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FINITE SETS AND SYMMETRIC SIMPLICIAL SETS

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Abstract. The category of finite cardinals (or, equivalently, of finite sets) is the symmetric analogue of the category of finite ordinals, and the ground category of a relevant category of presheaves, the *augmented symmetric simplicial sets*. We prove here that this ground category has characterisations similar to the classical ones for the category of finite ordinals, by the existence of a universal symmetric monoid, or by generators and relations. The latter provides a definition of symmetric simplicial sets by faces, degeneracies and transpositions, under suitable relations.

Introduction

The category $\tilde{\Delta}$ of finite ordinals (and monotone mappings) is the basis of the presheaf category $\tilde{\mathbf{Smp}}$ of augmented simplicial sets. It has well known characterisations, as:

(a) the free strict monoidal category with an assigned internal monoid;

(b) the subcategory of **Set** *generated by finite ordinals, their faces and degeneracies*;

(c) the category generated by faces and degeneracies, under the cosimplicial relations.

The last characterisation is currently used in the usual description of an augmented simplicial set as a sequence of sets with faces and degeneracies, subject to the (dual) simplicial relations. The restriction of this characterisation *(c)* to the category Δ of *positive* finite ordinals plays the same role for *ordinary* (non augmented) simplicial sets (while *(a)* cannot be so restricted).

Here, in Theorems 4.1 and 4.2, we give similar characterisations for the "symmetric analogue", the category $!\tilde{\Delta}$ of *finite cardinals*, with the same objects $n \geq 0$ and all mappings, equivalent to the (large) category of finite sets. $!\tilde{\Delta}$ is thus:

(a′) the free strict monoidal category with an assigned symmetric monoid;

(b′) the subcategory of **Set** *generated by faces, degeneracies and main transpositions*;

(c′) the category generated by faces, degeneracies and main transpositions, under the symmetric cosimplicial relations *(Section 3)*.

Again, the last characterisation gives a presentation of the non-augmented symmetric simplicial site $!\Delta$, and provides a definition of symmetric simplicial sets by faces, degeneracies and transpositions, under the dual relations.

To motivate the interest of such characterisations, let us recall that *symmetric simplicial sets*, i.e. the presheaves $X\colon !\Delta^{op} \to \mathbf{Set}$ on finite positive cardinals, have been studied

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in [4, 5], where a combinatorial homotopy theory has been introduced for their category $\mathbf{!Smp} = \mathbf{Set}^{!\Delta^{op}}$, extending a previous theory for simplicial complexes [3].

As a crucial advantage of the extension, we have a *fundamental n-groupoid* functor $\Pi_n\colon \mathbf{!Smp} \to n\text{-}\mathbf{Gpd}$ $(n \leq \omega)$ left adjoint to a *symmetric nerve* M_n, which yields a strong, simple version of the Seifert-van Kampen theorem: Π_n preserves all colimits. Analogously, a notion of (non-reversible) *directed homotopy* has been developed in the ordinary simplicial topos **Smp**, with applications to image analysis as in [3]; we have now a *fundamental n-category functor* $\uparrow\Pi_n$, left adjoint to a *nerve* N_n.

A classical reference on simplicial sets is May's book [11]. The characterisations of the category $\tilde{\Delta}$ of finite ordinals can be found in Mac Lane's text [10]. For monoidal categories, see [10] and Kelly's book [8]. The case $n = 1$ of the adjunction $\Pi_n \dashv M_n$ was already noted by Lawvere [9], and was at the origin of this research.

Notation. As usual, finite ordinals and finite cardinals coincide, and are constructed as $0 = \emptyset$, $n = (n-1) \cup \{n-1\} = \{0, 1, \dots n-1\}$. The term "graph" stands for *oriented graph.*

1. Reviewing the simplicial site

The category $\tilde{\Delta}$ of finite ordinals has a rich structure (cf. [10], VII.5, where $\tilde{\Delta}$, Δ are written as Δ, Δ^+, respectively); it is reviewed here as a leading frame for our symmetric analogue.

