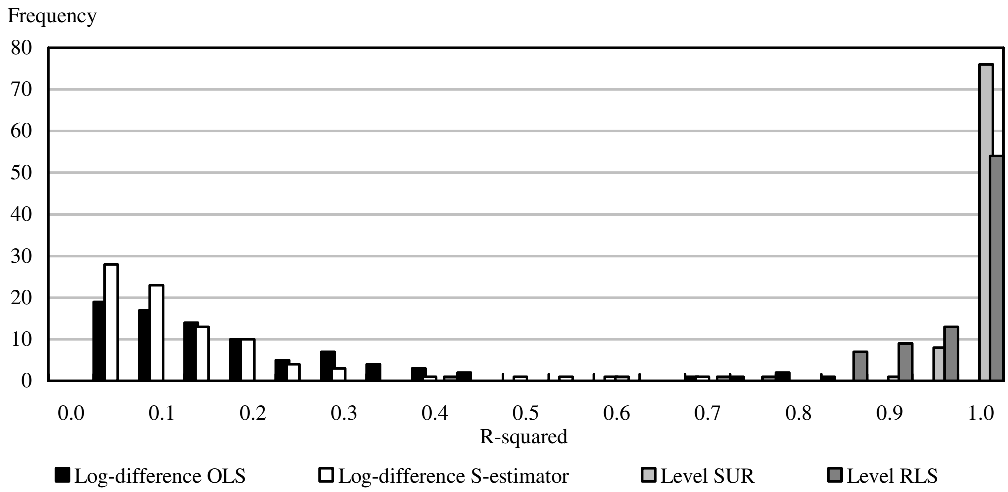
The log-difference equations have Durbin-Watson statistics that are near two on average, suggesting that, while serial correlation may be an issue for particular industries, it is not a concern overall. The log-difference equations exhibit lower R-squared statistics than the level equations and, for a number of industries, the regressions explain minimal amounts of the variation in the dependent variables.

Figure 18
Durbin-Watson D-statistic histograms



Note: OLS stands for ordinary least squares; SUR stands for seemingly unrelated regressions; and RLS stands for reweighed least squares.
Source: Statistics Canada.

Figure 19
R-squared statistic histograms



Note: OLS stands for ordinary least squares; SUR stands for seemingly unrelated regressions; and RLS stands for reweighed least squares.
Source: Statistics Canada.

The data suggest that the log-difference cost function estimates are more appropriate than the level equations. After accounting for the stochastic trend, the marginal cost saving estimates appear robust to trend specification errors, have a magnitude consistent with previous estimates for Canada and correspond to a statistically valid estimate from the production function. Moreover, for the chosen functional form, the simple and nominal GDP-weighted averages for the marginal cost saving estimates fall within the confidence intervals from all production function estimates.

What does the cost function tell us?

The cost function estimates suggest that, over time, additional public capital reduces private costs. The impact differs importantly across industries, where goods-producing industries tend to experience cost reductions while service industries experience minimal effects.

It is important to note that the way public capital is measured here influences the results. Only industries that pay for transportation services, which tend to be goods-producing industries, appear to be making use of public infrastructure. Service industries also make use of public infrastructure, however, they benefit from the density of urban environments and agglomeration effects, which are not captured in transportation costs. Service industries benefit indirectly, rather than directly.

Nevertheless, the cost-function estimates generate results similar to those previously obtained for Canada. The average cost savings from public capital for the business sector reported here are around 0.11 when nominal GDP weights are used and the preferable log difference approach is used. The elasticity would be -.08 if the log level equation were used and no corrections for aberrant observations made. The latter is similar to the 0.06 cost savings for the business sector from public capital reported in Harchaoui and Tarkhani (2003) where they use industry data sets, log level equations and estimate the impact of public capital directly, using a partial rather than a total cost function, as is done here.

6 Public capital's rate of return

A salient question that arises, once the marginal impact of public capital is estimated, pertains to what public capital's rate of return is. Identifying the rate of return is important because it informs researchers and policy makers about what the flow of public capital services may be.

