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Gino Cateau, Oleksiy Kryvtsov, Malik Shukayev, and Alexander Ueberfeldt

Canadian Economic Analysis Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
gcateau@bankofcanada.ca

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Abstract

Using the Bank of Canada's main projection and policy-analysis model, ToTEM, this paper measures the welfare gains of switching from inflation targeting to price-level targeting under imperfect credibility. Following the policy change, private agents assign a probability to the event that the policy-maker will revert to inflation-targeting next period. As this probability decreases and imperfect credibility abates, inflation expectations in the economy become consistent with price-level targeting. The paper finds a large welfare gain when imperfect credibility is short-lived. The gain becomes smaller with persisting imperfect credibility, turning to a loss if it lasts more than 13 years.

JEL classification: E31, E52
Bank classification: Monetary policy framework; Monetary policy implementation

Résumé

À l'aide de TOTEM, le principal modèle utilisé par la Banque du Canada pour l'élaboration de projections et l'analyse de politiques, les auteurs évaluent le gain de bien-être que procurerait l'adoption d'une cible fondée sur le niveau général des prix plutôt que sur le taux d'inflation en cas de crédibilité imparfaite. Une fois le nouveau régime en place, les agents privés assignent une probabilité à l'éventualité que les autorités rétablissent une cible d'inflation à la période suivante. À mesure que cette probabilité diminue et que la crédibilité de la banque centrale se renforce, les anticipations de taux d'inflation cadrent de plus en plus avec la poursuite d'une cible définie en fonction du niveau général des prix. Le gain de bien-être est considérable si le manque de crédibilité est de courte durée, mais il diminue si la banque centrale tarde à asseoir sa crédibilité et se transforme en perte si le manque de crédibilité persiste au-delà de treize ans.

Classification JEL : E31, E52
Classification de la Banque : Cadre de la politique monétaire; Mise en œuvre de la politique monétaire

1. Introduction

Over the last two decades, inflation targeting (IT) has been very successful in implementing low and stable inflation in many countries. Yet, many recent papers suggest that from a theoretical perspective, price-level targeting (PT) will lead to welfare gains.[1] However, there are at least two caveats to consider while evaluating these results to determine whether a central bank should move to PT *in practice*.[2] First, most of these theoretical results are derived in small-scale models which abstract from features of the economy that may be relevant for the welfare comparisons between PT and IT e.g. physical capital, international trade. Second, most of the papers ignore the transition costs that may arise due to a temporary destabilization of the private sector's beliefs once the policy-maker switches from IT to PT.

This paper uses ToTEM (Terms-of-Trade Economic Model), the Bank of Canada's main projection and policy analysis model, to answer the following question: How large are the welfare gains of switching from IT to PT under imperfect credibility? Hence we focus on a model that incorporates features that are important for the Canadian economy and informs policy-makers' decisions in practice. And we allow for imperfect credibility by assuming that once the switch to price-level targeting is made, private agents doubt that the policy-maker will be able to maintain the new regime. Specifically, they assign a positive probability weight to the event that the policy-maker will switch back to inflation targeting in the following period. With time, this weight eventually reaches zero so that private beliefs are fully consistent with price-level targeting.

Given the large size of ToTEM, solving for an equilibrium transition path following a

[1] See for example, Svensson (1999) or Vestin (2006).

[2] In view of its 2011 "Renewal of the inflation-control target" meetings with the Government of Canada, the Bank of Canada is currently investigating whether it should move from an inflation to a price-level target.

switch from IT to PT under imperfect credibility as defined above can be a daunting task. We overcome the computational challenge by modeling the probability of a policy reversal as a two-state Markov chain. In a low credibility state, the probability weight on reneging and switching back to inflation targeting next period is high, and it is low in a high credibility state. It is assumed that the high credibility state is absorbing so that private beliefs gradually converge to being fully consistent with price-level targeting. This method greatly reduces the computational burden of the numeric solution in a large-scale model such as ToTEM. We employ this method to study how the speed of convergence of beliefs affects the costs of switching from IT to PT.

We find that the welfare gains from switching to price-level targeting can be as high as half the standard deviation of CPI inflation as measured in Canada for the inflation targeting period. We also find that a minimum of 13 years of low credibility of the price-level targeting regime would be required to drive welfare gains negative.

This paper builds on Kryvtsov, Shukayev, Ueberfeldt (2008), who also study the implications of imperfect credibility for welfare gains of PT relative to IT. Their paper uses a simple Clarida, Gali, Gertler (1999) model with imperfect credibility modeled as a deterministic sequence. Similarly to this paper, they find that the switch to PT is welfare-improving, unless imperfect credibility is long-lasting. Quantitatively, however, the welfare gains found by Kryvtsov, Shukayev, Ueberfeldt (2008) are small, specifically, welfare gains are 5 times smaller than the ones found in this paper. The difference appears to be due to differences in the persistence of inflation: ToTEM is calibrated to match moments from the early 1980s onwards leading to a higher degree of inflation persistence than observed in the data during the inflation targeting era.[3] In contrast, the model used in Kryvtsov, Shukayev, Ueberfeldt (2008) is calibrated to match the data for the inflation targeting period. Large welfare gains of switching to PT are consistent with results in Cateau (2008), who uses ToTEM in an environment with full commitment.

The paper is organized as follows: Section 2 provides a non-technical summary of ToTEM followed by a formal generalized setup that is later used for the analysis of the discretionary policy problem. Section 3 outlines the policy problem and the solution method. Section 4 provides calibration details, the definition of the welfare measure, and the results. Section 5 concludes.