To begin with, $\tilde{\Delta}$ is a strict monoidal category, with respect to the ordinal sum $m + n$ (non-symmetric). The object 1 is an internal monoid

$$\begin{gathered} \partial : 0 \to 1 \leftarrow 2 : e \\ e(\partial + 1) = \mathrm{id} = e(1 + \partial), \qquad\qquad e(e+1) = e(1+e) \end{gathered} \tag{1}$$

with unit (or face) ∂ and multiplication (or degeneracy) e. Then, the terminal mapping $e^{(k)}\colon k \to 1$ appears to be an iterated multiplication, with

$$e^{(0)} = \partial, \qquad e^{(1)} = \mathrm{id}, \qquad e^{(2)} = e, \qquad e^{(3)} = e(e+1) = e(1+e)), \dots$$

and each monotone mapping $f\colon m \to n$ can be uniquely decomposed as a sum $f = e^{(m_0)} + \dots + e^{(m_{n-1})}$ of iterated multiplications, where $m_i = \#(f^{-1}\{i\})$, and $m = m_0 + \dots + m_{n-1}$.

The usual (co)faces and (co)degeneracies can be constructed with the structural maps ∂, e and the monoidal structure (for $0 \leq i \leq n$)

$$\partial_i^n = i + \partial + (n-i)\colon n \to n+1, \qquad e_i^n = i + e + (n-i)\colon n+2 \to n+1 \tag{2}$$

(the injective monotone map which omits i and the surjective monotone map which repeats i, respectively); the *cosimplicial relations* follow easily from the previous formulae:

$$\begin{aligned} &\partial_i \partial_j = \partial_{j+1} \partial_i \quad (i \leq j), && e_j e_i = e_i e_{j+1} \quad (i \leq j), \\ &e_j \partial_i = \partial_i e_{j-1}, \text{ or } 1, \text{ or } \partial_{i-1} e_j && (i < j \text{ or } i = j, j+1 \text{ or } i > j+1). \end{aligned} \tag{3}$$

A monotone mapping $f\colon m \to n$ has a canonical factorisation

$$f = \partial_{j_1} \cdot \partial_{j_2} \cdot ... \cdot e_{i_2} \cdot e_{i_1} \quad (m-1 > i_1 > i_2 > ... \geq 0;\ \ n > j_1 > j_2 > ... \geq 0) \tag{4}$$

by faces and degeneracies; every composite of faces and degeneracies can be put in canonical form, using the cosimplicial relations as rewriting rules (from the left).

Taking advantage of all this, the category $\tilde{\Delta}$ of finite ordinals is characterised as:

(a) the free strict monoidal category with an assigned internal monoid, 1;

(b) the subcategory of **Set** *generated by finite ordinals, their faces and degeneracies*;

(c) the category generated by the graph (2), subject to the cosimplicial relations (3).

2. Symmetric monoids

The category $!\tilde{\Delta}$ of finite cardinals (equivalent to the category of finite sets) has a strict monoidal structure $m+n$, the (categorical) sum of cardinals, with a canonical symmetry (provided by the sum)

$$\begin{gathered} s\colon\ m+n \to n+m, \\ s(i) = n+i \quad (0 \leq i < m), \qquad\qquad s(m+j) = j \quad (0 \leq j < n) \end{gathered} \tag{5}$$

which is not strict (note that the identity $m+n = n+m$ is not natural). Now, $(1; \partial, e)$ is a commutative monoid within this enriched structure, satisfying the obvious axioms

$$e(\partial+1) = \mathrm{id} = e(1+\partial), \qquad e(e+1) = e(1+e), \qquad es = e. \tag{6}$$

However, we want to be able to deal with "commutative" (or *symmetric*) monoids, within a mere monoidal category without symmetry (which is necessary for symmetric monads, cf. Section 6); this can be done by transferring the symmetry to the monoid itself. The object 1 is now viewed as an internal symmetric monoid, with a *unit* (or face) ∂, *multiplication* (or degeneracy) e and *symmetry* s,

$$\partial : 0 \to 1 \leftarrow 2 : e \qquad\qquad s\colon 2 \to 2 \qquad\qquad (s(t) = 1-t), \tag{7}$$

satisfying the axioms below (containing a Yang-Baxter condition on s, see [7] and references therein)

$$\begin{array}{ll} e(\partial+1) = \mathrm{id} = e(1+\partial), & e(e+1) = e(1+e), \\ ss = 1, & (s+1)(1+s)(s+1) = (1+s)(s+1)(1+s), \\ s(\partial+1) = 1+\partial, & es = e, \qquad s(1+e) = (e+1)(1+s)(s+1). \end{array} \tag{8}$$

(In the previous case the four new identities hold automatically, by the coherence theorem of symmetric monoidal categories and by naturality of s.)