The rate of return on public capital can be estimated by solving the equation describing the user cost of capital for the implied rate of return. Often, the user cost formula includes a term to account for capital appreciation over time. For public capital, this is not included because there is no market for public capital in Canada. Moreover, even if there were markets, the transaction costs would preclude these gains from being realized. When it is not possible to realize capital gains, they do not constitute an opportunity cost which should be accounted for when examining an asset's rate of return.

The relationship between the cost of capital and marginal revenue is described by the user cost of capital formula:

$$P_y \frac{\partial Y}{\partial G} = P_g (r_g + \delta_g) \tag{7}$$

where P_y is the price of output, P_g is the price of a new unit of public capital, r_g is the rate of return on public capital and δ_g is public capital's depreciation rate.

To relate $\frac{\partial Y}{\partial G}$ with $\frac{\partial \ln Y}{\partial \ln G}$ from the production function estimates using Equation(2), note that the logarithmic derivative can re-written as $\frac{\partial Y}{\partial G} = \beta_g \frac{Y}{G}$. As a result, Equation (7) can be re-arranged as:

$$r_g = \beta_g \left(\frac{P_y Y}{P_g G} \right) - \delta_g \tag{8}$$

where β_g is the elasticity of public capital, $P_y Y$ is nominal business sector GDP, $P_g G$ is the nominal value of the stock of public capital.

For the public sector, it is unclear that raising funds is costless. It is argued that the real cost of government funds is higher than the government bond rate—though for national accounts purposes, it is not obvious that the cost of raising funds is a relevant consideration in valuing the output that should be attributed to public output. The cost of raising capital can be accounted for by scaling the marginal revenue from an additional unit of public capital by the marginal cost of funds (MC_g). Measuring MC_g is beyond the scope of this paper; however, Usher (1986) provides an estimate of 1.6, which is employed here. Including the marginal cost of funds adjusts Equation (8) so that it takes the following form:

$$r_g = \beta_g \left(\frac{P_y Y}{P_g G} \right) \Big/ MC_g - \delta_g . \tag{9}$$

Because the argument about government efficiency is contentious, as is the size of the scaling factor, rates of return based on both Equation (8) and Equation (9) are reported. As well, it is important to note that there are 42 yearly rates of return calculated and that only the average rate of return is initially discussed.

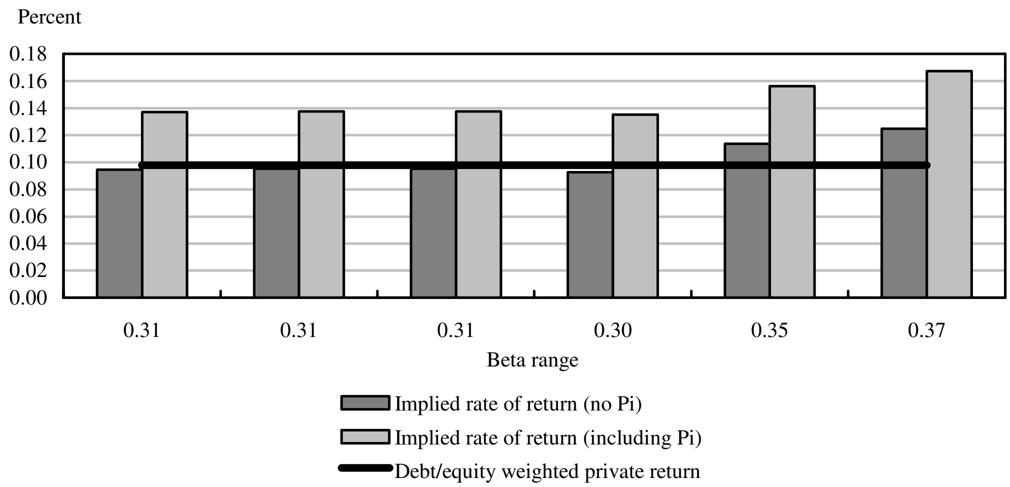
Before examining the estimates of public capital's rate of return, note that when the implied rate of return to private capital is calculated by solving the user cost of capital, the rate of return is close to average yearly TSE (Toronto Stock Exchange) returns over the 1962-to-2003 period (Figure 20). The implied return is calculated, including and excluding price appreciation for capital goods in the user cost of capital formula. The production function point estimates provide a reasonable estimate of the return to private capital, suggesting that the production function specification is adequate for inferring a private rate of return. The issue that remains is whether, and to what degree, a reasonable return to public capital can be estimated given the intertwined effects of public capital and multifactor productivity (MFP).