2. ToTEM: generalized setup

A. Brief description of ToTEM

ToTEM (Terms-of-Trade Economic Model) is the Bank of Canada's principal projection and policy-analysis model for the Canadian economy. It is a medium-scale open-economy Dynamic Stochastic General Equilibrium (DSGE) model with multiple goods and an endogenous monetary policy rule followed by the central bank. Optimizing behavior from households, firms, and the central bank yields a set of first-order conditions that dictate how these agents behave. This set of first-order conditions combined with market clearing conditions yields a system of dynamic nonlinear equations that characterize the behavior of the economy (see Murchison and Rennison (2006)). Since ToTEM is used not only for policy analysis but also for projections at the Bank of Canada, it is more elaborate than most standard models. The dynamics of 193 state variables is driven by 29 exogenous shock processes. What follows

[3] Inflation persistence in Canada decreased by more than a half in 1990s relative to 1980s. See Longworth (2002).

is a brief non-technical summary of ToTEM based on Cayen, Corbett, and Perrier (2006).

The production side of ToTEM is as follows. There are four types of final goods produced by domestic firms: consumption, investment, government and non-commodity export goods. To produce these goods, firms use a CES technology that combines capital with labor services, imported intermediate goods, and commodities. There is also a commodity sector. The commodities are produced by domestic firms by combining labor services with capital goods and a fixed factor that we refer to as land. All firms are allowed to vary their utilization rate, but this comes at a cost in terms of foregone output. The firms also face adjustment costs on the level of employment, on the change in investment and in terms of foregone output. It is assumed that final good producers are monopolistically competitive, which allows them to fix prices for more than one period as in Calvo (1983). The Calvo pricing framework is also used for modelling wage rigidities and import price rigidities as in Smets and Wouters (2002).

The demand side of ToTEM can be summarized as follows. Domestic households buy the final consumption goods as well as bonds from the (domestic) government and foreigners. They earn (after-tax) labor income from the labor services that they provide to the domestic firms and income from their holding of domestic and foreign bonds in the form of interest payments. They also receive transfers from the government. The government buys the final government goods from the domestic firms with tax revenues and distributes transfers to the domestic households. These expenditures are financed through tax revenues from labor income and indirect taxes. The model assumes that the government targets a desired level for the debt-to-GDP ratio, with some smoothing, and uses the tax rate on labor income as the policy instrument. Foreigners buy the commodities exports as well as the final non-commodity export goods. They sell intermediate imported goods to the domestic importers, and buy and sell bonds.

Foreign variables in ToTEM are presently generated with a semi-structural model. This model is exogenous with respect to the core of ToTEM in the sense that there is no feedback from domestic variables to the foreign variables. This is consistent with the assumption that Canada is a small open-economy. The foreign variables that enter in ToTEM are output and the output gap, inflation rate, interest rates (real and nominal) and real commodity prices.

Monetary policy in ToTEM is set according to a forward looking Taylor rule (see Cayen, Corbett, and Perrier (2006)), and it is assumed that the monetary authority in ToTEM can fully commit to its future policy actions. This implies that for any future history of shock realizations, the path of the nominal interest rate will be consistent with the policy rule. This is one of the key assumptions that we relax. In this paper, we deviate from the full commitment assumption in the sense that the monetary authority is choosing its policy on a period-by-period basis, optimizing its current-period objectives and taking the private expectations of the future variables as being beyond its control.[4] The remainder of this section lays out the generalized setup of ToTEM, which is then employed to solve the monetary policy problem under discretion.

[4] Although the model does not have an inflation bias as in Kydland and Prescott (1977) there is still a time inconsistency problem in this environment that leads to suboptimality of discretionary policies. See Clarida et al. (1999) for details.

B. Generalized setup

Our solution method is based on the linearized version of ToTEM.[5] The linearized model yields two sets of equations:

$$H_{1zy}y_t + H_{1zz}z_{t-1} + H_{2zz}z_t + H_{3zz}E_t z_{t+1} + B_z i_t = 0 \ , \tag{1}$$

$$H_{1yy}y_t + H_{1yz}z_{t-1} + H_{2yy}y_{t+1} + C_y \epsilon_{t+1} = 0 \ , \tag{2}$$

where

- i_t - the monetary authority's control variable;
- z_t - endogenous state variables that are to be determined within the model once the central bank sets his instrument at time t;
- y_{t+1} - state variables over which the central bank has no control other than through the influence of past predetermined z_{t-1}. If y_{t+1} do not depend on past z_t's (i.e. if $H_{1yz} = 0$ in terms of notation above), then y_{t+1} is a vector of exogenous state variables that influence the evolution of z_t; and
- ϵ_{t+1} - the innovations to y_{t+1}. These innovations are such that $E_t(\epsilon_{t+1}) = 0$ and $E_t(\epsilon_{t+1}\epsilon'_{t+1}) = I$.

Equation (1) typically results from the set of first-order conditions and market clearing conditions whereas (2) represents the law of motion of the driving processes and predetermined states. It will be convenient to rewrite the system above in terms of a state vector

[5] From here on, all variables are expressed as log- (or level-) deviations from a steady state. Unless otherwise noted, variables represent log deviations. See details in Murchison and Rennison (2006).