3. The symmetric site

After higher faces and degeneracies, we can also construct in $!\tilde{\Delta}$ the main transpositions s_i (the permutation which exchanges $i, i+1$, for $0 \leq i \leq n$)

$$s_i = s_i^n = i + s + (n-i)\colon\ n+2 \to n+2 \tag{9}$$

subject to the *Moore relations*:

$$s_i \cdot s_i = 1, \qquad s_i \cdot s_j \cdot s_i = s_j \cdot s_i \cdot s_j \ (i = j-1), \qquad s_i \cdot s_j = s_j \cdot s_i \ (i < j-1). \tag{10}$$

This is precisely the usual *Moore presentation* of the symmetric group S_{n+2}, the group of automorphisms of the set $n+2$: generators $s_i = (i, i+1)$, subject to the relations (10); see Coxeter-Moser [2], 6.2; or Johnson [6], Section 5, Thm. 3. (S_{n+2} also admits systems of two generators, e.g. the cyclic permutation $(0, 1, \dots n+1)$ and $s_0 = (0, 1)$; but then, the relations are complicated, cf. [2].)

Now, *faces, degeneracies and main transpositions form a system of generators for* $\tilde{\Delta}$: an arbitrary mapping $f\colon m \to n$ can be factorised as

$$f = h \cdot \rho, \qquad \begin{array}{l} h = f_0 + \dots + f_{n-1}, \\ (f_j = e^{(m_j)}\colon m_j \to 1, \quad m_j = \#(f^{-1}\{j\})) \end{array} \tag{11}$$

where $\rho\colon m \to m$ is a permutation and h is monotone; the latter is uniquely determined by f, as above, while ρ is not unique, generally: $h\rho = h\sigma$ iff $h\rho\sigma^{-1} = h$, iff $\rho\sigma^{-1}$ can be decomposed in a sum of permutations $\sigma_0 + \dots + \sigma_{n-1}$, coherently with the set-decomposition $m = m_0 + \dots + m_{n-1}$. (However, ρ is uniquely determined if we ask that ρ^{-1} be strictly monotone on each interval of the preceding decomposition of m; then, depending on conventions, ρ and ρ^{-1} are respectively called an $(m_0, \dots m_{n-1})$-*shuffle* and an $(m_0, \dots m_{n-1})$-*deal*, or vice versa.)

Our generators satisfy the *symmetric cosimplicial relations*, consisting:
- of the usual *cosimplicial relations* for faces and degeneracies (3),
- of the *Moore relations* for transpositions (10),
- of the following *mixed relations*

$$\begin{array}{lll} s_i\partial_j = \partial_j s_i, & s_i e_j = e_j s_i & (i < j-1), \\ s_i\partial_i = \partial_{i+1}, & s_i e_i = e_{i+1} s_i s_{i+1}, & \\ s_i\partial_j = \partial_j s_{i-1}, & s_i e_j = e_j s_{i+1} & (i > j), \end{array} \tag{12}$$

$$e_i s_i = e_i, \tag{13}$$

which again follow easily from the structural properties (8).

It follows easily that $s_i\partial_{i+1} = \partial_i$ and $s_i e_{i+1} = e_i s_{i+1} s_i$, so that the previous relations (12) can be viewed as rewriting rules for $s_i\partial_j\colon\ n+1 \to n+2$ and $s_i e_j : n+3 \to n+2$ ($i \leq n; j \leq n+1$), which allow one to transfer permutations to the right of monotone maps.