Previous studies that have estimated public capital's rate of return find that the return is large. This type of result is criticised as being improbable (Aaron 1990) and is one of the main reasons attention shifts from production function to the cost function estimates.

The implied rate of return for public capital generated in this paper is also large when the elasticity of public capital point estimates from the production function excluding MFP are used (Figure 20 Point Estimates). The $FGLS_1$ and $FGLS_2$ estimators for the production function imply that public capital's rate of return is over 50% if the marginal cost of raising funds is one. When Usher's marginal cost of funds estimate is employed the rate of return decreases to around 35%, but remains about four times larger than the average long-run government bond yield.

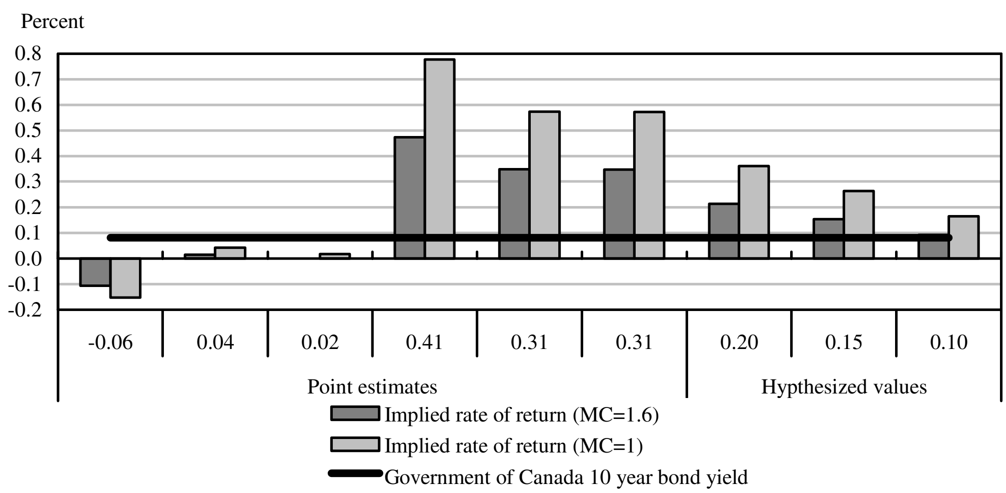
Although these large rates of return are criticized as implausibly large, they do warrant comment. First, if the elasticity of public capital estimate is capturing the enabling nature of public capital, which allows for agglomeration effects, or the effect of increasing the network of public roads that may generate significant externalities, then a large historical rate of return may be plausible. However, it would not be replicable because, as Fernald (1999) points out: "A . . . plausible interpretation . . . is that building one network may have a very high rate of return; but building a second network may have a very low marginal return" (p. 630).

Figure 20
Nominal rate of return to private capital



Source: Statistics Canada.

Figure 21
Public capital's implied nominal rate of return across estimates



Source: Statistics Canada.

Second, and more plausibly, the estimates in this paper are generated from a model that excludes MFP growth because it is difficult to disentangle the effect of MFP and public capital over time. Therefore, it is probable that if both effects could be separated, the marginal impact of public capital would be lower than estimated. Since a lower estimate will necessarily decrease the return from public capital, if MFP growth is reflected in the elasticity of public capital estimate, the rate or return is biased upward.

It is important to note that estimating the rate of return from public capital demands too much of a data series that increases at a fairly constant rate over time. In this setting, it may not be reasonable to assume the point estimate accurately represents the elasticity of public capital. The cost function estimates provide an external of source information that, as argued above, can be used to condition expectations about the magnitude of a reasonable elasticity estimate. The cost function includes the marginal impact of public capital and MFP, overcoming lack of variation in the public capital stock in the production function.