$X_t = [y_{t+1}, z_t]'$, such that

$$\begin{bmatrix} H_{1yy} & H_{1yz} \\ H_{1zy} & H_{1zz} \end{bmatrix} \begin{bmatrix} y_t \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} H_{2yy} & 0 \\ 0 & H_{2zz} \end{bmatrix} \begin{bmatrix} y_{t+1} \\ z_t \end{bmatrix}$$
$$+ \begin{bmatrix} 0 & 0 \\ 0 & H_{3zz} \end{bmatrix} \begin{bmatrix} y_{t+2} \\ E_t z_{t+1} \end{bmatrix} + \begin{bmatrix} 0 \\ B_z \end{bmatrix} i_t + \begin{bmatrix} C_y \\ 0 \end{bmatrix} \epsilon_{t+1} = 0 \ ,$$

or more concisely,

$$H_1 X_{t-1} + H_2 X_t + H_3 X_{t+1} + B i_t + C \epsilon_{t+1} = 0 \ . \tag{3}$$

Note that we include y_t in the time $t-1$ state vector X_{t-1} since in our set-up, the information that the policymaker has in hand when optimally determining i_t and z_t at time t is $F_t = \{X_{t-1}, X_{t-2}, ...\}$. This representation is consistent with Woodford (2003) and Vestin (2006) in which the cost-push shock u_t is known at time t.

For example, in Vestin (2006) the equilibrium system is

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t$$
$$u_{t+1} = \rho u_t + \xi_{t+1}$$

so that it can be written in the generalized form (1)-(2), where $z_t = \pi_t$, $y_{t+1} = u_{t+1}$, $X_t = [u_{t+1}, \pi_t]'$, $i_t = x_t$ and $\epsilon_{t+1} = \xi_{t+1}$.

To close the model, we need to characterize the monetary authority's choice of the path of policy instrument i_t. We undertake this task in the next section.

3. Policy problem in ToTEM

We assume a policy-maker that has no commitment technology and hence, sets policy *under discretion*. The policy-maker sets policy by choosing his policy instrument to maximize the value of his objective function. The goal of the policy-maker is to implement the 'best' monetary equilibrium from the set of feasible equilibrium outcomes. Here 'best' is defined by the objective function. As outlined in the previous section, the set of feasible equilibrium outcomes is characterized by (3), so we are left with specifying the objective function to define the policy problem. In this section, we first describe the inflation targeting and the price level targeting problem. The rest of this section is then devoted to characterizing imperfect credibility and solving the model under imperfect credibility.

A. Inflation targeting under discretion

Under inflation targeting, the policymaker sets the interest rate r_t (or equivalently the change in interest rate Δr_t since r_{t-1} is known at time t) to minimize a discounted weighted average of squared deviations of inflation, output gap and changes in the interest rate:

$$\min_{\{\Delta r_t\}} E_{-1} \sum_{t=0}^{\infty} \beta^t \{\pi_t^2 + \omega x_t^2 + \nu \Delta r_t^2\} \tag{4}$$

subject to the model's equilibrium equations and the initial state.

For our subsequent derivations, it will be convenient to rewrite the inflation targeting problem in matrix form as:

$$W_{IT}(X_{-1}) = \min_{\{i_t\}} E_{-1} \sum_{t=0}^{\infty} \beta^t \{z_t' Q_{IT} z_t + i_t' R_{IT} i_t\} \tag{5}$$

subject to (1)-(2) with y_0 and z_{-1} given, or more concisely, subject to (3) with X_{-1} given. In terms of our general set up, Q_{IT} is a matrix that picks π_t and x_t out of z_t and assigns appropriate weights, while i_t represents Δr_t.

Since the policy-maker chooses i_t *under discretion*, he solves (5) on a period-by-period basis. The optimal i_t in this case can be shown to be a function of only the predetermined state variables

$$i_t = F_{IT} X_{t-1}.$$

z_t will similarly be a function of X_{t-1} only,

$$z_t = n_{IT} X_{t-1},$$

and the economy will evolve according to

$$X_t = N_{IT} X_{t-1} + C_{IT} \epsilon_{t+1}. \tag{6}$$

B. Price-level targeting under discretion

Under price-level targeting, the policy-maker is delegated a loss function which penalizes squared deviations of the price level rather than inflation, i.e.,

$$\min_{\{\Delta r_t\}} E_{-1} \sum_{t=0}^{\infty} \beta^t \{(1 - \lambda_x - \lambda_i) P_t^2 + \lambda_x x_t^2 + \lambda_i \Delta r_t^2\}. \tag{7}$$

where P_t denotes the deviation of the aggregate log price level from the target and the weights λ_x and λ_i are optimally chosen to maximize social welfare, which as in Vestin (2006), corresponds to the IT objective (4).[6] In matrix form, the price-level targeting problem can be written as

$$\min_{\{i_t\}} E_{-1} \sum_{t=0}^{\infty} \beta^t \{z_t' Q_{PT} z_t + i_t' R_{PT} i_t\} \tag{8}$$

subject to (3) with X_{-1} given. As before, Q_{PT} is a matrix that picks P_t and x_t out of z_t and also assign appropriate weights, λ_x; and i_t represents Δr_t and $R_{PT} = \lambda_i$.

C. Imperfect credibility

We employ the above developed policy framework to conduct a policy experiment in which there is a one-time permanent switch from inflation targeting to price-level targeting.[7] It is assumed that (i) there is no commitment technology available to the policymaker; and (ii) upon the policy change, private sector assigns a positive probability to a policy reversal back to inflation targeting. The policy switch is *imperfectly credible.*

Specifically, let the policymaker switch from IT to PT in period 0. Assume that in period t private agents assign some probability $1 - \theta_t \in [0, 1]$ to the event that the policy in the following period is set according to IT, so that the remaining probability θ_t is assigned to the event that the policy in period $t + 1$ is set according to the price-level targeting. In the model, θ_t follows an exogenous (deterministic or stochastic) path that converges to unity within a certain period of time. Our goal is to study how the speed of convergence of beliefs

[6] Note that although the policymaker's objective under IT coincides with social welfare, the fact that the policy-maker has no commitment technology implies that there may exist other policies that, under discretion, deliver higher social welfare. Price-level targeting is typically one such policy, see Vestin (2006).