In general, given a strict monoidal category $(\mathbf{A}, +, 0)$ and an internal symmetric monoid $(a; \partial, e, s)$, the "multiples" $a + ... + a$ are linked by a system of maps (for $0 \leq i \leq n$)

$$\begin{aligned} \partial_i^n &= ia + \partial + (n-i)a\colon\ na \ \rightarrow\ (n+1)a, \\ e_i^n &= ia + e + (n-i)a\colon\ (n+2)a \ \rightarrow\ (n+1)a, \\ s_i^n &= ia + s + (n-i)a\colon\ (n+2)a \ \rightarrow\ (n+2)a, \end{aligned} \tag{14}$$

which satisfies the symmetric cosimplicial relations. As in Section 1, we write $e^{(n)} : na \rightarrow a$ the n-ary multiplication, inductively defined as

$$e^{(0)} = \partial, \qquad e^{(1)} = \mathrm{id}, \qquad e^{(n+1)} = e(e^{(n)} + 1) = e(1 + e^{(n)}); \tag{15}$$

one can easily deduce from (13), by induction, that $e^{(n)} \cdot s_i^{n-2} = e^{(n)}$.

4. Main results

4.1. Theorem. (The internal symmetric monoid) $!\tilde{\Delta}$ *can be characterised as:*
(a′) the free strict monoidal category with an assigned symmetric monoid, 1.

Proof. Let a strict monoidal category $(\mathbf{A}, +, 0)$ be given, together with an internal symmetric monoid $(a; \partial, e, s)$; we have to show that there is a unique strictly monoidal functor $F\colon\ !\tilde{\Delta} \rightarrow \mathbf{A}$ sending 1 to a and preserving the structure. We already know that the "multiples" $a + ... + a$ form a symmetric cosimplicial object (14). From Section 1, there is a unique strictly monoidal functor $F\colon\ \tilde{\Delta} \rightarrow \mathbf{A}$ sending 1 to a and preserving unit and multiplication; it operates in the obvious way on generators

$$F(n) = na, \qquad\qquad F(\partial_i^n) = \partial_i^n, \qquad\qquad F(e_i^n) = e_i^n. \tag{16}$$

Consider now the group S_{n+2} of automorphisms of $n+2$ in $!\tilde{\Delta}$; on the main transpositions $s_i^n = i + s + (n-i)\colon n+2 \rightarrow n+2$ we must have

$$F(s_i^n) = ia + s + (n-i)a = s_i^n\colon (n+2)a \rightarrow (n+2)a; \tag{17}$$

on the other hand, since this setting is consistent with the Moore relations (10), we have defined a sequence of group-homomorphisms $F\colon S_{n+2} \rightarrow \mathrm{Aut}((n+2)a)$, and extended the functor F to all bijections of $!\tilde{\Delta}$.

Take now an arbitrary mapping $f = h\rho\colon m \rightarrow n$, factorised as above (11): h is monotone and ρ is a permutation; we must set $Ff = Fh \cdot F\rho$; to show that the definition is correct, it is sufficient to verify that $Fh = Fh \cdot Fs_i$, for each main transposition s_i "acting within a summand" of $m_0 + ... + m_{n-1}$; taking for instance $0 \leq i < m_0 - 1$, we have (also by the identity $e^{(n)} \cdot s_i^{n-2} = e^{(n)}$, at the end of Section 3)

$$Fh \cdot Fs_i = (Ff_0 + ... + Ff_{n-1}) \cdot Fs_i = (Ff_0 \cdot Fs_i) + ... + Ff_{n-1} = Fh. \tag{18}$$

Last, we must prove that the extended mapping F preserves composition; let us begin showing that, for each monotone mapping $h: m \to n$ and each permutation $\sigma: n \to n$

$$\begin{array}{ccc} m & \overset{\tau}{\dashrightarrow} & m \\ {\scriptstyle h}\downarrow & & \downarrow{\scriptstyle k} \\ n & \xrightarrow{\sigma} & n \end{array} \tag{19}$$

one can find a permutation $\tau: m \to m$ and a monotone $k: m \to n$ such that the square above commutes, as well as its F-image in $\mathbf{A}$. One can assume that $\sigma = s_i$; let $h = h_u \cdot \ldots \cdot h_1$ be the canonical factorisation of a monotone map (4); applying the rewriting rules deriving from the mixed relations (12), we obtain a factorisation $\sigma h = k\tau$, with a monotone map $k = k_v \cdot \ldots \cdot k_1$ (canonical factorisation (4)) and a permutation $\tau = \tau_w \cdot \ldots \cdot \tau_1$ (product of main transpositions). The same relations hold in $\mathbf{A}$ (for its $\partial_i^n, e_i^n, s_i^n$) and F preserves the composition within monotone maps and within bijections, whence

$$F\sigma \cdot Fh = F\sigma \cdot Fh_u \cdot \ldots \cdot Fh_1 = Fk_v \cdot \ldots \cdot Fk_1 \cdot F\tau_w \cdot \ldots \cdot F\tau_1 = Fk \cdot F\tau. \tag{20}$$