Estimating the rate of return using a range of elasticity estimates of 0.2, 0.15 and 0.1, which are centered on the unweighted average S-estimate of the marginal cost savings from public capital, generates rates of return that are lower. And the weighted average of around 0.10 provides a rate of return of 17%. These rates of return are slightly above the long-run private rate of return on capital and the long-term government bond rate (Figure 20 Hypothesized Values).[7]

It should still be noted that there is uncertainty about the point estimates for the rates of return associated with the hypothesized elasticity of public capital coefficients taken from the cost function approach. For the rates of return of around 17%, excluding the marginal cost adjustment, a 95% confidence interval spans roughly 12 percentage points above and below that of the average estimates while the rates of return including the marginal cost of funds adjustment spans 8 percentage points above and below the average estimates.[8] In the prior case, the confidence interval surrounding the rate of return centered on the cost function average and weighted cost savings from public infrastructure include the long-term government bond rate.

The analysis of the bounds within which the true estimate of the rate of return falls considers only the statistical error that is associated with the estimation of the elasticity of business-sector cost with respect to public capital. There is, however, a non-systematic error that needs to be acknowledged—especially when comparing estimates of rates of return from different authors. Formula (9) for the rate of return involves three terms—the ratio of GDP to public capital ρ, the infrastructure costs elasticity β, and the depreciation rate δ. The use of different rates of depreciation will affect the estimate for the rate of return, both directly through δ and indirectly through the estimate of ρ. Lower depreciation rates applied to the perpetual inventory technique that is used to aggregate capital leads to a lower ρ.

The estimation of δ has important consequences for how large the rate of return estimate is. In addition to improving the accuracy of the elasticity estimate, improvements in estimating δ provide a second avenue along which more certain estimates for the rate of return can be generated. In the future, improved databases and estimation techniques may offer more accurate estimates for public capital's rate of return.

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7. Harchaoui and Tarkhani (2003) report a 12% rate of return from their elasticity estimate of 0.06 but use different depreciation rates that were adopted here, based on the most recent estimates that the productivity program has recently introduced.
8. For this estimate, we have used the average value of the ratio of gross domestic product to public capital in Equation (8). Additionally, we ignore the variance of the depreciation estimate.

7 Concluding remarks

Estimates of the rate of return to public capital that are found in the literature vary considerably. In order to evaluate what the rate of return for Canada might be, this paper builds on previous research at Statistics Canada (Harchaoui and Tarkhani 2003). It examines the extent to which different approaches and methodologies might provide a method of triangulating upon a central estimate of public capital's rate of return that may be used to inform the present discussions in the National Accounts community about how to account for the return that should be incorporated into estimates of value added in the public sector.

To do so, the paper uses a special data set derived from Statistic Canada's productivity accounts, a provincial and an industry panel data set, two different approaches (production function and cost function) and a number of different econometric techniques. Nevertheless, a need for additional research exists. Assumptions about the relationship between public capital and transportation costs, possible endogeneity problems in production and cost function estimation, the role of differing depreciation rates and methods for addressing parameter uncertainty all warrant further investigation.

The production function approach yields estimates for the elasticity of private capital and labour that accord well with their respective income shares. However, the elasticity of public capital and multifactor productivity (MFP) growth are difficult to disentangle. As a result, the range within which the rate of return likely lies is quite large.

The cost function approach presents an additional set of challenges. The time series properties of the data suggest that, in order to avoid spurious estimation results, the data should be first-differenced. In this framework there is less of a problem in separating out the impact of public capital from MFP; and, the resulting estimate of the elasticity of public capital (and the corresponding rate of return) falls within the confidence interval provided by the production function approach.

The estimated elasticity of cost savings from public capital is, on average, around 0.11. In both cases, the cost savings correspond to elasticity values that fall within the 95% confidence interval from production function estimates for the elasticity of public capital. They are also similar to the elasticity of cost savings from public capital of 0.06 reported in Harchaoui and Tarkhani (2003), who use a cost function, but estimate it in level, not first-difference, form. Moreover, after accounting for time series issues and unit heterogeneity, the results are robust across estimation methods.

Despite the 'triangulating' between the cost and production function approaches, there is still a range of plausible elasticity values. The weighted average from our preferred estimator suggests a 17% return. Nevertheless, the confidence interval around these estimates is plus or minus 12 percentage points, which leaves room for further refinement with improved databases.

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