[7] For simplicity, there is no change in the average inflation rate after the switch.

affects the costs of switching from IT to PT. Figure 1 lays out the timing of events in this model.

To incorporate imperfect credibility in the specification of the price-level targeting policy problem, we need to modify the expectations term in the constraint (3). Taking time t expectations of (3) yields

$$\begin{aligned} 0 &= H_1 X_{t-1} + H_2 X_t + H_3 E_t X_{t+1} + B i_t \\ &= H_1 X_{t-1} + H_2 X_t + H_3 \left\{ \theta_t E_t(X_{t+1}|PT) + (1-\theta_t) E_t(X_{t+1}|IT) \right\} + B i_t \\ &= H_1 X_{t-1} + \{ H_2 + (1-\theta_t) H_3 N_{IT} \} X_t + \theta_t H_3 E_t(X_{t+1}|PT) + B i_t \ , \end{aligned}$$

or

$$H_1 X_{t-1} + \tilde{H}_2(\theta_t) X_t + \theta_t H_3 E_t(X_{t+1}|PT) + B i_t = 0, \tag{9}$$

where $\tilde{H}_2(\theta_t) = H_2 + (1-\theta_t) H_3 N_{IT}$, and $E_t(X_{t+1}|PT)$ denotes expectations of X_{t+1} in period t conditional on price-level targeting policy in period $t+1$.[8] Hence, the problem of the policy-maker who conducts PT under imperfect credibility is that of choosing i_t to solve (8) subject to (9). In recursive form, the problem is written as follows:

$$V(X_{t-1}, \theta_t) = \min_{i_t} E_t \{ z_t' Q_{PT} z_t + i_t' R_{PT} i_t + \beta V(X_t, \theta_{t+1}) \} \tag{10}$$

[8]The advantage of defining θ_t as the *probability* is that it maintains the representative household assumption under which the linearized equilibrium system of equations 3 was derived.

subject to (9). To solve, we conjecture that

$$V(X_{t-1}, \theta_t) = X'_{t-1} P(\theta_t) X_{t-1} + r(\theta_t), \tag{11}$$

and let $z_t = h_z X_t$, where h_z is a matrix that picks out z_t from X_t. Under discretion the policymaker cannot affect expectations, so the first-order condition for i_t is

$$\begin{aligned} i_t &= R_{PT}^{-1} (\tilde{H}_2(\theta_t)^{-1} B)' (h'_z Q_{PT} h_z + \beta E_t P(\theta_{t+1})) \begin{bmatrix} E_t y_{t+1} \\ z_t \end{bmatrix} \\ &= F(\theta_t) \begin{bmatrix} E_t y_{t+1} \\ z_t \end{bmatrix} \end{aligned}$$

Further, we guess that the discretionary solution implies that

$$z_t = n(\theta_t) X_{t-1}.$$

Given this equation for z_t, the reduced form for the evolution of the economy will be

$$\begin{bmatrix} y_{t+1} \\ z_t \end{bmatrix} = \begin{bmatrix} -H_{2yy}^{-1} H_{1yy} & -H_{2yy}^{-1} H_{1yz} \\ \multicolumn{2}{c}{n(\theta_t)} \end{bmatrix} \begin{bmatrix} y_t \\ z_{t-1} \end{bmatrix} + \begin{bmatrix} -H_{2yy}^{-1} C_y \\ 0 \end{bmatrix} \epsilon_{t+1} = 0.$$

or

$$X_t = N(\theta_t) X_{t-1} + \tilde{C} \epsilon_{t+1}. \tag{12}$$

Hence,

$$i_t = F(\theta_t) N(\theta_t) X_{t-1}, \tag{13}$$

and

$$z_t = h_z N(\theta_t) X_{t-1}. \tag{14}$$

Substituting (11), (13), (14) and (12) into (10) gives us the following formula for $P(\theta_t)$:

$$P(\theta_t) = N(\theta_t)'[h_z' Q_{PT} h_z + F(\theta_t)' R_{PT} F(\theta_t) + \beta E_t P(\theta_{t+1})] N(\theta_t),$$

and for $r(\theta_t)$:

$$\begin{aligned} r(\theta_t) &= \beta E_t(r(\theta_{t+1})) + \\ &\quad \text{trace}(\beta E_t P(\theta_{t+1}) \tilde{C} \tilde{C}'). \end{aligned}$$

Finally, we solve for $N(\theta_t) = \begin{bmatrix} -H_{2yy}^{-1} H_{1yy} & -H_{2yy}^{-1} H_{1yz} \\ \multicolumn{2}{c}{n(\theta_t)} \end{bmatrix}$ by assuming rational expectations. For this we time shift equation (12) by one period forward and take expectations to obtain

$$E_t (X_{t+1} | PT) = E_t(N(\theta_{t+1})) X_t. \tag{15}$$

Then we substitute (15) and (13) in (9) to obtain

$$N(\theta_t) = -(\tilde{H}_2(\theta_t) + \theta_t H_3 E_t N(\theta_{t+1}) + BF(\theta_t))^{-1} H_1 \ . \tag{16}$$

Solving for equilibrium on the transition path after the switch from IT to PT involves solving (16) for a given path of θ_t. In Kryvtsov, Shukayev, Ueberfeldt (2008) it is assumed that θ_t follows a deterministic path converging to 1 within T periods. Solving for an equilibrium path then implies solving a nonlinear system of R^2T equations, where R has the rank of $N(\theta_t)$. For example, if $T = 40$ then for ToTEM the system contains $192^2 \cdot 40 = 1,474,560$ equations, which is computationally very demanding to solve. We resolve this computational issue by using the Markov chain idea.