Now, the functorial property for F follows easily: if the mappings $f = h\rho: m \to n$ and $f' = h'\sigma: n \to p$ are factorised as above, in (11), we rewrite $\sigma h = k \cdot \tau$ as in diagram (19), and

$$Ff' \cdot Ff = Fh' \cdot F\sigma \cdot Fh \cdot F\rho = Fh' \cdot Fk \cdot F\tau \cdot F\rho = F(h'k) \cdot F(\tau\rho) = F(f'f). \tag{21}$$

(For the last equality, note that $(h'k) \cdot (\tau\rho) = h'\sigma \cdot h\rho = f'f$ is an admissible factorisation of $f'f$, i.e. a permutation followed by a monotone map.) ∎

4.2. Theorem. (Presentation). *The category* $!\Delta^{\sim}$ *can also be characterised as:*

(b′) The subcategory of **Set** *generated by* faces *(*$\partial_i: n \to n+1$*),* degeneracies *(*$e_i : n+2 \to n+1$*) and* main transpositions *(*$s_i: n+2 \to n+2$*), where* $0 \le i \le n$*.*

(c′) The category generated by faces, degeneracies and main transpositions, under the symmetric cosimplicial relations *(Section 3).*

Proof. *(b′)* is already known and *(c′)* follows easily from the previous characterisation (Theorem 4.1). Let $!\Delta^{\sim}$ be defined by the presentation above. It is strictly monoidal: define the sum-functor in the obvious way ($\partial_i + q = \partial_i$, $p + \partial_i = \partial_{p+i}$, etc.) and check the consistency with relations. Then take a strict monoidal category $(\mathbf{A}, +, 0)$ with an internal symmetric monoid $(a; \partial, e, s)$; a strict monoidal functor $F: !\Delta^{\sim} \to \mathbf{A}$ sending 1 to a and preserving the structure is uniquely determined on generators, as in (16), (17): $F(n) = na$, $F(\partial_i^n) = ia + \partial + (n-i)a$, etc. Conversely, defining F in this way is obviously consistent with relations, since all of them can be deduced from the axioms (8) and the monoidal structure. ∎

5. Symmetric simplicial sets

Of course, the (non-augmented) symmetric simplicial site $!\Delta$ can be presented as the category generated by faces, degeneracies and main transpositions between positive cardinals, under the restricted relations.

Therefore, a symmetric simplicial set $X\colon !\Delta^{op} \to \mathbf{Set}$ can be assigned by the corresponding data $(X_n, \partial_i^n, e_i^n, s_i^n)$, where we write $X_n = X[n] = X(n+1)$, as usual; faces, degeneracies and main transpositions (for $0 \leq i \leq n$)

$$\partial_i^n\colon X_n \to X_{n-1}, \qquad e_i^n\colon X_n \to X_{n+1}, \qquad s_i^n\colon X_{n+1} \to X_{n+1} \tag{22}$$

are to satisfy the symmetric simplicial relations (dual to the ones considered in Section 3). Equivalently, one can assign a simplicial set $(X_n, \partial_i^n, e_i^n)$ and a right action of each symmetric group S_{n+1} on the component X_n ($x\rho = \rho^*(x)$), coherently with faces and degeneracies (i.e., the latter have to satisfy the dual *mixed relations* with the main transpositions, cf. (12), (13)).

The usual embedding of Δ in **Top** extends easily to $!\Delta$, forming a symmetric cosimplicial object with the same components, the standard topological simplices $!\Delta_n = \Delta_n$, and extended actions λ^* (for all mappings $\lambda\colon m \to n$)

$$(!\Delta_n, \lambda^*)\colon\ !\Delta \to \mathbf{Top}, \qquad \lambda^*((t_i)_{i=0,\dots n}) = (\sum_{\lambda i = j} t_i)_{j=0,\dots m}. \tag{23}$$

This model of $!\Delta$ in **Top** gives rise to the functor $!S_*$ of *symmetric singular simplices*

$$!S_*\colon \mathbf{Top} \to !\mathbf{Smp}, \qquad !S_n(X) = \mathbf{Top}(!\Delta_n, X) \tag{24}$$

where the transposition $s_i^n\colon !S_{n+1}(X) \to !S_{n+1}(X)$ amounts to a reflection of simplices, with respect to the symmetry hyperplane of the i-th, $(i+1)$-th vertices of Δ_n.