D. Methodology for computing an equilibrium

Assume that θ_t evolves according to a Markov chain over two states a low credibility state L with θ_L and a high credibility state H with θ_H. In practice, we will focus on the case of $(\theta_L = 0, \theta_H = 1)$. This means that in the low state, agents assign zero probability to monetary policy tomorrow following PT, and in the high state, agents's expectations are fully consistent with PT. How do the private sectors beliefs evolve? Here we assume that they follow a stationary transition matrix given by

$$\Pi = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix}. \tag{17}$$

Our focus will be on a special case where $q = 1$. In that case, the economy will eventually converge to all agents fully believing that PT will be the policy regime tomorrow. Under the outlined Markov setup, we obtain a system of equations which can be solved recursively for a fixed point.

Denote $F_L = F(\theta_L)$ and $F_H = F(\theta_H)$ and similarly, P_L, P_H, $\tilde{H}_{2L}$, $\tilde{H}_{2H}$, N_L,N_H, r_L, and r_H. Then the system of equations is given by

$$\tilde{H}_{2L} = H_2 + (1-\theta_L)H_3 N_{IT}$$

$$\tilde{H}_{2H} = H_2 + (1-\theta_H)H_3 N_{IT}$$

$$F_L = R_{PT}^{-1}(\tilde{H}_{2L}^{-1}B)'\{Q_{PT} + \beta[pP_L + (1-p)P_H]\}$$

$$F_H = R_{PT}^{-1}(\tilde{H}_{2H}^{-1}B)'\{Q_{PT} + \beta[(1-q)P_L + qP_H]\}$$

$$P_L = N_L'\{h_z' Q_{PT} h_z + F_L' R_{PT} F_L + \beta[pP_L + (1-p)P_H]\}N_L$$

$$P_H = N_H'\{h_z' Q_{PT} h_z + F_H' R_{PT} F_H + \beta[(1-q)P_L + qP_H]\}N_H$$

$$N_L = -(\tilde{H}_{2L} + \theta_L H_3 \beta[pN_L + (1-p)N_H] + BF_L)^{-1}H_1$$

$$N_H = -(\tilde{H}_{2H} + \theta_H H_3 \beta[(1-q)N_L + qN_H] + BF_H)^{-1}H_1$$

and

$$\begin{bmatrix} r_L \\ r_H \end{bmatrix} = \begin{bmatrix} 1-\beta p & -\beta(1-p) \\ -\beta(1-q) & 1-\beta q \end{bmatrix}^{-1} \begin{bmatrix} \mathrm{tr}([\beta[pP_L + (1-p)P_H]]\tilde{C}\tilde{C}') \\ \mathrm{tr}([\beta[(1-q)P_L + qP_H]]\tilde{C}\tilde{C}') \end{bmatrix}$$

Equilibrium now satisfies a nonlinear system of $2R^2$ equations. In ToTEM, this implies a system of 73,728 equations, which can now be solved.

4. Calibration, welfare measures and results

A. Calibration

A full-blown calibration of more than a hundred parameters in ToTEM under discretion is too costly and is not crucial for the purpose of this paper. Thus, we keep all parameter values as in the original ToTEM, except for standard deviations of innovations to exogenous shock processes, ϵ_t.[9] We recalibrate these standard deviations by matching moments predicted by the model in the discretionary inflation-targeting equilibrium to the corresponding moments in the Canadian data for the period Q1:1993 to Q2:2008. Specifically, we match the standard deviations of the core CPI inflation, 0.206%, the standard deviation of total CPI inflation, 0.339%, and the standard deviation of output level, 1.336%. We also tried matching the standard deviation of the nominal interest rate level, 0.325% points, but we made only partial progress.

It turns out that out of 29 exogenous shocks in the original ToTEM, recalibrating 5 selected shocks does a reasonable job in enabling us to match target moments: a shock to the rest of world output (`LYROW_SHK`), a wage mark-up shock (`LXW_SHK`), a shock to domestic output (`LY_RES_SHK`), a shock to government transfers to households (`TRANSF_R_SHK`) and a shock to consumption price (`LPC_SHK`).[10]

We consider three cases which correspond to low, medium and high weight on the

[9] Since our focus is on the transition dynamics, in the numeric simulations we abstract from growth components in ToTEM, so that all simulated time series are stationary.

[10] The 5 selected shocks explain the most of the variance of the 3 target moments. Due to complicated variance-covariance relationships among variables in ToTEM we could not match target moments with a smaller number of calibrated parameters.

output gap in the policymaker's loss function:

$$\frac{1}{1+\omega+\nu}\sum_{t=0}^{\infty}\beta^t(\pi_t^2+\omega x_t^2+\nu\Delta i_t^2). \tag{18}$$

That is, we calibrate the shocks for: (i) $(\omega,\nu) = (0, 0.1)$, and (ii) $(\omega,\nu) = (0.05, 0.1)$, and $(\omega,\nu) = (0.5, 0.1)$. Table 1 shows calibrated parameter values. In (18) π_t is given by quarterly core inflation, x_t denotes fluctuations in the ToTEM output gap[11], and Δi is the change in the interest rate from quarter t-1 to t.

The price-level targeting objective is

$$\sum_{t=0}^{\infty}\beta^t((1-\lambda_x-\lambda_i)P_t^2+\lambda_x x_t^2+\lambda_i\Delta i_t^2),$$

where P_t denotes the dynamics of price level and is defined as $P_t = \pi_t + P_{t-1}$ with π_t as the core inflation.

B. Welfare measures for IT and PT

Since the PT solution under discretion involves the objective (7), which differs from the social welfare criterion, (5), we need to compute the implications of the PLT discretionary solution for social welfare.

Recall that the social welfare criterion is

$$E_{-1}\sum_{t=0}^{\infty}\beta^t\{z_t'Qz_t+i_t'Ri_t\},$$

[11] In ToTEM potential output is a composite of total labor input and capital utilization gaps in each of the production sectors.

and that under PLT, the optimal setting of the instrument is

$$i_t = F(\theta_t)N(\theta_t)X_{t-1},$$

$$z_t = h_z N(\theta_t)X_{t-1},$$

and the reduced form model,

$$X_t = N(\theta_t)X_{t-1} + \tilde{C}\epsilon_{t+1}. \tag{19}$$

We derive social welfare under PLT recursively as follows:

$$\begin{aligned} W_{PT}(X_{-1}, \theta_0) &= E_{-1} \sum_{t=0}^{\infty} \beta^t \{z_t' Q z_t + i_t' R i_t\} \\ &= E_{-1} \sum_{t=0}^{\infty} \beta^t \{X_{t-1}' N(\theta_t)' [h_z' Q h_z + F(\theta_t)' R F(\theta_t)] N(\theta_t) X_{t-1}\} \\ &= X_{-1}' N(\theta_0)' [h_z' Q h_z + F(\theta_0)' R F(\theta_0)] N(\theta_0) X_{-1} \\ &+ \ \beta W_{PT}(X_0, \theta_1) \end{aligned} \tag{20}$$

We conjecture that

$$W_{PT}(X_{t-1}, \theta_t) = X_{t-1}' G(\theta_t) X_{t-1} + g(\theta_t).$$

By substituting (19) in (20) and solving, we obtain

$$G(\theta_t) = N(\theta_t)' [h_z' Q h_z + F(\theta_t)' R F(\theta_t) + \beta E_t G(\theta_{t+1})] N(\theta_t),$$

and

$$g(\theta_t) = \beta E_t(g(\theta_{t+1}))+$$
$$\text{trace}(\beta E_t G(\theta_{t+1})\tilde{C}\tilde{C}').$$

Under the Markov Chain assumption for the evolution of θ_t, we further obtain

$$G_L = N_L'\{Q + F_L'RF_L + \beta[pG_L + (1-p)G_H]\}N_L$$
$$G_H = N_H'\{Q + F_H'RF_H + \beta[(1-q)G_L + qG_H]\}N_H$$

and

$$\begin{bmatrix} g_L \\ g_H \end{bmatrix} = \begin{bmatrix} 1-\beta p & -\beta(1-p) \\ -\beta(1-q) & 1-\beta q \end{bmatrix}^{-1} \begin{bmatrix} \text{tr}(\beta[pG_L + (1-p)G_H]\tilde{C}\tilde{C}') \\ \text{tr}(\beta[(1-q)G_L + qG_H]\tilde{C}\tilde{C}') \end{bmatrix}.$$

Recal that $W_{IT}(X_{-1})$ denotes the value of the social welfare loss (4) implied by the inflation targeting (IT) policy given the initial state X_{-1}, and $W_{PT}(X_{-1}, \theta_0; \Pi)$ is the value of the period 0 social welfare loss for the price-level targeting (PT) policy given the initial state (X_{-1}, θ_0) and the transition matrix Π. Following Kryvtsov, Shukayev, Ueberfeldt (2008) we evaluate the welfare difference between the two policy regimes as an *equivalent permanent reduction in the standard deviation of inflation* that would make the social loss under IT equal to that under PT with full credibility, $W_{PT,full}$.[12] That is, the welfare losses for IT and for PT under discretion, in our metric, are measured (in percentage points) as

$$\Delta W_{IT}\left(X_{-1};\Pi\right) = 100\left[\sqrt{\left(1-\beta\right)W_{IT}\left(X_{-1}\right)} - \sqrt{\left(1-\beta\right)W_{PT,full}}\right], \tag{21}$$

$$\Delta W_{PT}\left(X_{-1},\theta_L;\Pi\right) = 100\left[\sqrt{\left(1-\beta\right)W_{PT}(X_{-1},\theta_L;\Pi)} - \sqrt{\left(1-\beta\right)W_{PT,full}}\right].$$

This welfare metric has the advantage that it allows welfare losses from the policy switch to be directly compared with the actual standard deviation of inflation, observed in the data. It is also well suited for comparing welfare under non-stationary policy rules.[13]

C. Results

We first compare welfare under IT and PT given our calibration and the optimized weights for the price-level targeting objective. Here we focus on the *stationary* dynamics under IT and PT, that is when IT (or PT) is always in place and the dynamics are not affected by transition dynamics or initial conditions. Table 3 provides target moments and welfare losses for the low weight on the output gap (results for other cases are similar). The welfare loss from being in the IT regime (relative to being in the PT regime) is 0.14% in units of the standard deviation of quarterly inflation.[14] PT dominates IT due to the *expectations channel* effect previously noted in the literature. Namely, when a shock pushes the current price level above the target, future inflation is expected to be lower than usual in order

[12] Switch to PT with full credibility corresponds to initial state $(X_{-1}, 1; \Pi)$ and $\theta_t = 1$ for all t, that is when private beliefs are fully consistent with PT in all periods.

[13] See details in Kryvtsov, Shukayev, Ueberfeldt (2008).