Its left adjoint is the (symmetric) geometric realisation functor $!\mathbf{Smp} \to \mathbf{Top}$: the realisation of the symmetric simplicial set X is the coend $\int^{[n]} X_n \cdot !\Delta_n$ (of the inner functor $!\Delta^{op} \times !\Delta \to \mathbf{Top}$). By Yoneda, the realisation of $!\Delta[n]$ is $!\Delta_n = \Delta_n$.

6. Symmetric comonads

A comonad (K, ∂, e) in the category $\mathbf{A}$ is a comonoid in the category End($\mathbf{A}$) of endomorphisms of $\mathbf{A}$, with the strict monoidal structure of composition

$$\begin{array}{ll} \partial\colon K \to 1, & e\colon K \to K^2, \\ \partial K \cdot e = \mathrm{id}K = K\partial \cdot e & eK \cdot e = Ke \cdot e; \end{array} \tag{25}$$

it generates an augmented simplicial object in End($\mathbf{A}$), and - by evaluation - a functor $K_*\colon \mathbf{A} \to Sm\tilde{p}(\mathbf{A})$ with values in the category of augmented simplicial objects on $\mathbf{A}$ (cf. [1])

$$\begin{array}{ll} K_*(X) = ((K_{n+1}(X)), (\partial_i^n), (e_i^n)) & (n \geq -1;\ 0 \leq i \leq n), \\ \partial_i^n = K^{n-i}\partial K^i\colon K^{n+1} \to K^n, & e_i^n = K^{n-i}eK^i\colon K^{n+1} \to K^{n+2}. \end{array} \tag{26}$$

For a category $\mathbf{C}$, the category of augmented simplicial objects $\tilde{Smp}(\mathbf{C})$ has a well known comonad, given by the shift (or decalage) $\mathbf{K}$

$$\begin{array}{rl} \mathbf{K}: \tilde{Smp}(\mathbf{C}) \to \tilde{Smp}(\mathbf{C}), & \mathbf{K}X = ((X_{n+1}), (\partial^{n+1}_{i+1}), (e^{n+1}_{i+1}))_{n \geq -1}, \\ \partial: \mathbf{K}X \to X, & \partial = \partial^{n+1}_0: X_{n+1} \to X_n, \\ e: \mathbf{K}X \to \mathbf{K}^2X, & e = e^{n+1}_0: X_{n+1} \to X_{n+2} \end{array} \tag{27}$$

with counit ∂ and comultiplication e consisting of the discarded faces and degeneracies.

In fact, $\tilde{Smp}(\mathbf{C})$ is the cofree category-with-comonad on $\mathbf{C}$, with respect to the forgetful functor $| - |$ from categories with a comonad to categories. The counit-component $|\tilde{Smp}(\mathbf{C})| \to \mathbf{C}$ sends the augmented simplicial object X to X_{-1}, while the unit-component $\mathbf{A} \to \tilde{Smp}(|\mathbf{A}|)$ is the functor K_* considered above.

Similarly, a *symmetric comonad* (K, ∂, e, s) will be a symmetric comonoid in End($\mathbf{A}$): with respect to the previous structure, in (25), we have to add a *symmetry* $s: K^2 \to K^2$, satisfying

$$\begin{array}{lll} ss = 1, & \multicolumn{2}{l}{sK \cdot Ks \cdot sK = Ks \cdot sK \cdot Ks,} \\ \partial K.s = K\partial, & se = e, & eK \cdot s = Ks \cdot sK \cdot Ke. \end{array} \tag{28}$$

By the characterisation theorems of Section 4, it generates an augmented symmetric simplicial object in End($\mathbf{A}$) and a functor $K_*: \mathbf{A} \to !\tilde{Smp}(\mathbf{A})$. Again, $!\tilde{Smp}(\mathbf{C})$ is the cofree category with symmetric comonad over $\mathbf{C}$.

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