[14] This welfare loss is the (appropriately weighted) average of welfare losses defined in (21) over all possible initial states X_{-1}. Our results do not hinge on particular value of the initial state X_{-1}.

to revert the price level back to the target. This in turn counteracts the current inflation increase, due to the standard New Keynesian Phillips Curve relationship. In effect, price-level targeting creates an automatic stabilizer that works via the effect of expected inflation on current inflation.[15]

The advantage of PT over IT is equivalent to reducing the historical CPI inflation in Canada by almost half a standard deviation. Our historical reference point is the quarterly CPI inflation in Canada over the Q1:1993 to Q2:2008 period, which is 0.34%. Hence, when the timing and transition costs of the regime switch are ignored, the advantage of PT over IT is substantial. This finding is consistent with results in Cateau (2007) for full-commitment monetary policy in ToTEM, where the advantage of PT over IT ranges between 0.02% and 1.6% depending on the parametrization of the loss function.

Regime-switching - in our case, from IT to PT - may entail costs associated with a sluggish adjustment of private beliefs. We next ask: By how much does the cost of transition lessen the long-run advantages of the regime change from IT to PT? Specifically, we consider the effect on welfare of a one-time permanent policy change from IT to PT in period 0. Our methodology is based on the assumption that the parameter guiding the extent of imperfect credibility, θ_t, follows a two-state Markov process with transition matrix Π from (17). We further assume that $\theta_L = 0$, $\theta_H = 1$. That is the low (high) state of credibility corresponds to zero (full) credibility of the new PT regime. Moreover, we assume that $q = 1$, that is the high state is an absorbing state: once full credibility is achieved, it remains. Under these assumptions, the expected time from period 0 until the full credibility is achieved is given by

[15] See Svensson (1999), Woodford (2003), Yetman (2005), Vestin (2006), Gaspar, Smets, Vestin (2007) for the discussion of the expectations channel under price-level targeting.

$\frac{1}{1-p}$. We then conduct a number of simulations varying p from 0 to 1.

Figure 2 considers the case of a zero weight on output gap stabilization in the loss function. The dashed line plots the welfare loss of IT relative to PT under full credibility, $\Delta W_{IT}(X_{-1})$ while the solid line plots the welfare loss of PT under imperfect credibility relative to PT under full credibility, $\Delta W_{PT}(X_{-1}, \theta_L; \Pi)$. Both losses are expressed in percentage points of an equivalent permanent reduction in the standard deviation of inflation that would make the social loss equal to that under PT with full credibility.

The line showing welfare losses for the PT regime under imperfect credibility slopes up, meaning that losses rise with the time it takes for expectations to become fully consistent with the PT regime. These losses on the transition path to the new PT regime (i.e. transition costs) stem from the fact that shortly after a change to PT, private beliefs are still aligned with the old regime - IT. This means that the expectations channel, whereby expectations of muted price-level fluctuations decrease the necessity for large movements in the policy instrument - is weakened. This, in turn, implies that the monetary authority has to move the policy instrument excessively in order to implement the new price-level targeting regime. Extra volatility of the nominal interest rate, according to (5), leads to welfare losses. The longer it takes for expectations to converge to full credibility, the longer the expectations channel will be ineffective, and therefore the larger will be welfare losses. Figure 2 illustrates that point. If it takes expectations more than 13 years to become fully consistent with the PT regime, the costs of transition outweigh the long-run benefits of PT. In that case, it is not worthwhile to switch from IT to PT.[16]

[16]When higher weights are assigned to output gap stabilization in the loss function, the cutoff time at which the switch to PT becomes undesirable is longer.

We conclude that the benefits of switching from inflation- to price-level targeting depend on the speed with which private expectations accommodate the policy change. We find that for ToTEM, the long-run benefits of PT outweigh the cost associated with the time that it takes for private beliefs to become consistent with the new PT regime, unless this time is longer than 13 years. Carroll (2003) and Mankiw, Reis, Wolfers (2003) find from the expectations survey data for the U.S. that it takes about one year for households to accommodate macroeconomic information. Therefore we conclude that it is likely that adopting PT in Canada would be welfare improving. The quantitative effect may diminish due to transition costs.[17]

The advantages of switching to PT that we find using ToTEM are five times larger than those that Kryvtsov, Shukayev, Ueberfeldt (2008) found using a Clarida, Gali, Gertler (1999) model, where they are 1/10 of the standard deviation of inflation in Canada. The difference in results is due to the amount of persistence of inflation and output gap fluctuations in the two models. Serial correlation of CPI inflation in ToTEM is 0.92 matching inflation persistence in Canada from 1980:Q1 to 2008:Q3, the calibration period in Murchison and Rennison (2006). For the inflation targeting period, the serial correlation of the CPI inflation is 0.65.[18] Kryvtsov, Shukayev, Ueberfeldt (2008) showed that the benefits of PT over IT increase disproportionately with inflation persistence: they are below 0.1% if the serial correlation of inflation is below 0.8 but rise quickly to 0.23% as inflation persistence goes up to 0.96. Hence,

[17] We verified the the robustness of the results with respect to alternative calibrations of the size and persistence of the main driving shocks in the model, as well as alternative specifications of the loss function. Main results are not sensitive to alternative ways of calibrating the underlying shocks or alternative definitions of the social welfare as used in the literature. Appendix with robustness details is available upon request.

[18] We used year-to-year quarterly inflation rates to get rid of higher frequency noise. See also Longworth (2002), who documents the decrease in inflation persistence in 1990s relative to 1980s.

our results using ToTEM with its current parametrization can be interpreted as an upper bound on the benefits of PT over IT.

5. Conclusion

This paper uses ToTEM, the Bank of Canada's main projection and policy analysis model, to measure the potential benefits of moving from an inflation targeting regime to a price-level targeting regime. Given that a transition is likely to destabilize the private sector's expectations regarding the monetary policy regime (at least temporarily), we introduce an adjustment of credibility into the model. The large size of the model forces us to model credibility as a time-stationary Markov chain. We find that the welfare gains from switching to price-level targeting can be as high as half the standard deviation of inflation as measured in Canada for the inflation targeting period. We also show that only very long spells of imperfect credibility, 13 years and more, can undermine the benefits of switching to price-level targeting.

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Table 1: Standard deviation of shocks in percentage points

Shock	$(\omega,\nu)=(0,0.1)$	$(\omega,\nu)=(0.05,0.1)$	$(\omega,\nu)=(0.5,0.1)$
LYROW_SHK	0.011	0.011	0.013
LXW_SHK	0.012	0.020	0.022
LY_RES_SHK	0.465	0.549	0.570
TRANSF_R_SHK	0.250	0.320	0.370
LPC_SHK	0.252	0.260	0.260

Note: ω and ν are weights on output gap and change in the nominal interest rate in the welfare criterion (5).

Table 2: Optimized weights for PT objective

	$(\omega,\nu)=(0,0.1)$	$(\omega,\nu)=(0.05,0.1)$	$(\omega,\nu)=(0.5,0.1)$
λ_x	0.947	0.962	0.971
λ_i	0.024	0.027	0.029

Note: ω and ν are weights on output gap and change in the nominal interest rate in the welfare criterion (5).
λ_x and λ_i are weights on output gap and change in the nominal interest rate in price-level targeting objective (8).

Table 3: IT vs PT when weights are $(\omega, \nu) = (0, 0.1)$

Standard deviation	Data	IT	PT
Core inflation, π_t	0.206*	0.205	0.033
Total CPI inflation	0.339*	0.337	0.254
Log output	1.336*	1.335	1.083
Output gap, x_t		0.857	0.131
Interest rate, i_t	0.325	0.543	0.308
Interest rate change, Δi_t	0.155	0.286	0.237
Welfare loss relative to PT, % points of std(π_t)		0.142	0

Note: All entries are in % points. * denotes calibration targets. Moments and welfare are computed for stationary dynamics under IT and PT. Welfare is measured as equivalent permanent reduction in the standard deviation of inflation that would make the social loss under IT equal to that under PT.

Figure 1: Timing of events in ToTEM under imperfect credibility

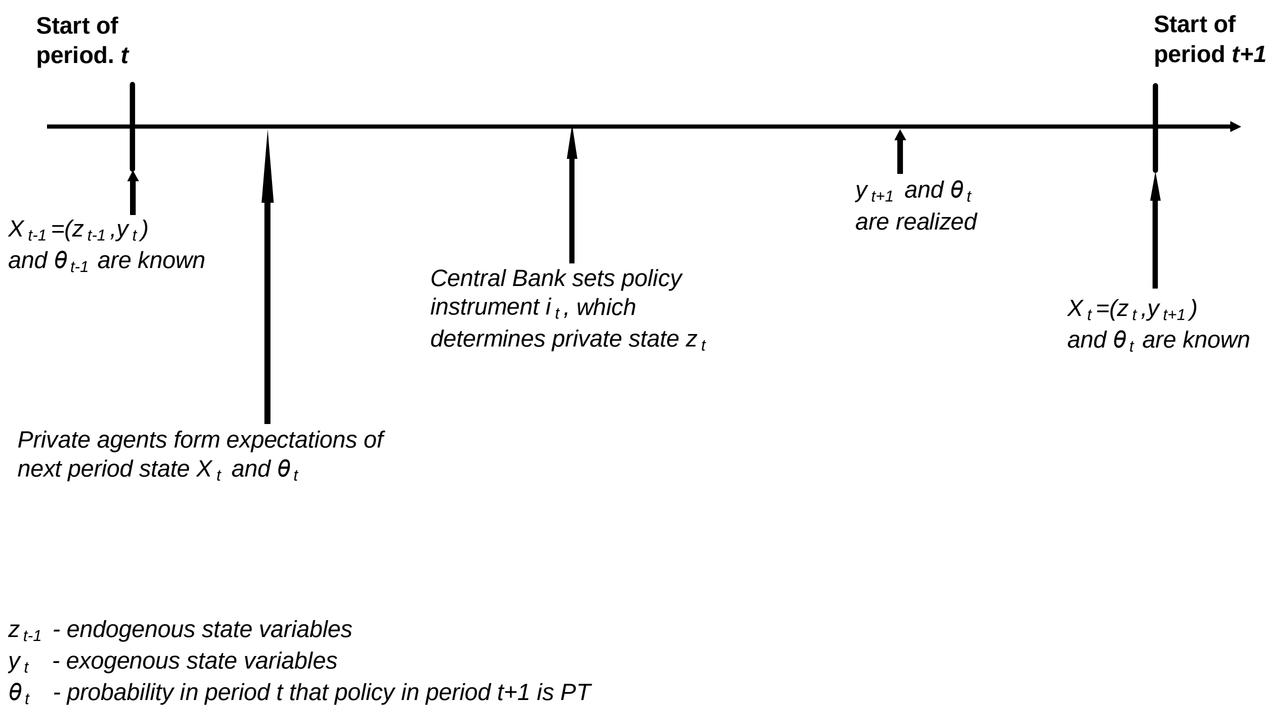

Figure 2: Welfare losses as a function of expected time of reaching high credibility state, $(\omega, \nu) = (0, 0.1)$